Risk calculation of interest rate swaps for Cinnober Financial Technology AB

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Abstract

In this thesis, we put forward a possible solution for Cinnober Financial Technology AB, to be able to calculate the risk of a portfolio of interest rate swaps in real time using risk modeling. The chosen instrument is plain vanilla swap and the time horizon is one day. The main assumptions have been that the risk of having a portfolio of plain vanilla interest rate swaps as an intermediary between various parts can be modeled based on historical data. The main approach in modeling has been a straight forward calculation based on Principal Component Analysis (PCA). Thereby daily Value at Risk (VaR) at 95% level of confidence has been calculated using Monte Carlo-simulated future data.

The PCA approach has been put into stress testing, where the main reason has been to study the robustness of the approach. The argue is that stress testing combined with VaR gives a more comprehensive picture of risk. While VaR gives a picture of the risks in an everyday market environment, stress testing instead gives a picture of the risk in an abnormal market.

When it comes to real-time risk management, the calculations should be optimized in order to reduce the latency. Two methods have been suggested in this paper that need further preparation and implementation, Parallel Vector Computing and Incremental Computing. These methods make it possible to calculate the risk sufficiently fast. Thereby real-time risk management is not longer a dream but a reality that should be required from any participant in the finance sector.

Acknowledgements

We would like to express our thanks to the people that have made this paper possible. First we thank our great supervisors Filip Lindskog at KTH and Magnus Sandström at Cinnober. They have always shown an interest in our work and have been a strong force in making us go forward. Some names that we would also like to mention are Bengt Jansson for his expertise, Jicun Zong for his great interest in risk calculation, Gunnar Mjöberg for his lingual advices and the management at Cinnober Financial Technology AB for letting us writing the thesis at their premises.

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Chapter 1

Introduction

The credit crisis is demonstrating that financial institutions have had insufficient levels of controls for managing risk. Hence there is a growing demand for faster and more effective risk management and more transparency on the financial markets. The faster risk management is closely related to the word "Real-Time" risk management, implying the calculation of the risk *on trade* or shortly after the trade has been made. It is not an easy task and this puts pressure on skilled and flexible technical solutions to support the business development and maintaining the competitiveness.
The competition has also increased the demand on trading platforms. With new ways of trading such as algorithmic trading, that is, entering trading orders with computer algorithms deciding on aspects of the order, there has been a high pressure on high performance and clearing technology.
The pressure is also on governments, who are faced with one of the toughest economic challenges in history to stabilize the global economy. They are in a position where they have to support the financial institutions in their work for effectiveness, with liquidity in order to ensure a restart of the credit flow. However, the future market looks bright for risk management and clearing with various business development opportunities for organisations with systems capable of fast and efficient risk management as well as the flexibility to clear a wide range of instruments.

1.1 Cinnober Financial Technology AB

Cinnober Financial Technology AB (Cinnober) is a Swedish-based company founded in 1998 by four former employees of OMX and SEB. They had a vision to develop a fast and flexible platform for financial transactions using the Internet and Java.
Today Cinnober has over 170 employees and is operating all around the world, providing exchanges, banks and other financial institutions with its platform TRADExpress. All new solutions have a layered design with TRADExpress as the base, the business logic on the top of the platform layer and with the final step of customization for the customer to tailor the system to its own business model.

1.1.1 TRADExpress RealTime Clearing

Due to the high demands for risk management, Cinnober sees the growing market for real-time risk management. In doing so, Cinnober has launched a real-time clearing system, called TRADExpress RealTime Clearing. The system targets established and emerging clearing organizations and exchanges.
New risk measures are calculated and published for each new trade that is matched. It is built to be flexible, with a so called "plug-in" architecture, in order for new instruments to be configured, new risk models to be implemented and new asset classes to be added to the system without code changes.[1]

Chapter 2

Problem statement

The aim of this thesis is to find a way of implementing a new instrument, an Interest Rate Swap called "Plain Vanilla Swaps", into the TRADExpress RealTime Clearing.
In order to implement a new instrument for the clearing system, the risk calculation in real time has to be possible. Being of interest overall, is the risk exposure of the clearing house over a short time interval, by holding portfolios of interest rate swap contracts. Cinnober requests a Value at Risk (VaR) calculation, with a given confidence level, in real time for a portfolio of Plain Vanilla Swaps.
There has not been much written on this topic, neither has there been any intuitive method for calculating the risk of interest rate swaps. Therefore another aim of this thesis is to create an intuitive method for such a calculation.

Chapter 3

Theory

To understand the pricing of swaps, it is important to be familiar with zero coupons and yield curves. In this chapter there is a short briefing of how the swaps market looks like and a closer look at different types of swaps. Furthermore various risk measures are introduced as well as important properties. The chapter finally ends with the risk measure of interest, called Value at Risk. VaR is a risk measure that was developed to respond to the demand for aggregating various sources of market risk, that is, risk coming from fluctuations in the market, into a single quantitative measure. A short introduction is also given to risk measure that is used as a complement to the Value at Risk measure, called Expected Shortfall or Conditional Value at Risk.

3.1 Zero Coupon Bond

A bond is a debt security and is a formal contract to repay borrowed money with interest (coupons) at fixed intervals and/or repay the principal at a later date, usually at expiration date called maturity. A *zero coupon bond* is a bond that has no periodic interest payments, i.e., no coupon payments and is therefore called "zero" coupon bond. It is basically a bond where the buyer pays Z_0 today (i.e. time $t = 0$) and receives the amount 1 at the time of maturity. There is nothing random about a zero coupon bond, both the price Z_0 and the amount 1 received day T are known today. Although the randomness takes part when looking at buying/selling a zero coupon bond in the future $(T - \Delta t)$ with maturity at time T, where $0 < \Delta t < T$. Examples of zero coupon bonds include U.S. Treasury bills, U.S. savings bonds or any other type of coupon bond that does not make any periodic interest payments. [6]
Zero coupon bonds are usually used to analyze the yield curve (described in next section), that is, the relation between the interest rate (cost of borrowing) and the time to maturity (expiration date of the contract). This is because any fixed income security (an investment that provides a return in the form of fixed periodic payments and the return of principal amount at the time of maturity), can be broken down into individual cash flows and viewed as a portfolio of zero coupon bonds. The fixed income security can then be priced by each individual cash flow discounted by the appropriate rate implied by zero coupon bonds.

3.2 Yield Curve

As implied earlier, the yield curve is the relationship between interest rate and time to maturity of the debt and is commonly an increasing function of time. The short-time interest rate is decided by the government securities and the long time rates are derived from the inflation expectations and can as well be interpolated. Depending on the maturity preferences of the investor, the yield curve is a snapshot in time of the yields that are available. The most important curves are those derived from the government bond markets, that represent bonds paying regular coupons. [6]
However from these coupon paying bonds, it is possible to derive the yields (the curve) that would be required on a bond that does not pay coupons, so called *zero coupon bond.* From this zero coupon curve and by calculating forward rates of interest, hence constructing forward yield curves, it is possible to get an insight into market expectations about future interest rates.

3.3 Derivatives

A derivative is a financial instrument with a value that is based on, or derived from, a less complex underlying variable. The underlying could be a security, an asset or a market index. It is plainly a contract between two or more parties with a value determined by fluctuations and future performance of the underlying asset. A mathematical explanation of a derivative is a *function* of the price for an underlying variable, for example a stock price [2].
The most popular and common types of derivatives are *Futures Contracts, Forward Contracts, Options* and *Swaps.*
A futures contract obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. The contract is settled daily.
A forward contract is a contract that obligates the holder to buy or sell an asset for a predetermined delivery price at a predetermined future time. An option gives the buyer of the option the right, but not the obligation, to buy or to sell a specified asset (underlying) on or before the option's maturity, at an agreed price.
And a swap is an agreement to exchange cash flows in the future according to a prearranged procedure. All these types of derivatives can be customized according to a certain agreement. Since derivatives are contracts, almost anything can be used as a derivative's underlying asset. It is generally used to hedge risk, that is, constructing a counter trade designed to reduce the total risk [6].

3.3.1 Players on the derivatives market

There are different types of players in the market. The so called *Hedgers* are mainly interested in the underlying instrument, for example a baking company might buy wheat futures to help estimate the cost of producing its bread in the months to come.
The *Speculators* are concerned with the profit to be made in buying or selling the contract at the most opportune time. Listed derivatives are traded on organized exchanges or

markets. Other derivatives are traded over-the-counter (OTC), i.e., off-exchange trading directly between two parties, and in private transactions.

3.3.2 The derivative considered in this paper

In this paper, a common construction of the last derivative mentioned, the Swap, will be analyzed. The construction is a swap of interest rates between two parties and called *Interest Rate Swap.* The parties agree on exchanging a fixed rate of interest on a certain *Notional Principal* , which is a predetermined amount in a given currency, for a floating rate of interest on the same notional principal. [6]

There are many reasons for constructing such a contract and a few of them are:

- The parties are expecting an increase or a decrease in interest rates
- One party wants to match the interest flows between assets and loans
- Reallocate interest structure in the balance sheet

3.4 Swaps

Although the swap already has been introduced, there is need to give a brief background of it and what type of different swaps that are used in the market.
The first swap contracts were negotiated in the early 1980s between IBM and the World Bank. However, despite their relative youth, swaps have exploded in popularity. In 1987, the International Swaps and Derivatives Association (ISDA) reported that the swaps market in the US had a total notional value of $865.6 billion, and by 2009 this figure exceeded $341.86 trillion. [5]
Swaps are, unlike most standardized options and futures, customized contracts OTC-traded between private parties. Hence it is not an exchange traded instrument. [3]

In the early days, the swaps took place between interested parties, where the banks acted as brokers and arrangers, and charged the counterparties with an arrangement fee. As the market started to grow, the banks started to act as dealers and hedging their swap deals. Swaps now occupy a position of central importance in the OTC derivatives market and this is mainly due to their great flexibility. [6]

Because the swaps occur on the OTC market and that there is a large risk of a counterparty defaulting on the swap, the market is dominated by big firms and financial institutions. As described previously, a swap is an agreement between two parties to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way they are to be calculated. The calculation of the cash flow involves the future value of an interest rate, an exchange rate or other market variables. Most swaps are constructed as very simple products. The most common and simple swap, called *Plain Vanilla interest rate swap*, is a contract between two parties, called *counterparties*, with an agreement to exchange, on designated future dates, two streams of interest rate payments. A party agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal, for example $10 million, to the other counterparty. In return it receives interest at a floating rate on the same notional principal for the same period of time. The floating rate is usually linked to the London Interbank Offered Rate (LIBOR), which is the rate offered by banks on Eurocurrency deposits (that is, the rate at which a bank is willing to lend to other banks). [6]

3.4.1 Market and size

The 2009 semi-annual OTC derivatives statistics report, published by *Bank for International Settlements* (BIS), consists of information about the size and the structure of different derivatives market in the G10 countries and Switzerland. According to this report, interest rate swaps are the second largest single-currency derivatives by instrument, counterparty and currency. The notional amount outstanding and the gross market value at the end June 2009 was $341.89 trillion respectively $13.93 trillion [5].

By definition, the notional amount outstanding is a measure of market size and a reference from which contractual payments are determined in derivatives markets and not a measure of the true amount at risk. The later is a function of the price level and the volatility of the underlying instrument, the duration and liquidity of the contract and the creditworthiness of the two counterparties. Furthermore gross market value is the sum of the absolute values of all open contracts and provides a measure of the scale of financial risk in derivatives transactions [4].

3.4.2 Different types of Swaps

As swaps are OTC instruments and in general refer to a contract where counterparties agree to change their cash flows, there is no limit for how many different contracts that can be invented. This study will focus on interest rate swaps being the most common type of swaps. There are many different types of interest rate swaps. Counterparties may exchange a fixed rate for a floating rate, floating rate for a floating rate or fixed rate for a fixed rate. They may also stand in same currency or different currencies. For example when the counterparties exchange fixed for fixed they should stand in different currencies as otherwise the swap contract will be meaningless. [6]

The main different types of interest rate swaps are:

- Fixed for floating rate swap, same currency
- Fixed for floating rate swap, different currencies
- Floating for floating rate swap, same currency
- Floating for floating rate swap, different currencies

- Fixed for fixed rate swap, different currencies

3.4.3 The type of swap considered in this paper

As mentioned earlier the most common Interest Rate Swap, the Plain Vanilla interest rate swap, will be considered. The most common floating rate used in a Plain Vanilla is LIBOR, *London Interbank Offer Rate*, a daily reference rate based on the interest rates set by banks borrowing unsecured funds from other banks in the London money market. [6]
Whereas the fixed payment is computed on the notional principal at the swaps' contract rate. The agreement is settled due to the fact that each party thinks it will have a good chance to gain something by exchanging cash flows.

3.5 Risk measures

The basic questions to be answered are, *"What is risk?"* and *"How can it be measured?"*
The answer to the first question concerns the results of future events and is related to the negative consequences, where the positive consequences are more referred to as *chance*.
To be able to answer the second question, the time interval of interest payments has to be decided when analyzing the portfolio and then decide from what perspective the risk is to be measured.
Consider two time points, time 0 being now and a future time point 1 (a measure in units of Δt) and three different perspectives a player can have.

First we have the ***investor***, who wants to use the initial capital V_0 to form a portfolio of financial instruments. The investor has a criterion of what future net worth is acceptable at a certain time and seeks the optimal portfolio among those with acceptable values.
The ***risk controller*** decides whether the risks taken within a position are acceptable, and if not the positions have to be modified in order to be acceptable.
The last perspective, is of the ***regulator***, who wants to form rules to prevent banks from taking too much risk and thereby threatening the financial stability and on the other hand allows companies to be profitable. The regulator specifies the minimum buffer capital requirement that the company must set aside in order to be allowed to continue with its business.
To determine the amount of cash kept in reserve in order to make the risks acceptable a number has to be put to the risks and for that reason there are various types of risk measures used in the financial business. Risk measures are basically defined as a mapping from a set of random variables to the real numbers ($+\infty$). The random variables represent the risk at hand, that is, the amount of buffer capital requirement, and the used notation for a risk measure associated with any random variable X is $\rho(X)$, where X in this case represents the net worth at time T_1 [9].

Well known examples of risk measures are *value at risk* (VaR) and *expected shortfall* (ES), where both measures will be explained in 3.5.2 and 3.5.3. In recent years the attention has turned towards coherent risk measurement, which is a risk measure that satisfies four important properties to be discussed here next.

3.5.1 Properties

We have already introduced the risk measure ρ, which is associated with a certain random variable, in this case the final (at time 1) net worth, X:

$$X = V_1 - V_0(1 + r_f) \tag{3.1}$$

is the difference between the value of the portfolio at time 1 and the value of the portfolio today, calculated with the money at time 1, where r_f is the relative return on the risk free zero coupon bond with maturity at time 1.
The function ρ is then chosen so that $\rho(X) \leq 0$, meaning that X is an acceptable final net worth and that $\rho(X)$ is the minimum *number of units* of this currency at time 0 that needs to be added to the position and buying risk free zero coupon bonds until time 1 in order to make the position acceptable.
This is called a risk measure and it determines the economic capital or buffer capital that for example a broker holding a portfolio of collateral, sets aside. To be a useful tool when managing risk, such a measure should satisfy some natural properties. Further four different properties will be evaluated which are some of those properties that should be covered when choosing a good measure. The four properties are called *translation invariance*, *monotonicity*, *positive homogeneity* and *subadditivity*.

Translation invariance (T): Can be written as

$$\rho(X + \alpha(1 + r_f)) = \rho(X) - \alpha \ ; \quad \forall \alpha \in \mathbf{R} \tag{3.2}$$

which states, by adding a certain amount of cash, α, of a risk free asset (zero coupon bond) to a risky position, the risk decreases by the very same amount. Hence, if having an unacceptable position, it can become acceptable by adding the amount equal to the buffer capital to the position in order to make the position acceptable:

$$\rho(X + \rho(X)(1 + r_f)) = \rho(X) - \rho(X) = 0. \tag{3.3}$$

Monotonicity (M): Can be written as

$$If \ X_2 \leq X_1 \quad \Rightarrow \quad \rho(X_1) \leq \rho(X_2) \tag{3.4}$$

which states that if one of two positions has a greater future net worth for sure, then that position is less risky. Hence choosing a less risky position should result in less buffer capital needed to be set aside.

Positive homogeneity (PH): Can be written as:

$$\rho(\lambda X) = \lambda \rho(X) \tag{3.5}$$

for all $\lambda \geq 0$. This says that the risk increases at least linearly in the size of the position, that is, if a position is doubled, then the buffer capital should double as well. This property holds in markets that that are perfectly liquid.

Sub additivity (S): Can be written as:

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \tag{3.6}$$

which says that diversification should be rewarded and that the sum of two stand alone risks for two assets is always greater or equal to the risk of the sum of the two assets. This means that the joint risk is less since the both positions will not experience their worst losses at the same time period if they are not perfectly correlated. It also means that two units of an entity should be required to put aside less buffer capital than the sum of the buffer capital for the two units considered as separate entities. Hence, it does not encourage companies to break up into parts in order to reduce the amount of buffer capital.

A risk measure ρ that satisfies the properties **T**, **M**, **PH**, **S** is a coherent risk measure and thus a useful tool for managing risk.

3.5.2 Value at Risk

An attempt to provide a single number summarizing the total risk in a portfolio of financial assets is the Value at Risk measure and it is widely used within financial risk management due to the easy understanding. The VaR of the final net worth X at time 1 at the confidence level $1 - \beta$, where the significance level $\beta \in (0, 1)$, is:

$$\mathrm{VaR}_\beta(X) = \min\,\{m : \Pr(m(1 + r_f) + X < 0) \leq \alpha\}. \tag{3.7}$$

As shown in the Formula (3.7), the buffer capital that should be set aside (VaR) is the smallest amount of money (m) that if added to the position now and invested in a risk free asset, ensures that the probability of a strictly negative final net worth at time 1 is not greater than β. This means that with the probability β, losses will exceed the buffer capital, which results in β usually getting very close to zero since only very unlikely events should cause losses being greater than the buffer capital set aside. [6]

The portfolio manager: Suppose a portfolio manager holds a portfolio with a VaR equal to \$100 (expressed as an absolute number amount) at a 95% confidence level. This means that in five days out of 100 trading days there will be a loss more than \$100.

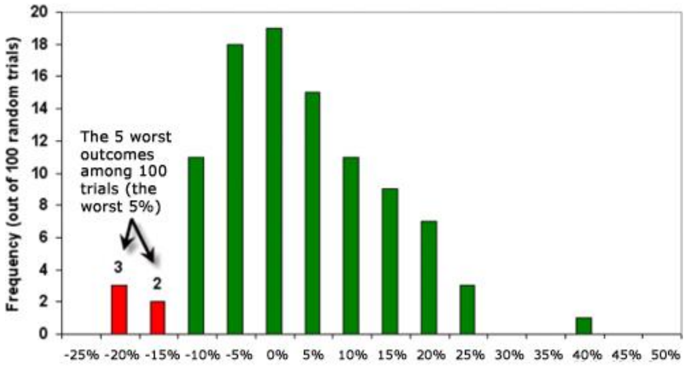

Figure 3.1: Empirical Value at Risk

There are some drawbacks with this risk measure. One of them is that VaR fulfills all properties of a coherent risk measure except for the subadditivity property, thus not rewarding diversification. However for some distributions such as the normal distribution VaR fulfills subadditivity property and in that case is a coherent risk measure.
Another drawback is that VaR does not give any information about how the losses greater than the buffer capital are distributed. It cannot specify how much the potential losses will be. One interesting graphical illustration is how the tails of the distribution look like. If the tails decay faster than an exponential distribution, it is called *light tailed* and if the tails are not exponentially bounded, it is called *heavy tailed* .

3.5.3 Expected Shortfall

Although VaR has become a standard risk measure for financial risk management due to its conceptual simplicity, it has several limitations. For example it does not show how bad the losses may be when things go wrong, in other words it disregards any loss beyond the VaR level (usually referred to as the "tail risk"). It is also not coherent, since it is not subadditive. *Expected Shortfall* (ES), sometimes called *Conditional Value at Risk* (CVaR), is the conditional expectation of loss given that the loss is beyond VaR level and for X (as defined earlier) Expected Shortfall at level α is the average value at risk for levels $p \leq \alpha$, i.e.,

$$ES_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha \mathrm{VaR}_p(X) dp$$

By definition, this is the conditional expectation of loss given that the loss ($L = -X/(1 + r_f)$) is beyond the VaR level.

$$ES_\alpha(X) = E[L | L \geq \mathrm{VaR}_\alpha(X)]$$

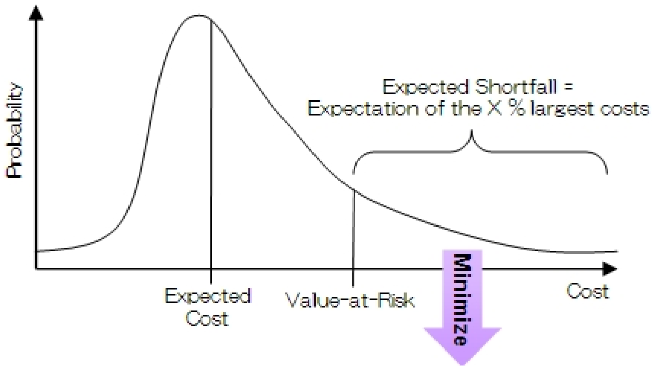

Figure 3.2: Expected Shortfall (Conditional Value at Risk)

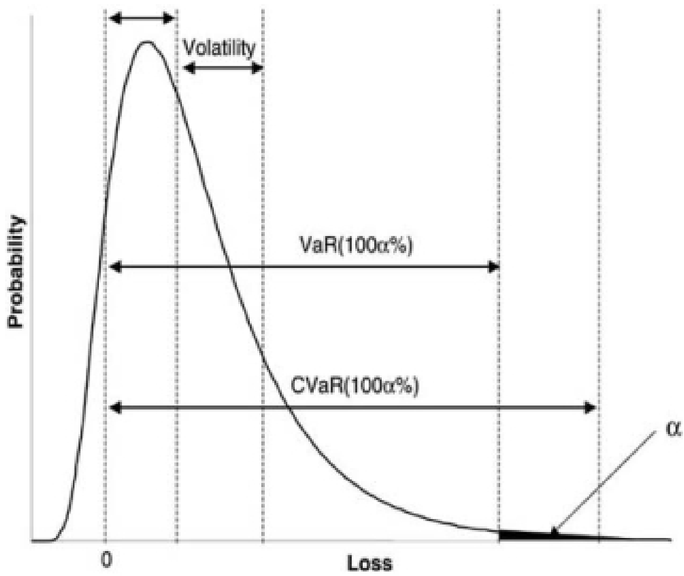

Figure 3.3: Value at Risk and Expected Shortfall

Chapter 4

Plain Vanilla Interest Rate Swaps

A plain vanilla interest rate swap is an agreement between two parties to swap periodic interest rate payments on predetermined settlement dates. In this chapter the cash flow stream as well as how to price the swap will be discussed. Furthermore for pricing the swap, it will be necessary to describe the relationship between swaps and bonds.

4.1 Cash Flow

As we already introduced the "Plain Vanilla" interest rate swap, we want to look closer at how the cash flow works. We have two parties in an agreement of exchanging cash flows. In the Plain Vanilla case the two cash flows are paid in the same currency. Party A agrees to pay party B a predetermined, fixed rate of interest on a notional principal (e.g. $10 million) on specific dates for a specific period of time. Concurrently, party B agrees to make payments based on a floating interest rate to party A on the same notional principal on the same specific dates for the same specified time period. The specified dates are called settlement dates, and the periods between are called settlement periods. Hence in the beginning of the settlement period, the floating rate is usually determined for the first payment.

As mentioned earlier, the floating rate is based on a marked index LIBOR. On the other hand, the fixed rate of interest is normally expressed as a spread over, for example the U.S. Treasury bonds of a similar maturity. When the floating and the fixed payments are due on the same date (usually between 1 and 15 years), being generally the case, only the netted amount changes hands. Hence the principal does not at any point change hands, which is why it is referred to as a *notional amount*. Furthermore bilateral netting of interest rate payments is a fundamental characteristic of swaps, and is clearly designed to mitigate credit risk (that is, the risk that one part defaults). [12]

4.2 Swap pricing in theory

The difficult part of the agreement is the pricing of the swap. The interest rate swaps are typically set so that the *present value* of the counterparty payments is equal to the present value of the payments to be received.

The present value is a way of comparing the value of cash flow now with the value of cash flow in the future. The rule of thumb is that "One dollar today is worth more than a dollar tomorrow".

Since an interest rate swap is just a series of cash flows occurring at known future dates, it can be valued by simply summing the present value of each of these cash flows. When calculating the present value, it is necessary to first estimate the correct *discount factor* (df) for each *period* (t) where a cash flow occurs. Discount factors are derived from investors' perceptions of interest rates in the future and are calculated using for example, LIBOR. The interesting part is to find out the fixed component, that is, the *swap rate*, in the contract to fulfill a possible agreement. As mentioned earlier, the theory is based on the present value (PV) of the payment being equal to the present value of the payment to be received by the counterparty. [12]

$$\mathrm{PV(fixed\ rate)} = \mathrm{PV(floating\,rate)} \tag{4.1}$$

and

$$\mathrm{PV(fixed\ rate)} = \mathrm{(Swap\,Rate)(notional\,principal)} \sum_{t=1}^{N} \frac{\mathrm{days}_t}{360}\, df_t \tag{4.2}$$

by using (4.1) and (4.2) the following is derived:

$$\text{Swap Rate} = \frac{\text{PV(floating rate)}}{\text{(notional principal)} \sum_{t=1}^{N} \frac{\mathrm{days}_t}{360} df_t} \tag{4.3}$$

Market participants are also interested in the *swap spread*. It is used as a benchmarking tool, where the known swap rate today is compared to a U.S Treasury Security of comparable maturity. It is basically the difference between the theoretical swap rate calculated as in formula (4.3) and the rate offered through other comparable investment instruments with comparable characteristics (that is, similar maturity).

Ex: Simple contract between two companies
First we assume that we have two companies, A and B in the market that are interested in a swap contract. Suppose that company A has earlier borrowed \$100 million at LIBOR plus 10 basis points (bp). One basis point is one hundredth of 1%, so the rate is LIBOR plus 0.10%. In the swap, company A agrees to pay company B a fixed interest of 5% and receive LIBOR. Company A has now a cash flow that nets out to an interest rate payment of 5.1% instead of LIBOR plus 10bp. From earlier, company B has a fixed payment to an outside lender of 5.2% on an amount of \$100 million and wants to use a swap to transform its liability. In the exchange with company A, it receives fixed 5% and pays LIBOR, and company B has now a cash flow that nets out to an interest rate payment of LIBOR plus 20bp instead of the fixed rate of 5,2%, see figure (simple contract).

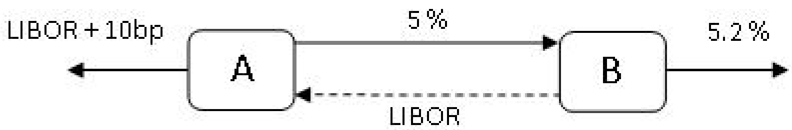

Figure 4.1: Simple contract

Ex: Role of the Clearing House
Usually two companies, as in the previous example A and B, do not directly arrange a swap, mainly because of the credit risk. Therefore they each deal with an intermediary financial institution usually a *Clearing House* (CH) or a so called a *Central Counterparty* (CCP). The "*Plain Vanilla*" fixed-for-floating is usually arranged such that the CCP earns between 3bp to 4bp (0.03% or 0.04%) per year on a contract between two parties. If we still assume the notional amount is \$100 million, the CCP earns between \$30,000 and \$40,000 per year on the agreement. B ends up borrowing 5.115% instead of 5.1% and A ends up borrowing a LIBOR plus 21.5bp instead of LIBOR plus 20bp.

Note that the financial institution has two separate contracts with each company. If for instance company B defaults, the CCP has to find another counterparty as soon as possible. There the main risk for the CCP is the loss of interest rate change during the default until they find a new counterparty. Note also that if both companies default in this case, the CCP will not receive future earnings of 30bp. Looking at the perspective of the CCP, it always wants to minimize its market risk (i.e. changes in the market such as prices and interest rates). CCP has to act as an intermediary and leave the risk to the counterparties A and B.

Due to the liquid market for interest rate swaps, which means that there always is a demand for a contract on the market if a counterparty defaults, the CCP will be able to find another party taking over the credit risk easily. This means that the CCP always has to hedge each contract it sets up with each counterparty. The loss will be the value change in the price of the contract during a certain time, where we will consider the time to be over one day. [6]

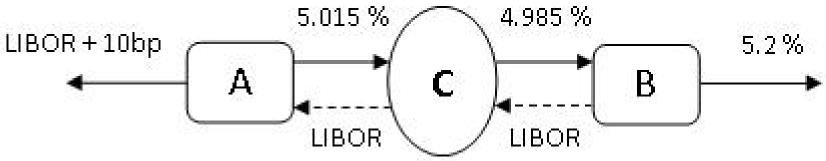

Figure 4.2: Involvement of an intermediary financial institution

Cash flow transformation for A:
A_{before}: LIBOR + 0.1%
A_{after}: LIBOR + 0.1% + 5.015% - LIBOR = **5.115%**

Cash flow transformation for B:
B_{before}: 5.2%
B_{after}: 5.2% + LIBOR - 4.985% = **LIBOR** + **0.215%**

4.3 Relationship between swaps and bonds

As mentioned earlier there is a fixed part and a floating part in an interest rate swap. Since the fixed interest rate payments are known in terms of actual amounts, they can be seen as the sum of zero-coupon bonds. Suppose the fixed rate of interest is r_{fixed} and the zero coupon price day t that matures at time T is $Z_{t,T}$ where (t,T)>0 and where there are N payments, we get the present value of the fixed leg to be:

$$\text{PV(fixed)} = r_{\text{fixed}} \sum_{i=1}^{N} Z_{t,T_i}$$

Note that this is usually multiplied by the notional principal, but here we have scaled it to one.
The floating part, shown in Figure 4.3, is a bit more complicated. Suppose we have the LIBOR that is paid at a certain time in the future, T. Since the LIBOR rate, r_{LIBOR}, is the interest rate paid on a fixed-term deposit, we can say that $\$1 + r_{\text{LIBOR}}$ at T is equal to \$1 at time $T - \Delta t$. This means that a single floating rate at time T is equal to a withdrawal of \$1 at the same time and a deposit of \$1 at time $T - \Delta t$, where Δt is the period of exchange.

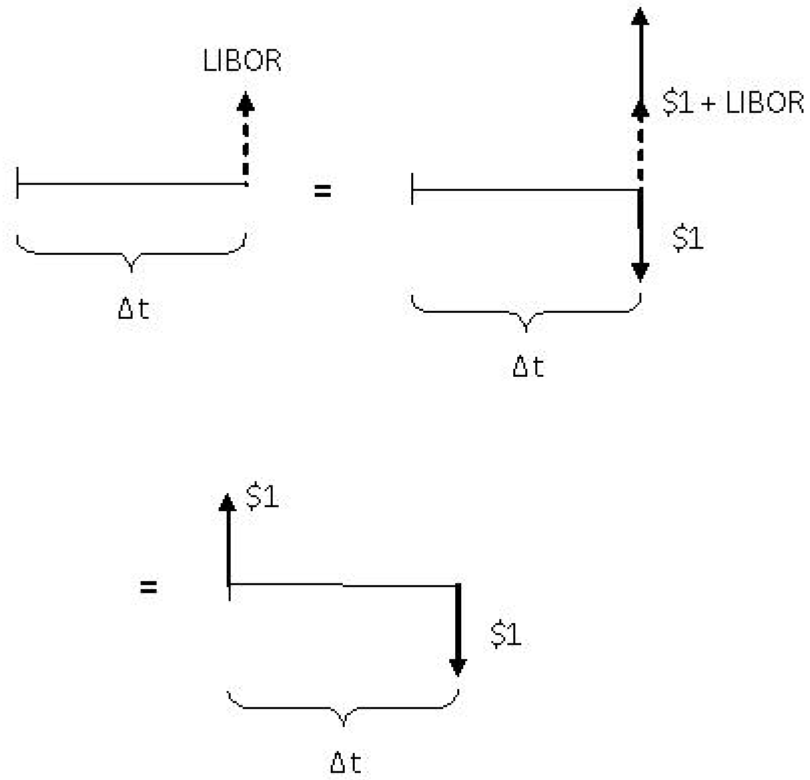

Figure 4.3: Schematic diagram of a single floating leg

If adding up all the floating rates in the future, as shown in Figure 4.4, there will be a cancellation of all \$1 cash flows except for the last and the first exchange. Hence we will have \$1 today and \$1 at time T,

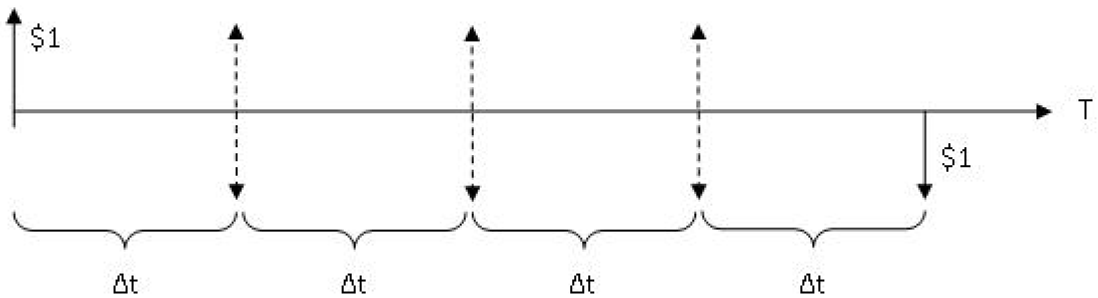

Figure 4.4: Schematic diagram of the floating leg

and the present value of the floating leg will be $\$1 - Z_{t,T}$.

$$\text{PV(floating)} = 1 - Z_{t,T_N}$$

Hence the value of the contract for the fixed receiver is the difference between PV(fixed) and PV(floating):

$$\text{Value of contract} = \mathrm{r}_{\text{fixed}} \Delta t \sum_{i=1}^{N} Z_{t,T_i} - 1 + Z_{t,T_N} \tag{4.4}$$

The fair price of the swap exists when the present value of the fixed and the floating rates are equal, to avoid arbitrage, therefore we set them equal in order to get the fair price, which is the swap rate (fixed rate of interest).

$$1 - Z_{t,T_N} = \mathrm{r}_{\text{fixed}} \Delta t \sum_{i=1}^{N} Z_{t,T_i}$$

Hence, the swap rate is then calculated to be

$$\mathrm{r}_{\text{fixed}} = \frac{1 - Z_{t,T_N}}{\Delta t \sum_{i=1}^{N} Z_{t,T_i}}$$

An interesting part of swaps is that their fair prices ($\mathrm{r}_{\text{fixed}}$) determine the yield curve and not vice versa. In practice, given the $\mathrm{r}_{\text{fixed}}$ at time T, Equations (4.5) and (4.6) are used to calculate the prices of zero coupon bonds and thus the yield curve. For the first discount factor we solve

$$\mathrm{r}_{\text{fixed}}(T_1) = \frac{1 - Z_{t,T_1}}{\Delta t \sum_{i=1}^{N} Z_{t,T_1}}$$

and finally

$$Z_{t,T_1} = \frac{1}{1 + \mathrm{r}_{\text{fixed}}(T_1)\Delta t} \tag{4.5}$$

After finding the first discount rate we use the same technique for the $(j+1)$th factor.

$$Z_{t;T_{j+1}} = \frac{1 - \mathrm{r}_{\text{fixed}}(T_{j+1})\Delta t \sum_{i=j}^{N} Z_{t,T_i}}{1 + \mathrm{r}_{\text{fixed}}(T_{j+1})\Delta t} \tag{4.6}$$

This can be done and put into a graph to obtain the zero coupon yield curve. [21]

Chapter 5

Current situation in the academic world

Interest rate swaps have been in the market for three decades and therefore they are considered as new instruments. The industry has been growing enormously since the start and therefore a demand for new solutions that can be backed up with academic studies has been huge. There are several companies that specialize in IRSs and provide post-trade and pre-trade risk solutions for traders. In the academic world, IRSs have been studied for 30 years and different approaches for calculating the risk associated with trading IRSs has been purposed. Interest rate derivatives are still a topic on which many academics and people from industry find difficult to agree on. In this section we will review the academic papers related to the calculations of IRSs.

5.1 Recent studies

Of all the papers studied, we chose three to be presented in this section. The first paper is from 1988, the second from 1998 and the third from 2003. The timeline is chosen on purpose to show a trend in the academic world. This section will give a clear picture of how researchers have started their journey of modeling IRSs and where the trend has been moving during past thirty years.

The revolutionary research of Litterman and Scheinkman [20] in 1988 is the main inspiration source of choosing Principal Component Analysis as a factor analysis method used in this study. They studied the common factor that influences on US government bond returns. They studied the duration concept that is used to describe the parallel shift movements of the yield curve. They compared the result from the duration analysis to PCA and found that it is not only the parallel shift that affects the movements of the curve but also two other factors, *steepness* and *curvature*, that together with parallel shift can describe 96% of the movements in the yield curve. The conclusion of the paper was that the PCA method is more accurate than the duration analysis.

Niffikeer, Hewins and Flavell [17] in 1998 decomposed the IRS yield curve of 10 major currencies into their common factors and found that the first two factors, parallel shift and rotation, explain between 97.1% and 98.6% of the variations in the IRSs. Further, they introduce *synthetic* factors, which is the main contribution of this paper. They model the two explanatory factors as synthetic factors in order to develop an innovative approach to calculate Value at Risk for a portfolio of interest rate swaps.

Manca [18] in 2003 discusses the application of the PCA to some interest rate derivatives that react to the swing and tilt of the forward yield curve. He studies Constant Maturity Swaps. He approves that PCA is a useful tool in sorting out the data and in reducing the size of the data. He also emphasize that based on empirical evidence, it is sufficient to use three components to model the movements of the yield curve. The three components are the intercept of the yield curve, the slope and the curvature.

5.1.1 Our study

The main contribution of this paper is that it is reproducible both for academics and for practitioners. We decide not to use the duration analysis but PCA. The rational behind this is that duration analysis only approves the parallel shift in the yield curve as the explanatory factor. We find it more accurate and more time effective to use PCA which yields three explanatory factors with a higher explanation level. We first prove that three components are sufficient to explain the movements in the yield curve, using PCA analysis. Second we model our modified data using historical analysis, Monte Carlo simulation and distribution analysis to be able to introduce a new calculation method of Value at Risk for a portfolio of IRSs with different maturities.

The result is meant to be used by a clearing house to be able to set a marginal level for swap contracts committing to.

Chapter 6

Method

Having discussed earlier the core problems of predicting the future values with the right risk measure, it is essential to prepare a strategy for solving the problems. No one can predict the future, but what can be done is to model the data in hand (e.g. from past history) and use that for future predictions and try to build as reliable models as possible with the lowest achievable estimation error, and modeling is an attempt to see trends and similarities in different data sets.

When analyzing the portfolio of interest rate swaps in one currency for the CCP, the goal is to find the cost of change in interest rates with 95% confidence level.
The first natural step is to model the zero coupon rates since the interest rates swaps are based on zero coupon rates, see Formula (4.4). Therefore the zero coupons need to be modeled in order to simulate future values. Further the simulated future values will provide a picture of how different scenarios can look like and thus make it easier to find the worst case scenario.
The value at risk (VaR) for each portfolio will be the amount of money held as security, by the CCP from each member, and is called *margin*. By adding up the margins, the CCP will have the total security required from all its members.

Hence, the chosen risk measure is value at risk at a 95% confidence level over a one day horizon. The largest VaR for each portfolio is called the worst case scenario at 95% level over one day, which means that, on average, in five days out of 100 trading days, there will be a loss larger than the amount held as security. The following approaches are used in this study to solve the problem:

1. **Preparation of data**
2. **Use PCA to reduce the data set**
3. **Model the zero coupon rates**
4. **Simulate future values using Monte-Carlo simulation**
5. **Calculate VaR**

The following sections describe each step in details.

6.1 Preparing the data

To get a reliable model that is capable of capturing the most important changes in a data set, it requires a rearrangement of the data to suit the purpose of the model.
Usually when analyzing the stock market, the risk factor changes are set to be the logarithmic return over one day (i.e. log of the difference in spot price of a stock from yesterday and today). The reason for this is because the stock market is volatile and in order to make the data approximately stationary and identically independently distributed (IID), which makes the model more reliable. In this case, when looking at interest rates, they don't fluctuate in the same manner as stock prices and therefore it is unnecessary to make the data stationary. The procedure is simply to look at the differences between two interest rates as a measure of change. Therefore the preparation of the data is done by taking the differences between the interest rates at the times t and $t\text{-}1$. Hence the difference between two interest rates of two different times will be regarded as a measure of change.

6.2 PCA

The prepared data can now be analyzed in various ways. A common approach is Principal Component Analysis (PCA) which is used for finding patterns in data of high dimensions and also used to handle the risk arising from groups of highly correlated market variables. It tries to reduce the dimension of the data and define a set of uncorrelated components, so called principal components (PCs), that account for most of the variability of the original data.

6.2.1 The linear algebra behind PCA

PCA is based on linear algebra and it helps to explain high dimensional data, with correlated factors, in terms of a few uncorrelated factors. PCA is more of a data rotation tool than a model.
Let $\mathbf{f}$ be a column vector of different factors and Σ_f the covariance matrix of $\mathbf{f}$

$$\mathbf{f} = (f_1, ..., f_m)^T$$

$$\mathbf{\Sigma}_f = \mathrm{Cov}(\mathbf{f})$$

A symmetric and positive semidefinite matrix (with non-negative eigenvalues) Σ_f can be written as a product of a diagonal matrix $\boldsymbol{D}$ with non-negative eigenvalues $(\lambda_1, ..., \lambda_m)$ and an orthogonal matrix $\boldsymbol{O}$ whose columns are eigenvectors of Σ_f, orthogonal and of length one. Then:

$$\mathbf{\Sigma}_f = \mathrm{Cov}(\mathbf{f}) = ODO^T$$

To make the forthcoming calculation easier it is assumed that the columns of $\boldsymbol{O}$ and $\boldsymbol{D}$ are ordered in a way that the diagonal elements are in a descending order, without any loss in the data. Now $\mathbf{f}$ has to be transformed so that the main axes are parallel to the coordinate axes.

By choosing:

$$\mathbf{f}^* = O^T(\mathbf{f} - E[\mathbf{f}])$$

one can see that

$$\mathrm{Cov}(\mathbf{f}^*) = E[O^T(\mathbf{f} - E[\mathbf{f}])(\mathbf{f} - E[\mathbf{f}])^T O] = O^T \mathrm{Cov}(\mathbf{f}) O = D,$$

i.e. the components of $\mathbf{f}^*$ are uncorrelated and have variances $\lambda_1 \geq ... \geq \lambda_m$.
The geometric interpretation of this transformation is that $\mathbf{f}$ has now rotated until the main axes are parallel to the coordinate axes. Here $\mathbf{f}^*$ is the principal component transform of $\mathbf{f}$ and can be considered as a rotation and a re-centering of $\mathbf{f}$ [9].

To show the transformation from correlated components to uncorrelated components of $\mathbf{f}$, we put $\mathbf{e_k}$ as the kth standard unit vector in R^m, such that:

$$\sum_{k=1}^{m} f_k \mathbf{e_k} = \mathbf{f} = OO^T\mathbf{f} = E[\mathbf{f}] + O\mathbf{f}^* = E[\mathbf{f}] + O\sum_{k=1}^{m} f_k^* \mathbf{e_k} = E[\mathbf{f}] + \sum_{k=1}^{m} f_k^* \mathbf{o_k}$$

indicates that the components of $\mathbf{f}$ are correlated when expressed in terms of the standard basis $\mathbf{e_1}, ..., \mathbf{e_m}$ for R^m, however uncorrelated when expressed in the alternative orthonormal basis $\mathbf{o_1}, ..., \mathbf{o_m}$ of R^m.

To measure how much of the total variance of $\mathbf{f}$ is explained by the first few principal components, it is essential to define the total variance.

$$\sum_{k=1}^{m} \mathrm{var}(f_k^*) = \sum_{k=1}^{m} \lambda_k = \sum_{k=1}^{m} \mathrm{var}(f_k)$$

By letting $\sum_{k=1}^{m} \mathrm{var}(f_k)$ be the total variance of $\mathbf{f}$, the ratio

$$j \longmapsto \frac{\sum_{k=1}^{j} \lambda_k}{\sum_{k=1}^{m} \lambda_k}$$

represents the percentage of total movements that can be explained by the first j principal components.
It is noticed that the ratio is close to one for $j = 1$ which indicates that movements of $\mathbf{f}$ is mainly in the direction of the first component, o_1, and that

$$\mathbf{f} \approx E[\mathbf{f}] + f_1^* \mathbf{o_1}$$

is a reasonably accurate approximation. In other words, the first principal component is the standardized linear combination of $\mathbf{f}$, which has maximum variance among all combinations. Principal components are orthogonal [7].

Setting up historical data for interest rates with different maturities, different factors (or principal components) describing the rate movements can be found. The idea is then to express the observed interest rates as a linear sum of the factors by solving a set of simultaneous equations. Then the quantity of a particular factor in the interest rate changes on a particular day is the factor score for that day. The importance of a factor is then measured by the standard deviation of its factor score. The factors are also set so that the factor scores are uncorrelated. Furthermore the variance of the factor scores, that is, the square of the standard deviation, has the property that they add up to the total variance of the data. And to see the importance of the factors, it is possible to calculate how much each factor accounts for the total variance of the original data and to pick those factors that account for most of the data. From that, it is now possible to relate the risks in a portfolio of interest rate dependent instruments to movements in these factors instead of considering all factors [9].

6.3 Modeling

Now it is time to model the factors that can explain most of the changes in data. Factors to be modeled are derived from the PCA. Now it is essential to analyze the factors and find their location-scale family. The approach here is to find which family they belong to and then estimate their location and scale.

6.3.1 Finding the distribution

Determination of the distribution family can be done in several ways. One approach is to plot histograms of the data in order to see symmetric tendencies and also analyze how the data is distributed around the extremities, often referred to as the tails. The measure of asymmetry is called *skewness* whiles the measure of tail thickness is called *kurtosis*. One example illustrating a bell-shaped histogram with the skewness 0 and kurtosis of 3 is the normal distribution.

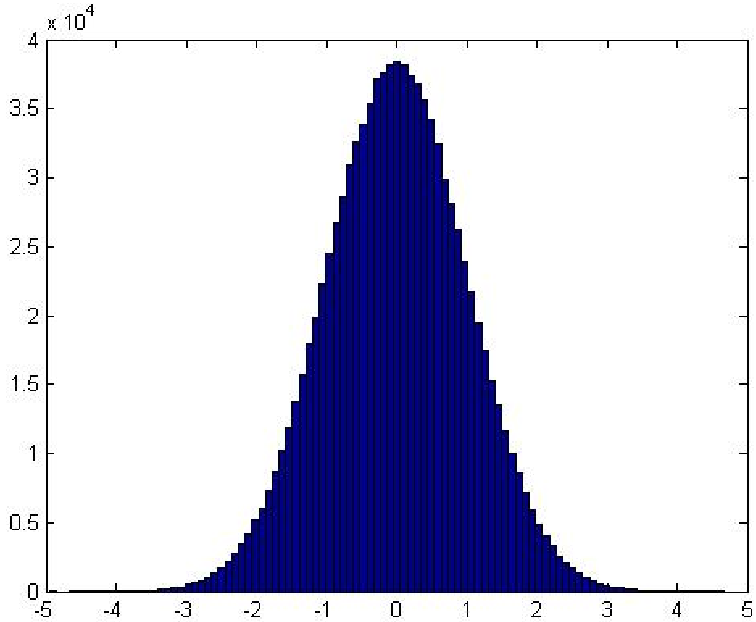

Figure 6.1: Histogram over a normal distribution

Another approach is to make quantile-quantile plots (qq plots). In order to study the level of heaviness, qq plots are useful tools. In probability theory, a quantile function of a probability distribution is the inverse F^{-1} of its cumulative distribution function F. The idea is to test the data against a variety of reference distributions and choose the one that fits best, then estimate the parameters and make another qq-plot to look at the goodness of fit.

Defining a sample in descending order as $X_{n,n} \leq X_{n-1,n} \leq ... \leq X_{1,n}$, and the reference distribution F, then points of the qq plot are derived from:

$$\left\{\left(F^{-1}\left(\frac{n-k+1}{n+1}\right), X_{k,n}\right) : k = 1, ..., n\right\}$$

by putting

$$\left[n\left(1-\frac{n-k+1}{n+1}\right)\right]+1=\left[\frac{n}{n+1}k\right]+1=k$$

the points of the qq plot can be written as follows:

$$\left\{\left(F^{-1}\left(\frac{n-k+1}{n+1}\right),F_n^{-1}\left(\frac{n-k+1}{n+1}\right)\right):k=1,...,n\right\}$$

where the empirical quantile is on the y axis and the reference quantile is on the x axis. A linear plot indicates that the data has a distribution, which is a linear transformation of the reference distribution, that is, from the same location-scale family $F_{\mu,\sigma}(x)=F((x-\mu)/\sigma)$. Further, the parameters of location and the scale of the distribution should be estimated.

If the plot is not linear and it curves up at the left and/or down at the right, the data has lighter tails than the reference distribution and the opposite occurs if the data has heavier tails than the reference distribution [9].

6.3.2 Estimating distribution parameters

The next step is to suggest a parametric distribution F_θ where θ can be estimated using maximum likelihood estimation (MLE). Given a known distribution, MLE finds the value of parameters of that distribution. These estimates are more likely than any other estimates.
The approach is to maximize the likelihood function and solve a series of equations to get the estimated values, where the likelihood function is a function of distribution parameters such that each distribution has its own likelihood function with different parameters.
Estimated parameters of the chosen distribution should be tested against the original data set to determine how good they fit.

6.4 Monte Carlo simulation

When the result from the section above is satisfying, the next step is to simulate future values. There are many methods for simulating future values, given a known distribution. One of the most popular methods is called Monte Carlo simulation.
Quantile functions are used in the Monte Carlo method in order to simulate non-uniform random numbers. A quantile function as mentioned before is the inverse F^{-1} of its cumulative distribution function F and is defined as following, for the probability α, $0<\alpha<1$:

$$F^{-1}(\alpha)=\min\{x\in R: F_\alpha(x)\geq\alpha\}$$

The approach here is to use the quantile function of the distribution to be simulated and apply it on a sample from a uniformly distributed function.
This will result in non-uniform random numbers based on the desired distribution. This method allows the randomization of a large number of scenarios for a future yield curve and thereby the change in values of our portfolio of IRSs. A large number of scenarios will provide a reasonably good approximation of a portfolio's distribution. Then the lowest quantile of this distribution can be used as a measure of risk, VaR. [15]

6.5 Calculating VaR

Measuring VaR

For calculation of Value at Risk there are three common approaches and there are numerous variations within each approach. These are known as the *Empirical Method*, *Variance-Covariance Method* and *Monte Carlo Simulation.*

Empirical method:
The first method is based on the assumption that the changes in the market from today until tomorrow can be described by previous changes in its past, that is, that history will repeat itself, from a risk perspective.
The procedure is that the historical changes during a certain time interval of interest is recognized and sorted ordered from worst to best in order to create a histogram and chose the certain quantile of the confidence level of interest. We recall the quantile function:

$$F^{-1}(\alpha) = \min\{x \in R : F_\alpha(x) \geq \alpha\}$$

and denote X to be the net worth of the portfolio and L to be the discounted loss. Then the VaR at α level is defined as:

$$VaR_\alpha(X) = F_L^{-1}(1-\alpha)$$

One advantage of a historical method is that it is non-parametric, meaning that it does not require assumptions for probability distributions. However, one disadvantage is that the worst case in the future can not be worse than the past. Also there is a need for large amounts of relevant data, which are not always available.

Variance-Covariance method:
The second approach is an analytical method and based on the assumption that the short term changes in the market are normally distributed. It means that there is a requirement for only two parameters, being the expected value and the standard deviation of the changes. Based on the parameters, a normal distribution curve can be plotted and the quantiles can be analyzed by counting the number of standard deviations of interest by multiplying the standard deviation with the normal quantiles for each confidence level.

Describing this method mathematically, the future net worth X is a linear function of vector $\mathbf{Z}$ so that $X = g(\mathbf{Z})$. Then the $\mathbf{Z}$ vector is assumed to have a multivariate normal distribution.

$$\mathbf{Z} \sim N_d(\mu, \Sigma)$$

Now the estimated VaR can be defined as:

$$VaR_\alpha(X) = c + \mathbf{w}^T\mu + \sqrt{\mathbf{w}^T\Sigma\mathbf{w}\mathbf{\Phi}^{-1}}(1-\alpha)$$

for some $c > 0$ and some vector $\mathbf{w}$ so that

$$\mathbf{w}^T\mathbf{Z} \sim N(\mathbf{w}^T\mu, \mathbf{w}^T\Sigma\mathbf{w})$$

From the formulas above, the two parameters, mean and variance then are $c + \mathbf{w}^T\mu$ and $\mathbf{w}^T\Sigma\mathbf{w}$ respectively. These parameters should then be estimated to get an estimated VaR.

Advantages of the method are its simplicity and swiftness. One drawback is that it is unrealistic to make an assumption of normal distribution if the data is not normally distributed. This assumption will lead to underestimation of risk. [15].

Monte Carlo method:
This method uses Data Generation Processes (DGP) and simulates future values. The basic idea is to generate large amounts of data given a known distribution function. The generated data yields an empirical risk factor distribution from which a relevant quantile can be read. [8] From here this approach is the same as the historical method.
The procedure is to generate N (a large number) independent loss samples $L_1, ..., L_N$ from F and from the empirical distribution:

$$F_N(x) = \frac{1}{N}\sum_{k=1}^{N} \mathbb{I}_{[L_k,\infty]}(x).$$

The estimated VaR can therefore be written as:

$$VaR_\alpha(X) = F_N^{-1}(1-\alpha) = L_{[N\alpha]+1,N}$$

Advantages of using the Monte Carlo method is that it is adjusted for many different distributions and gives full distribution values, not only a specific percentile. On the other hand, the method is more accurate for non-linear instruments and also requires a lot of computational power due to very large amounts of data that are simulated.

6.5.1 The chosen method for calculating VaR

In this study, the method used for calculating the VaR is the Monte Carlo method. The reason is the need for large amounts of simulated data in order to cover as many scenarios as possible, which is an advantage compared to the historical method. We have had access to powerful machines which makes it easier to handle large amounts of data. We concluded this to be a major advantage.

The historical method is rejected due to the paradox of the future worst case scenarios, as it does not allow future values to be worse than the past. Finally the parametric approach is rejected due to its underestimation of risk. The normally distributed data assumption in parametric approach is not always suitable.

Chapter 7

Results

7.1 Data

The selected data are historical US swap rates from 3rd of July 2000 until 8th of March 2010. The daily data for IRSs with maturity in 1, 2, 3, 4, 5, 7, 10 and 30 years are downloaded from Oesterreichische Nationalbank. Further, we derive the zero coupon rates from IRSs using Equations (4.5) and (4.6). The daily differences between zero coupon rates are then calculated to achieve a new matrix of data which can be modeled. The time horizon is chosen to be one day so that the VaR which will be achieved later on is a daily VaR.

7.2 Results from PCA

Changes in the data set are analyzed by defining a set of components or factors that explain the majority of movements.

Table 7.1 shows the explanatory level of the principal components. It is shown as a cumulative sum of the principal components' variances. The result shows that 99.0579% of the variance in data is explained by the three first principal components where these components stand for 93.0828%, 5.0822% and 0.8929% respectively.

Cumulative sum of the principal components' variances (%)
93.0828 , 98.1650 , 99.0579 , 99.4955 , 99.6936 , 99.8083 , 99.9148 , 100.0000

Table 7.1: Cumulative sum of components

Figure 7.1 shows the first three components which stand for 99.0579%of the movements in original data.

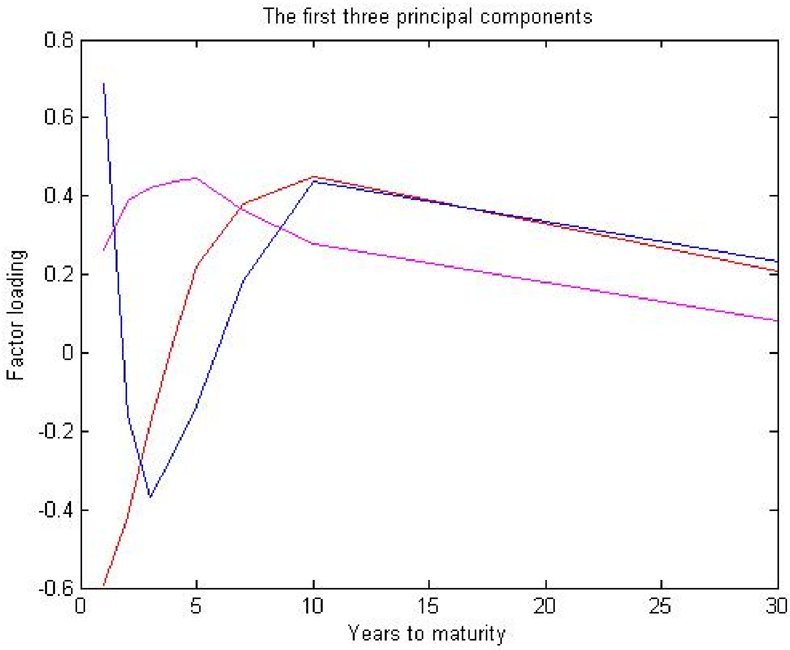

Figure 7.1: The first three PCs

Table 7.2 shows the eight principal components that together describe 100% of the movements in data. The second column, PC1 corresponds to a roughly *parallel shift* in the zero coupon rate curve. A parallel shift shows how all interest rates shift by the same amount. In other words, they shift up or down in parallel to the original curve. For example if the 3-year zero coupon rate increases by 0.4212 bp, then the 4-year zero coupon rate increases by 0.4353 basis points. The second principal component, third column, shows a *twist* in the curve because there is a change in steepness of the curve. Zero coupon rates with maturities between 1-year and 3-year move in one direction and zero coupon rates with maturities between 4-year and 30-year move in the opposite direction. The third factor, fourth column, is called a *bowing* of the curve because there is a positive-negative-positive pattern across the data. Short-term rates and long-term rates move in the same direction while the mid-term rates move in the opposite direction.

Maturity	**PC1**	**PC2**	**PC3**	**PC4**	**PC5**	**PC6**	**PC7**	**PC8**
1-year	0.2598	-0.5965	0.6918	0.1977	0.2390	0.0407	-0.0113	-0.0071
2-year	0.3841	-0.4229	-0.1686	-0.2842	-0.7036	-0.2454	0.0490	0.0828
3-year	0.4212	-0.1831	-0.3784	-0.2658	0.2769	0.6863	-0.0387	-0.1616
4-year	0.4353	0.0352	-0.2569	-0.1059	0.5560	-0.6318	-0.0390	0.1491
5-year	0.4456	0.2284	-0.1126	0.7268	-0.2178	0.0349	-0.3629	-0.1674
7-year	0.3665	0.3703	0.1832	0.0965	-0.0702	0.1803	0.6802	0.4307
10-year	0.2805	0.4455	0.4298	-0.4331	-0.0840	-0.0894	-0.0688	-0.5751
30-year	0.0810	0.2069	0.2299	-0.2705	-0.0517	0.1619	-0.6287	0.6328

Table 7.2: Principal components

7.3 The model

The aim of this section is to model the first three factor scores related to the first three principal components. After finding an appropriate model, it is possible to make simulation that can account for future changes in zero coupon rates.
To do so, the factor scores need to be observed. Figure 7.2 shows the first three factor scores from 3rd of July 2000 until 8th of March 2010. It is noticed that the scores are roughly stationary, approximately independently and identically distributed making it possible to model.

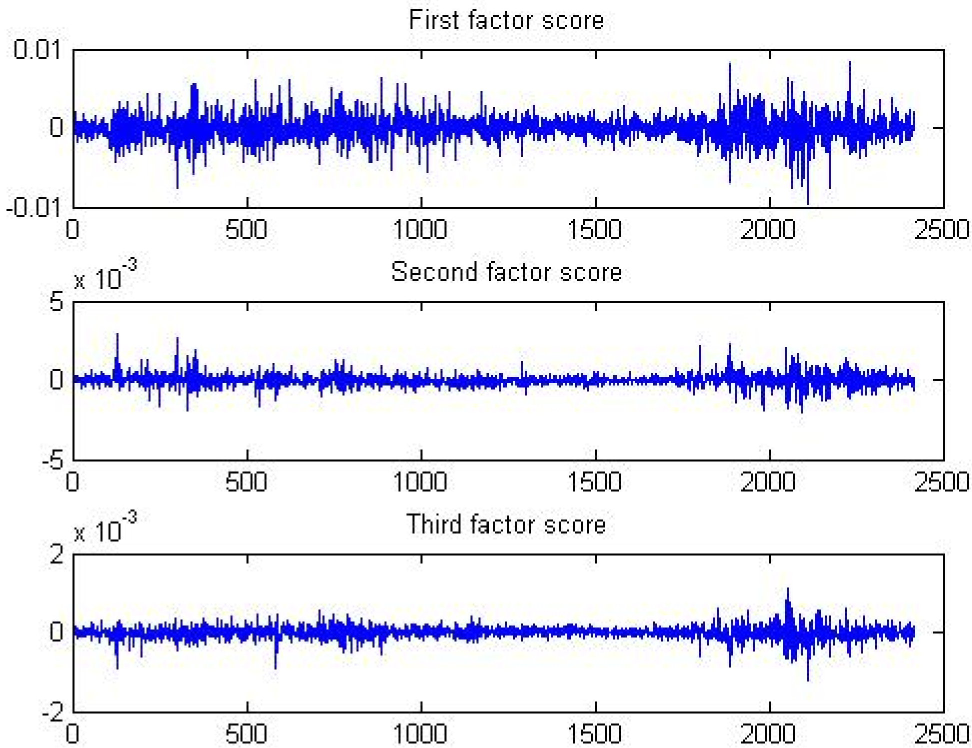

Figure 7.2: The first three factor scores

7.3.1 Histograms

By analyzing the histograms over the factor scores, we can identify the marginal distributions, shown in Figure 7.3. The histogram of normal distributed data should be bell shaped with skewness (a measure of asymmetry) of zero and kurtosis (a measure of tail thickness) around 3 [8]. Here the histograms generated from our three factor changes are compared with a histogram of a normal distributed data.

Factor Scores	Skewness	Kurtosis
First factor	-0.0456	5.7613
Second factor	0.4664	9.7135
Third factorr	-0.4403	8.7807

Table 7.3: Skewness and Kurtosis of factor scores

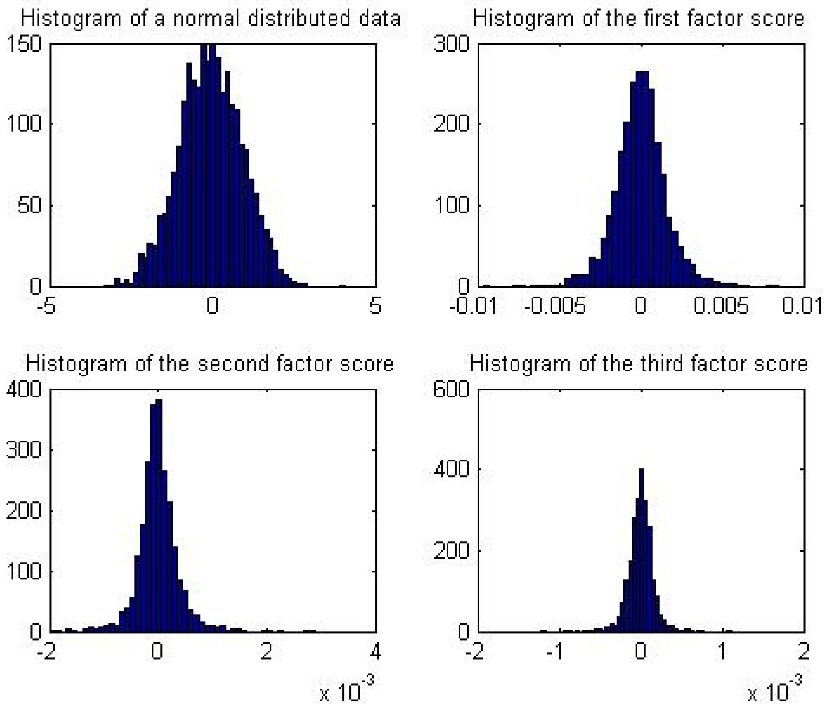

Figure 7.3: Histograms of factor scores

Factor scores show no sign of a normal distribution. Therefore fitting a normal distribution to data is not appropriate due to the large probability of high losses. This indicates that the distribution may have heavier tails than the normal distribution. Thus, it might be appropriate to try to model the marginal distributions using the t distribution.

7.3.2 Quantile-quantile plots

In the above section, it was noticed that the data set has a heavier tail than a normal distribution. In order to study the level of heaviness, quantile-quantile plots are used. Figure 7.4 shows three combinations of factor scores against each other. The conclusion that can be made is that they all look roughly linear indicating that they are from the same family of distributions. It is also notable that the first factor has a heavier tail than the second and the third factors. These plots curve down to left and up to right implying that the data on the x axis (first factor) has a heavier tail the the data on the y axis, in this case second and third factors. The plot between second and third factors is linear indicating that these two factors should have equal or very close heaviness.

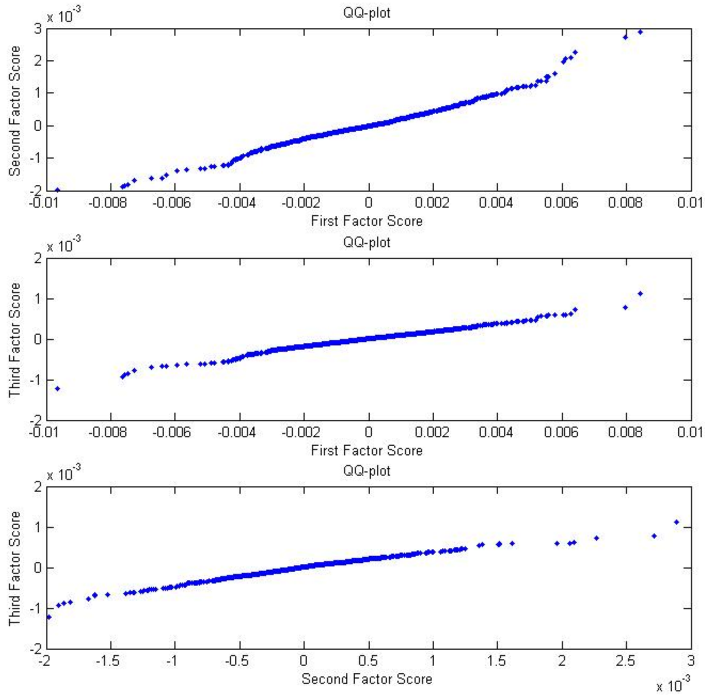

Figure 7.4: QQ plot of three factor scores

In Figure 7.5, the first factor is plotted against different distributions in order to determine the distribution fitting best. The chosen distributions are a normal distribution and t distributions with 3, 4.5 and 6 degrees of freedom. The observations indicate that the t distribution with 4.5 degrees of freedom explains the distribution of the first factor best. The normal distribution plot curves up left and down o right implying means that the first factor has a heavier tail than normal.

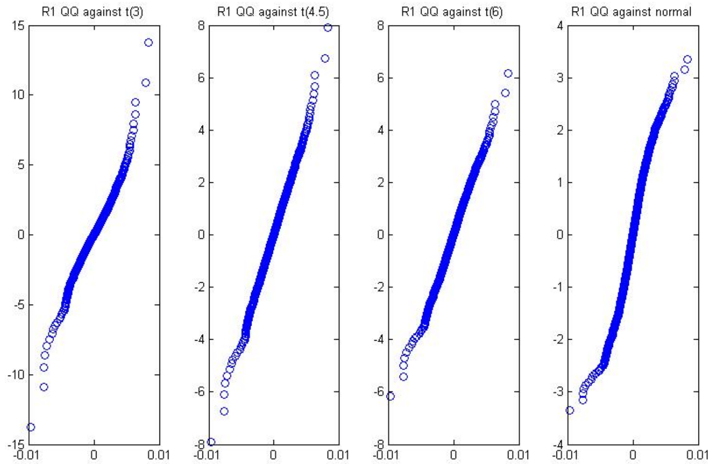

Figure 7.5: QQ plot of first factor against different distributions

The next factor to study is the second factor. From Figure 7.4 it is expected that this factor should have a t distribution with a lower degree of freedom than the first factor. Therefore in Figure 7.6, second factor is plotted against a normal distribution and t distributions with 2.3, 3 and 4 degrees of freedom. With no surprise the t distribution with 3 degrees of freedom is the most suitable one.

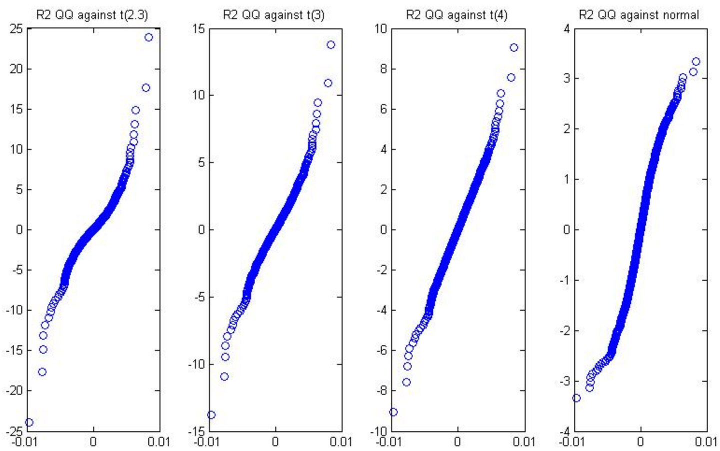

Figure 7.6: QQ plot of second factor against different distributions

As discussed earlier the, third factor should have a t distribution with a degree of freedom around 3. The third factor is tested against normal distribution and t distributions with 3, 3.5 and 4.5 degrees of freedom. Figure 7.7 proves that the most suitable distribution for the third factor is t distribution with 3.5 degrees of freedom.

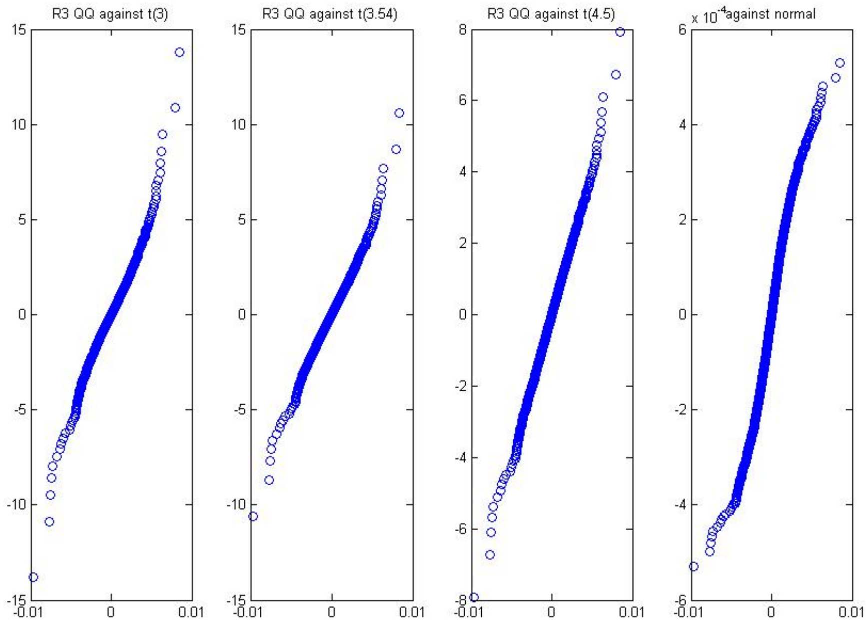

Figure 7.7: QQ plot of third factor against different distributions

7.3.3 Parameter estimation

Knowing the distribution, it is time to look at the parameter estimation. Two approaches are used to find the parameters. One is the known maximum likelihood estimation and the other is one of MATLAB's built-in functions called DFITTOOL, based on MLE however with some minor exceptions.

DFITTOOL opens a graphical user interface for displaying fitted distributions to data. After fitting a distribution it is possible to analyze the distribution and see the parameter estimation of the chosen distribution.

The maximum likelihood estimation is also done in MATLAB where the approach is to maximize the likelihood function and solve a series of equations to get the estimated values. A likelihood function is a function of distributions' parameters so each distribution has its own likelihood function with different parameters.

The results from DFITTOOL and MLE are as follows:

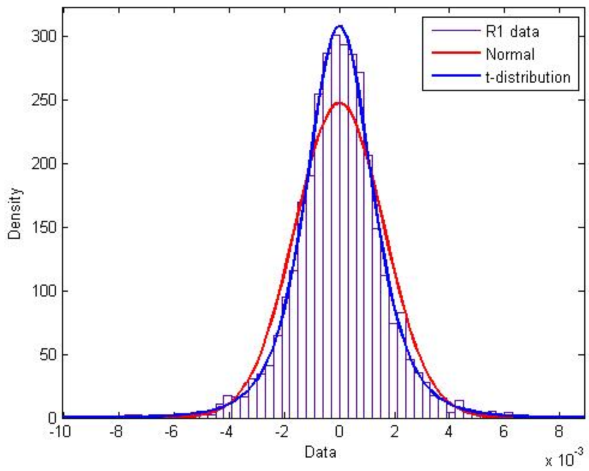

Figure 7.8: First factor's distribution

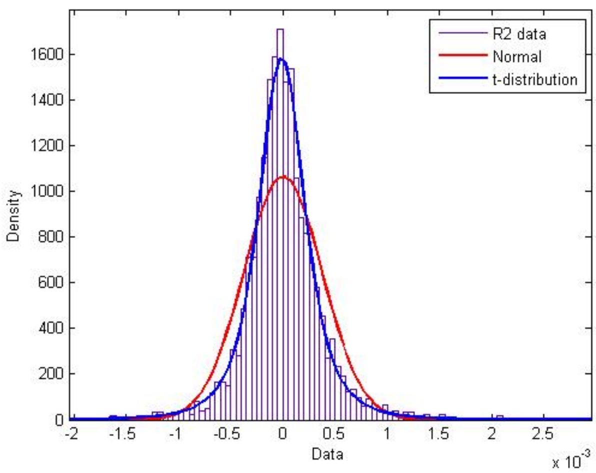

Figure 7.9: Second factor's distribution

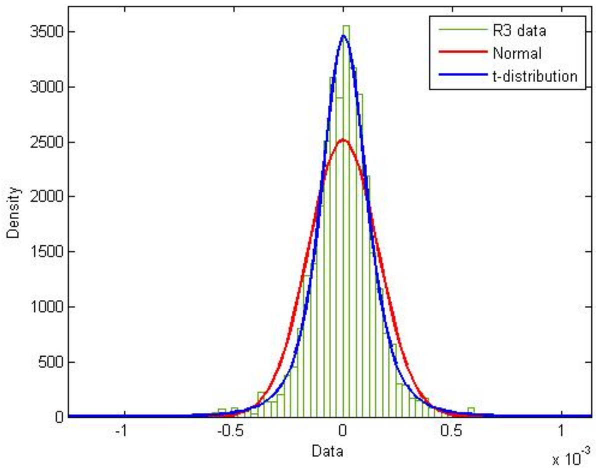

Figure 7.10: Third factor's distribution

Factor scores	First factor	Second factor	Third factor
Optimal degree	4.57	2.92	3.54

Table 7.4: Optimal (ML) degree of freedom of t distribution

A comparison between results from these two approaches shows no difference in the results.

7.4 Simulated future values

The results from PCA show that there are three uncorrelated t distributed factors having different parameters. The uncorrelated variables can be simulated both as dependent and independent variables. The chosen method in this study is the independent version. Figure 7.11 illustrates the difference between dependent and independent simulated samples. To the left, the dependent sample (circular shape) is illustrated and to the right graph, the independent sample (star shape).

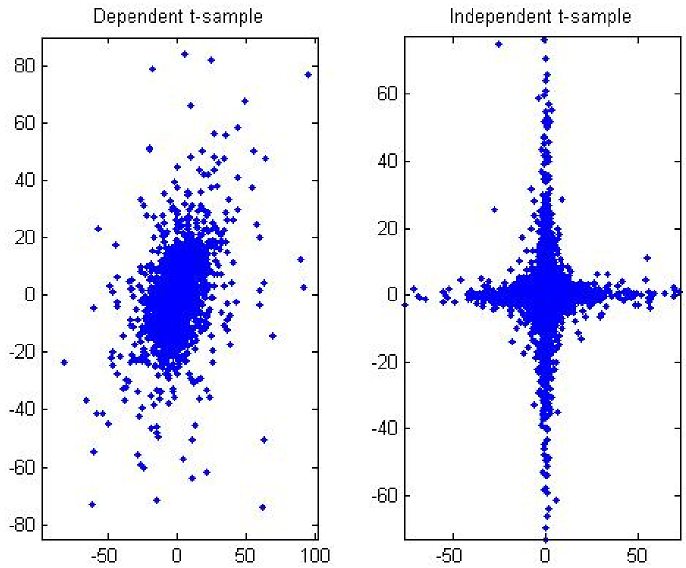

Figure 7.11: Independent and dependent t-simulated samples

We use Monte Carlo simulation to simulate 10000 independent samples for each factor with respect to their different parameters. Furthermore the simulated values are the zero coupon rates and have to be transformed into discount factors. The reason behind this transformation is that we need discount factors for calculating the value of an interest rate portfolio. The independence of the three factors, illustrated in Figure 7.12, can be compared to the previous figure, where it is possible to see a star-shaped structure in the simulated data.

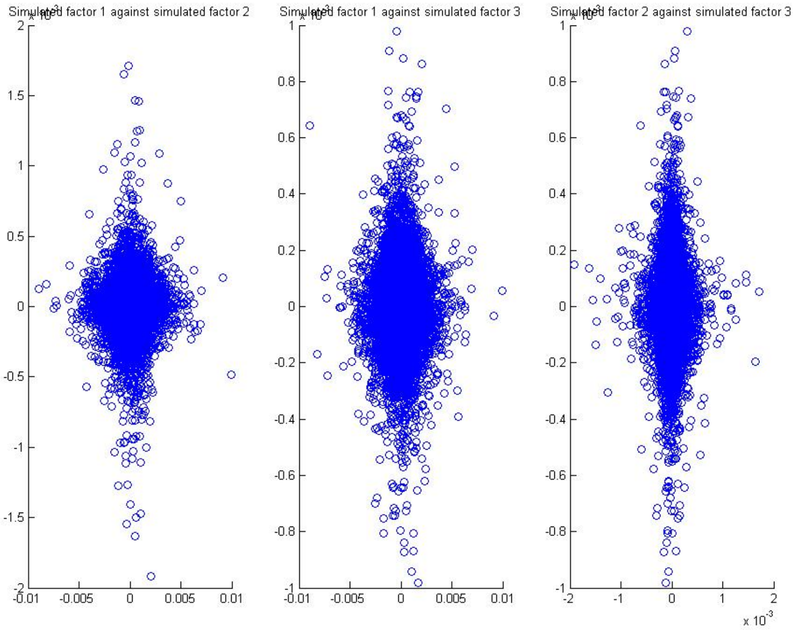

Figure 7.12: Scatter plot of the simulated data

Let us introduce y_i as follows:

$$y_i = t_{\nu_i}^{-1}(t_\nu(x_k)) \qquad \text{for k} = 1, 2, 3$$

where x_i is simultaneous t distributed with different degrees of freedom (ν). When using a large ν the final result is very similar to what we get from our model, but when using a small ν the final result is completely different. This behavior is expected. The reason is when ν approaches infinity, the $\mathbf{Y}$ will have a cumulative normal distribution, where $\mathbf{Y} = (y_1, y_2, y_3)$.

An illustration of how the distribution looks like as a for each ν can be followed in the Figures 7.13 7.14, 7.15 and 7.16.

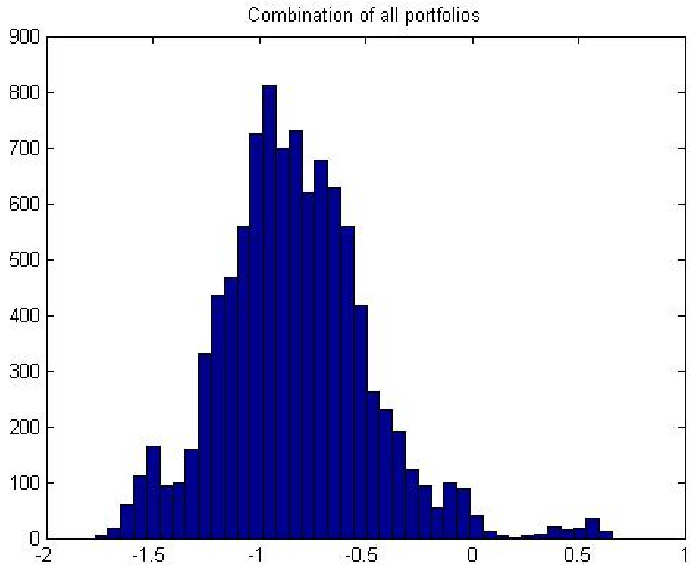

Figure 7.13: Distribution as a function of $\nu = 2.2$

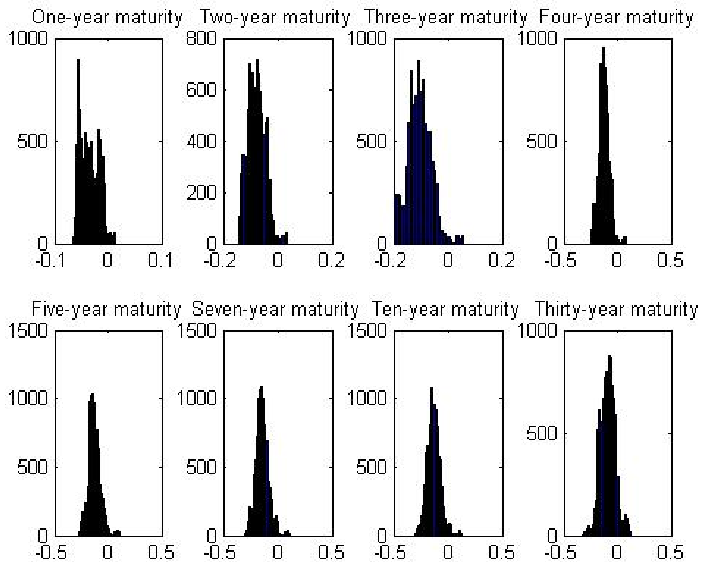

Figure 7.14: Distribution as a function of $\nu = 2.2$ for each maturity

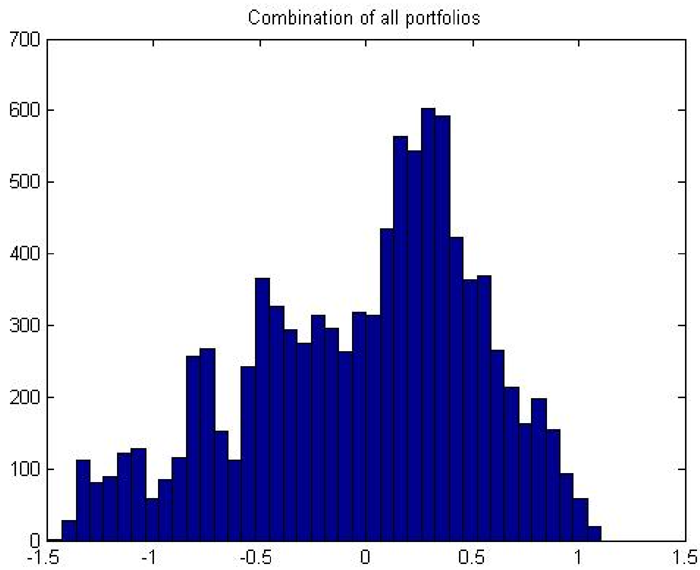

Figure 7.15: Distribution as a function of $\nu = 100$

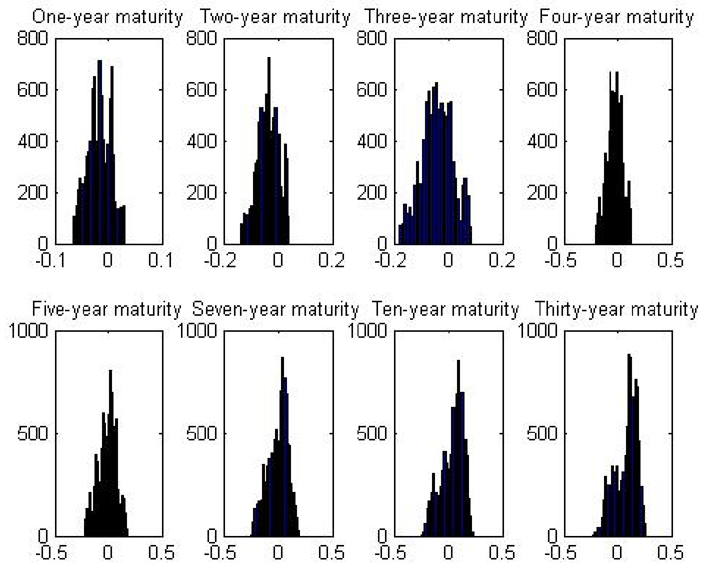

Figure 7.16: Distribution as a function of $\nu = 100$ for each maturity

7.5 The VaR

To calculate VaR of a portfolio of IRSs the empirical approach is used on simulated data. The time horizon is one day and the confidence level used is 95%. The value of a swap contract is calculated using (4.4). The value of contract is calculated for portfolios of one maturity, i.e. a portfolio of swap rates with maturity in one year. The values are sorted in descending order and the 95% quantile is picked as a daily VaR for that portfolio.

Calculating one maturity at a time has the advantage of looking at each maturity per se and being able to compare short time portfolios with long-time and middle-time portfolios. A CCP can then combine different portfolios to match its actual portfolio. It also gives a possibility to combine different portfolios and find an appropriate combination that suits the CCP's risk preferences.

The results are:

Different Portfolios	**Value-at-Risk**
1-year Maturity	1.88%
2-year Maturity	3.89%
3-year Maturity	6.49%
4-year Maturity	9.40%
5-year Maturity	13.10%
7-year Maturity	14.24%
10-year Maturity	17.40%
30-year Maturity	18.61%
Sum	85.01%
95%-VaR of sum of portfolio values	80.05%

Table 7.5: VaR at 95% level for different portfolios

The VaR increases as the maturity increases. It is more risky to tie up the capital for longer periods, hence the investor will loose alternative investments that could result in higher returns. Therefore the investor requires a higher return when borrowing for a long period compare to a short period, as a compensation.

It is also notable that the sum of all Value-at-Risks is greater than the Value-at-Risk for the combined portfolio. The combined portfolio is a portfolio of all maturities with equal amount invested in each, as expected from VaR measure satisfying the property of subadditivity, explained in section 3.5.1.

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$$

In the following three figures, the distribution of portfolio values is illustrated. Figure 7.17 shows the distribution of short-term portfolios with maturities one, two and three years. Mid-term portfolios and long-term portfolios are shown in Figures 7.18 and 7.19 respectively.

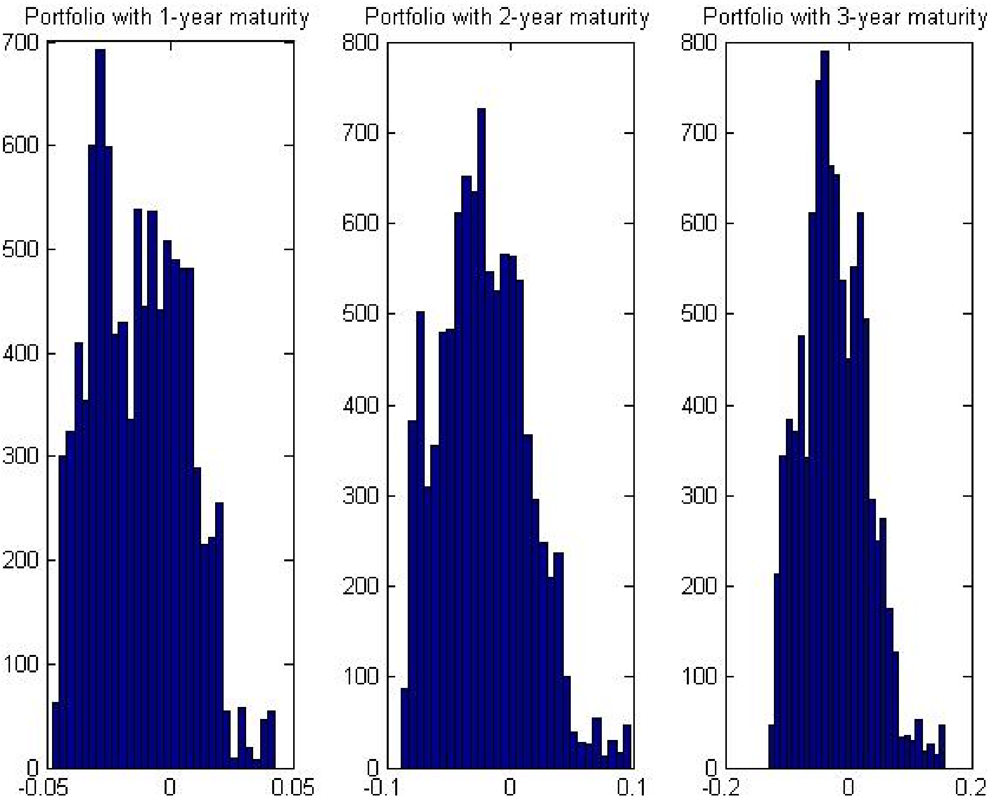

Figure 7.17: Distribution of short-term contracts with maturity from one to three years

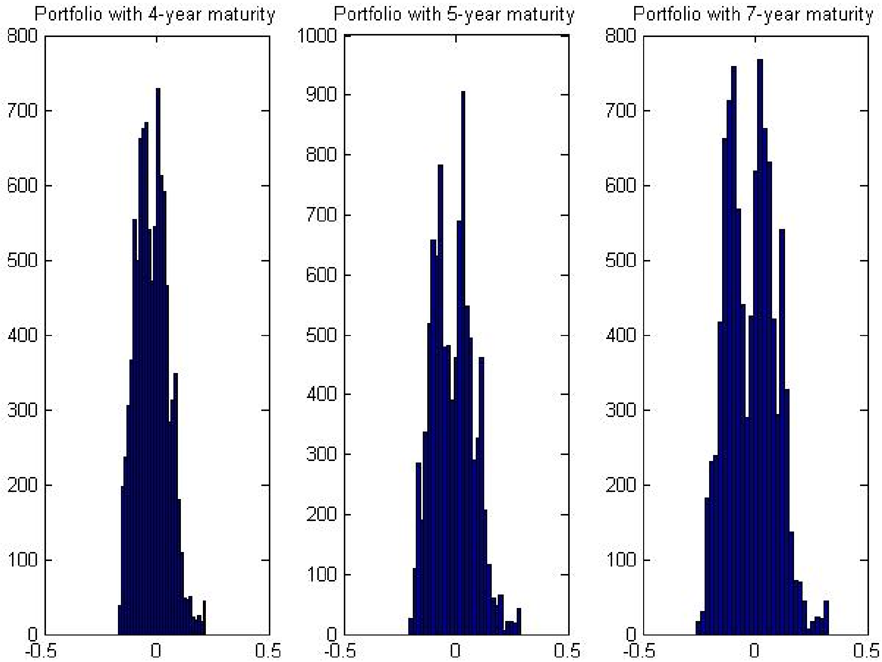

Figure 7.18: Distribution of mid-term contracts with maturity from four to seven years

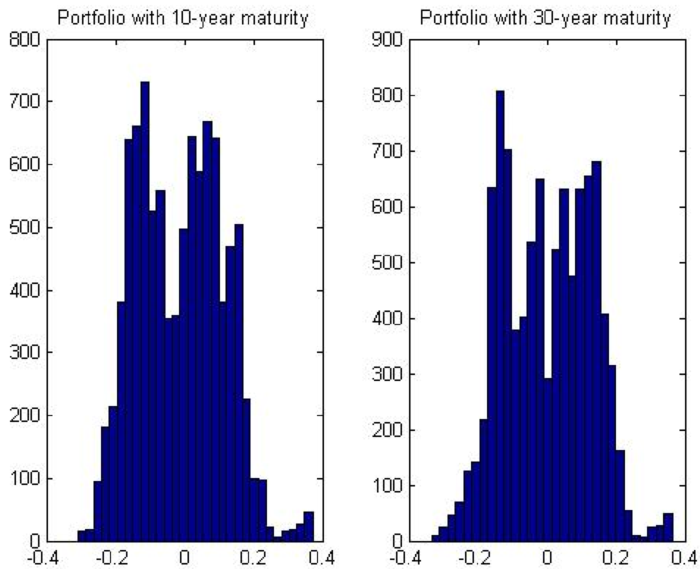

Figure 7.19: Distribution of long-term contracts with maturity from ten to thirty years

The graphs look roughly symmetrical as expected. Due to the fair value of the contract it is expected that there is no arbitrage during the life of the contract which means that value of the contract to a fixed receiver should be equally negative as positive. This is of course not the case in reality. Both sides of the swap enter the contract with different future market expectations and with hope for a positive outcome. It is natural that the value of the contract angles and is not exactly symmetrical.

Figure 7.20 illustrates the distribution of the sum of separate values, the combined portfolio. As we add up all the values, it is expected that the distribution is more symmetrical than the individual distributions. This is confirmed in the Figure 7.20. The 95% VaR for the combined portfolio is 80.05%. This is the total risk a CCP could face at a 95% level when having an equal notional amount in each portfolio.

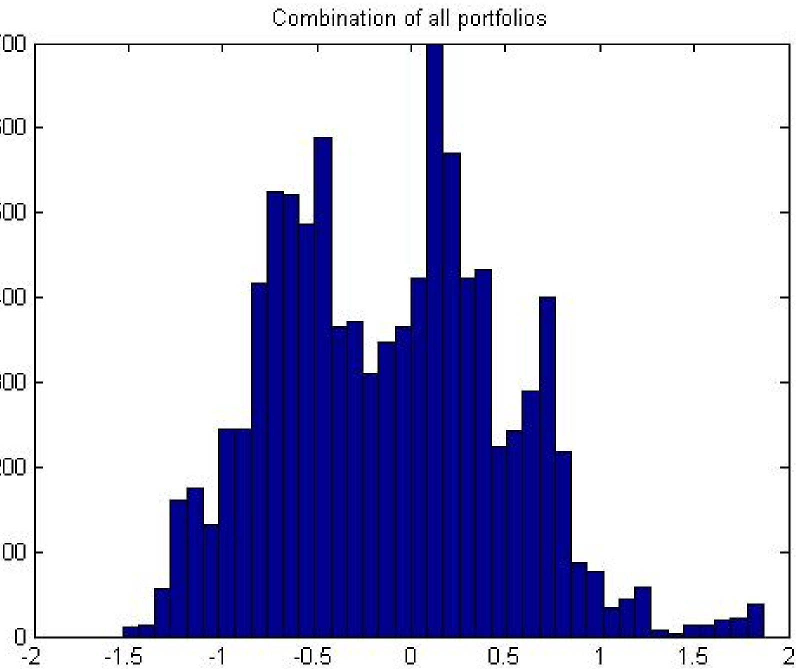

Figure 7.20: Distribution of the sum of separate values

Table 7.6 illustrates the differences in the final results when using different ν values, indicating that the VaR in this model is a function of degrees of freedom. Using a large $\nu = 100$, the VaR for each maturity is very close to the model predictions. On the other hand, having a small $\nu = 2.2$, the VaR becomes negative, indicating that the most of the mass is on the left side of the centre. It is also notable that each VaR is very close to each other when $\nu = 2.2$.

VaR for different Portfolios	**Our model**	$\nu = 2.2$	$\nu = 100$
1-year Maturity	1.88%	-0.60%	1.83%
2-year Maturity	3.89%	-2.73%	3.078%
3-year Maturity	6.49%	-3.83%	6.56%
4-year Maturity	9.40%	-4.26%	10.45%
5-year Maturity	13.10%	-3.74%	15.11%
7-year Maturity	14.24%	-4.23%	16.55%
10-year Maturity	17.40%	-2.83%	19.14%
30-year Maturity	18.61%	-1.99%	20.28%
Sum	85.01%	-20.29%	86.12%
95%-VaR of sum of portfolio values	80.05%	-24.40%	79.44%

Table 7.6: VaR at 95% for different ν values

One can also see that the VaR, for $\nu = 100$, are almost the same as obtained VaR from our model. On the other hand, the small differences between each VaR are because the Monte Carlo simulations have not converged enough.

Chapter 8

Discussion

The main approach in this thesis has been to generate, using the Monte Carlo simulation, a large amount of future interest rates according to a certain distribution (t distribution). Using the distribution, the risk has been calculated according to a standard approach based on the value of each contract. However there are alternative approaches to generate future interest rates. The two most popular approaches will be introduced and discussed in this section. Both approaches are very similar to our approach and they are worth mentioning.

8.1 Alternative approaches

The first approach is the Ho-Lee tree, introduced in 1983. Ho-Lee tree is a non-arbitrage model, which is a model designed to be consistent with today's term structure, for short interest rates. Armed with a set of expressions, the whole evolution of the interest rates can be mapped out. However for the purpose of this paper, there are a set of drawbacks in this model. When having a long-time perspective like maturities of 30 years, short-time interest rate models are not relevant especially as the Ho-Lee tree grows exponentially in the structure. It is also a stiff model due to the fact that every following interest rate follows the "zero position". Also, for example, if the interest rates will boost enough, it will affect the whole structure. It is usually a good model if used over a short-time period, for example a five-year horizon.
The other model used for future interest rate calculation is called Hull-White or Mean Reversion model which can be characterized as the Ho-Lee model with a mean reversion. It has the same amount of analytic traceability as Ho-Lee and is easier to implement due to its multivariate property. However it has some drawbacks since it is a short-term model and when looking at long-time horizons it is not suitable.

8.2 Real Time Risk Management

Looking at real-time risk management, the calculations have to be optimized in order to be sufficiently fast.

There are many ways of doing so and two interesting approaches suggested as further studies are Parallel Vector Computing and Incremental Computing.

Parallel Vector Computing:
After simulating 10000 scenarios and creating a 10000×8 matrix, the value of the portfolio is derived by multiplying the simulated matrix with the portfolio size 8×1. The running time for calculation of a rectangular matrix like the one mentioned, that is, calculating a m×n matrix with a n×q matrix, is O(mnq), that is, O(10000×8×1).
Because this layout puts pressure on the program memory, it is possible to gain substantial performance through the use of parallel computing. It operates on the principle that large problems can be divided into smaller ones, which are then solved concurrently (in parallel).
This could be interesting to look at for further studies and for future implementations of the model suggested in this paper.

Incremental Computing:
When modifying a single trade in a portfolio, following the suggested model, all calculations have to be done again to produce the final matrix. If there would be an incremental calculation where only the trade of interest could be recalculated and the rest kept unchanged, the calculations would run much faster.

8.3 Stress testing

In addition to calculating VaR, it is also essential to stress test the result to get a sense of robustness. Stress testing involves measuring the sensitivity to huge or sudden changes in the market. What would happen to a portfolio if all interest rates fall by 0.25%? What would the VaR be in that case? These are typical questions that can be answered by performing a stress test. Regulators all over the world perform stress testing for banks and insurance companies on a regular basis to avoid big crashes in the market.

In this study, the stress scenario is that all rates in the market, with different maturities, fall at the same time. Two scenarios have been studied. First it is assumed that interest rates all over the market drop by 1 bp (0.01%). As mentioned earlier, the first component from PCA stands for 93% of all changes in interest rate movements. Therefore, the first principal component is then set to be minus 1 bp for all maturities in order to see the effect of such a scenario.

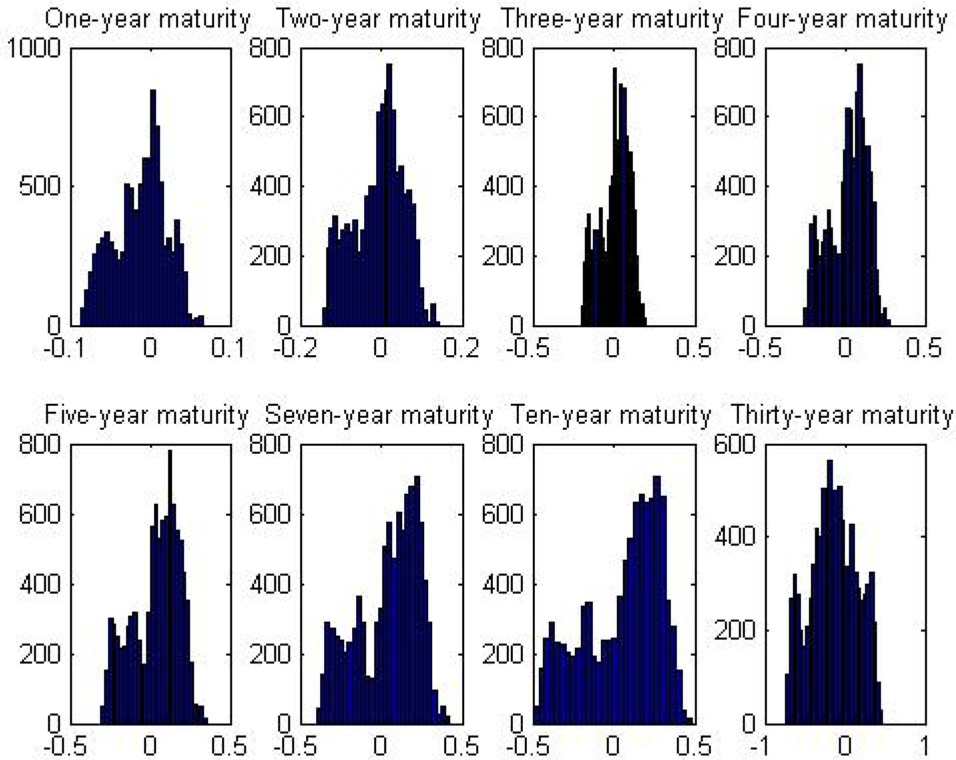

Figure 8.1: Distribution of contract values under 1 bp stress

Figure 8.1 illustrates how different portfolios react to the stress of 1 bp, which means all of the rates drop by 1 bp. It is noticeable that even such a small change does affect the value of all contracts. But there is still a sense of symmetry in most of the contracts. The VaRs for all individual portfolios are shown in the Table 8.1. The sum of all VaRs is more than 100%, implying these contracts will not tolerate a drop of 1 bp in interest rates, and if this scenario occurs, there will be huge losses to take care of.

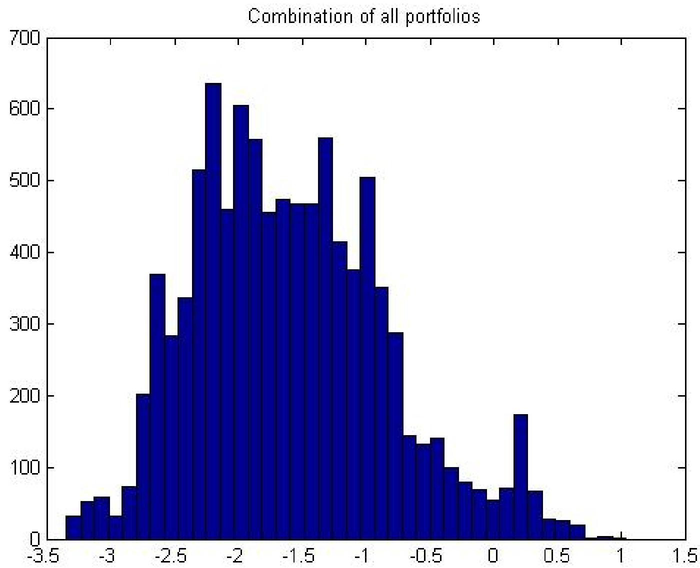

Figure 8.2: Distribution of sum of contract values under 1 bp stress

Figure 8.2 illustrates the combined portfolio's distribution under stress. Most of the mass is lying in the left part of the graph which is the loss side.

Different Portfolios	**Value-at-Risk**
1-year Maturity	3.77%
2-year Maturity	8.39%
3-year Maturity	13.21%
4-year Maturity	17.96%
5-year Maturity	23.15%
7-year Maturity	28.59%
10-year Maturity	34.87%
30-year Maturity	35.49%
Sum	165.44%
95%-VaR of sum of portfolio values	117.18%

Table 8.1: VaR at 95% level for different portfolios under 1 bp stress

The other stress level being studied in this paper is a drop in all interest rates by 25 bp which will put the contracts under an even greater pressure. The results, as expected, show a very low or no level of tolerance. In Figure 8.4, more than 90% of the mass is on the loss side. A drop in rates by 25 bp will result in a huge crash in the market. The combined portfolio in Figure 8.4 proves this as well. In Table 8.2, the results from calculating the VaRs are shown and the sum of all VaRs is more than 425%.

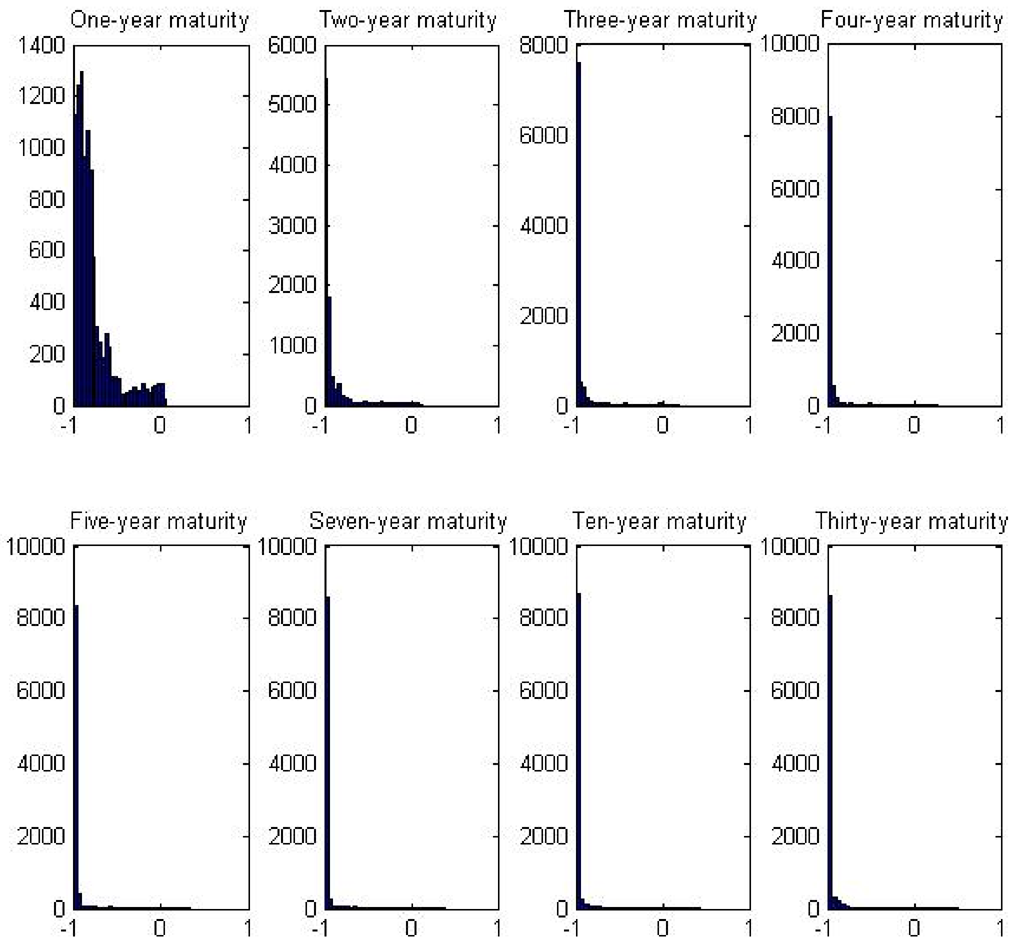

Figure 8.3: Distribution of contract values under 25 bp stress

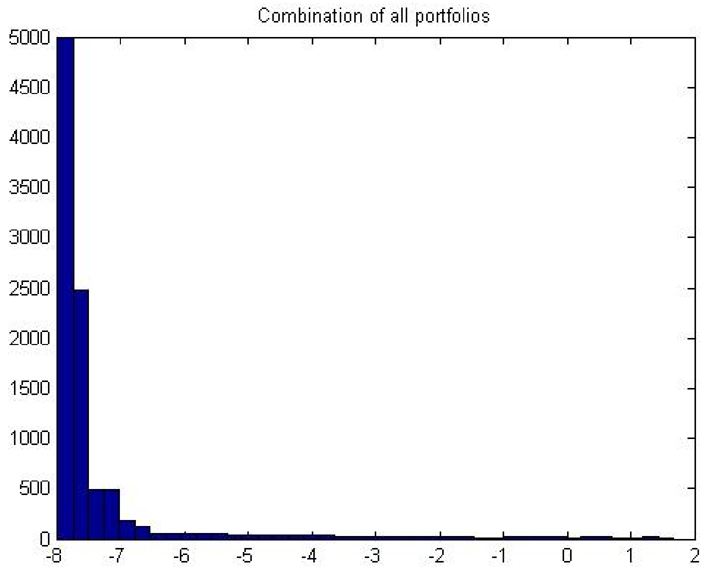

Figure 8.4: Distribution of sum of contract values under 25 bp stress

Different Portfolios	Value-at-Risk
1-year Maturity	-18.97%
2-year Maturity	-32-93%
3-year Maturity	-42.98%
4-year Maturity	-50.99%
5-year Maturity	-56.79%
7-year Maturity	-66.31%
10-year Maturity	-74.12%
30-year Maturity	-82.66%
Sum	-425.64%
95%-VaR of sum of portfolio values	-425.96%

Table 8.2: VaR at 95% level for different portfolios under 25 bp stress

Chapter 9

Conclusions

In this thesis, the problem of how to calculate the risk for plain vanilla interest rate swaps has been addressed. The main attribute is that the risk of having a portfolio of plain vanilla interest rate swaps as an intermediary between counterparties can be modeled based on historical data. The main approach in modeling has been a straightforward calculation based on Principal Component Analysis.
Other approaches have been introduced as common models in the area of forecasting future interest rates with focus on short-term rates. These approaches are then rejected because the maturities used in this paper are up to thirty years and therefore a short term model is not suitable.

The PCA approach has been stress tested, and the rational behind this is to study the robustness of the model. The reason has been that stress testing combined with VaR gives a more comprehensive picture of risk. While VaR gives a more comprehensive picture of the risks in an everyday market environment, stress testing gives a picture of the risks in an abnormal market.
The Monte Carlo simulation is used to simulate future values but the method has some disadvantages. One is that the simulation process is very time consuming and in real-time risk management a faster approach is preferred.
One way to speed up the calculation process is to optimize it by performing Parallel Vector Computing when working with large matrix multiplications and using Incremental Computing for post-modifications of a portfolio.

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