Characteristics of High-Frequency Trading

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23 June 2011

Abstract

In this thesis we investigate some properties that have been suggested in the literature to be characteristic of High Frequency Trading (HFT). These properties are examined on a data set consisting of 28 days of tick market data from the Swedish exchange Burgundy. A statistical analysis of the properties is conducted to determine if they seem to be correlated with one another.

Acknowledgements

I would like to thank my supervisor Filip Lindskog for his guidance and feedback. I would also like to thank Gustav Ryd for his ideas and for being an invaluable sounding board. I wish to thank Lars-Ivar Sellberg for coming up with the subject of this thesis and his inputs. I am immensely grateful to Burgundy for providing the data for this thesis. Finally I would like to thank my colleagues at Scila, family and friends for their continuous support.

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1 Introduction

During the last decade great advances in computer technology has been made and allowed for the growth of the electronic financial market. It has also opened up the possibility for a new class of trading called algorithmic trading. An algorithmic trading system utilizes computers to analyze market data with advanced mathematic models and to generate trading signals. The system can be designed to make trading decisions on its own or to just optimize the execution of already made decisions or both. The term algorithmic trading includes a wide range of trading strategies, some are well known while others, for natural reasons, are non-public and carefully guarded secrets.

A special class of algorithmic trading that has grown over the last few years is high-frequency trading (HFT). According to a report from Aite Group in February 2009 (cited in Aldridge, 2009) HFT made up for over 60% of the traded volume at financial exchanges in 2009.

Even though HFT has received much attention lately no common definition has yet been publicly recognized. This is perhaps not that surprising since HFT can involve a variety of trading strategies. Aldridge (2009) suggests that HFT is characterized by the following properties:

- Short position-holdings (seconds, minutes or hours but less than a day)
- No or small overnight positions
- A large number of trades with a small profit per trade
- Analysis of tick market data

In fact the last two properties may be seen as a result of the first two properties, which suggests that short position holding times and no overnight positions might be used as a definition of HFT. Another property that characterizes HFT is the submission of numerous orders (Wheatley, 2010).

A HFT strategy aims at identifying small temporary mispricing in the market and to trade upon them. Since the mispricing is small the trading opportunity quickly disappears and therefore speed of execution is of great importance. This has led to an arms-race among HFTs to have the latest in computer technology and even to pay exchanges for placing their server close to the exchange's matching engine with the purpose of shaving off execution time.

After the "Flash Crash" in May 6, 2010, where HFT is claimed to have contributed (Lauricella, Scanell, & Strasburg, 2010), HFT has been a hot topic among regulators, market supervisors and in financial discussions. It has been accused of causing volatility in the market and that its advanced technology gives an unfair advantage to other investors. Whether these accusations are true or not remain unanswered but it is clear that monitoring HFT is necessary.

As noted above, two of the key characteristics of HFT are short position-holdings and no overnight positions. Therefore it may seem like the simplest and most effective way to monitor HFT would be to look for market participants with short position-holding times with little or no overnight positions. However, the possibility to trade the same security at multiple exchanges makes it difficult to monitor positions unless having access to full trading information from multiple exchanges at the same time. This is typically not the case.

The idea behind this thesis is to investigate what other properties HFT is characterized by and hence to find other ways of monitoring HFT. More specifically, this thesis will investigate and compare some different methods that measures properties that has been suggested in the literature to be characteristic of HFT. The different methods will be run on 28 days of tick market data (i.e. a complete set of data containing every order and transaction entered) from the Swedish exchange Burgundy. An analysis of the different methods will be conducted to determine if they seem to be correlated with one another.

The outline of the thesis is as follows:

- Chapter 2 gives an overview of the electronic market place.
- Chapter 3 gives an overview of some of the basic HFT strategies.
- Chapter 4 gives an overview of some properties that the literature suggests to be typical for assets suitable for HFT. The chapter also describes how these properties will be measured.
- Chapter 5 gives an overview on how information asymmetry will be measured.
- Chapter 6 gives an overview of some properties that the literature suggests to be typical of HFT-traders. The means to measure these properties is also described.
- Chapter 7 gives a description of the data that has been used.
- Chapter 8 describes the methodology used when determining what properties are typical of HFT.
- Chapter 9 presents all results.
- Chapter 10 gives a summary of the results found.

2 The electronic marketplace

An electronic financial marketplace is a place for electronic exchange of assets, e.g. stocks, between a buyer and a seller.

2.1 Orders and order types

An order entered by a market participant at an exchange is an instruction to either buy or sell an asset. There are a number of different order types available, each with its own characteristic. The order types allowed differ among exchanges. Below follows an overview of the most common order types.

2.1.1 Market orders

A market order is an order with the instruction to trade at the best available price, no matter how good or bad it might be. I.e. a participant entering a market order does not specify a price that he is willing to trade at; only the number of shares that he wants to execute (called the volume of the order). Market orders get the highest priority and are the first to be executed. An active market order never rests in the order book but is immediately executed upon insertion.

2.1.2 Limit orders

A limit order is an order with the specification of the maximum or minimum price that the participant is willing to buy or sell at. The limit order will remain in the order book until being matched or canceled.

2.1.3 Non-active orders

All orders entered need not be immediately active and valid for trading. Some orders become valid first at a specific time of the day. Other orders, such as stop orders, become active first when the trade price reaches a certain limit.

2.1.4 Hidden volume orders

This is an order where only part of the orders total volume is visible to the other market participants.

2.2 Concept of order book

Every asset at an exchange has an order book, sometimes more than one. This is where all the orders to trade the asset are contained and matched, even non-active orders. Orders that are not immediately matched upon insertion will remain in the order book until they can be executed or are canceled by the trader or the trading system. The difference between the current best bid price and best ask order price in the order book is called the spread.

2.3 Market participants

There are several participants at an exchange, each with different roles. The main categories are market makers, customers and brokers.

2.3.1 Market makers

A market maker is a dealer who often receives rebates and privileges by the exchange for providing liquidity in an order book. I.e. market makers provide a spread tighter than a maximum value and at least a minimum volume on the bid and ask side of the order book. The market maker makes money of the spread.

2.3.2 Customers

Customers include both retail and institutional customers who trade an asset either for purpose of investment or speculation.

2.3.3 Brokers

A broker is an individual or firm that arranges transactions between a buyer and a seller and takes a commission for doing this.

3 Common HFT strategies

Aldridge (2009) mentions the following four HFT strategies as being the most popular:

- Automated liquidity provision with a typical holding position of less than 1 minute.
- Market microstructure trading with a typical holding position of less than 10 minutes.
- Event arbitrage trading with a typical holding position of less than 1 hour.
- Deviations arbitrage with a typical holding position of less than 1 day.

3.1 Automated liquidity provision

Liquidity provision, also known as market making or inventory trading, are strategies that involve optimal placing and pricing of limit orders on the bid and ask side of the order book in order to make money from the spread. Moreover, many exchanges provide liquidity rebates for providing liquidity.

Market making strategies share the following key characteristics:

- Almost only limit orders are used.
- The gain per execution is small why these strategies rely on moving in and out of position a large amount of times over a day.
- They operate at high frequencies in order to be able to enter a multitude of trades.

3.2 Market microstructure trading

These strategies involve processing market data in order to extract information and to trade upon this information. The type of information could for example be order book imbalances, trading activity, etc.

3.3 Event arbitrage trading

Certain macroeconomic events yield predictable responses by the market. This set of strategies attempts at making money out of this predictability.

3.4 Statistical arbitrage

These strategies look at exploiting deviations from statistical relationships. Two examples are deviation between two highly correlated stocks or between a derivative and its underlying.

4 Finding assets suitable for HFT

There are many assets that are suitable for HFT. Some are however more suitable than others. This chapter will describe the methods that will be used to find the assets which are most suitable for HFT.

According to Aldridge (2009) the two primary requirements for an asset to be suitable for HFT are:

- High liquidity to be able to quickly move in and out of positions.
- Sufficient volatility so that price movement exceeds transaction costs.

Another asset property of importance that is discussed by Aldridge is the level of predictability of an asset's price changes.

The methods that will be used to measure these properties are described below.

4.1 Measuring predictability

A HFT system aims to consistently generate income over a large number of trades. To accomplish this, the system tries to predict future prices. A complete random price process is therefore of little interest. While a price process may appear random for time intervals of days, this need not be the case for other intervals, e.g. minutes or seconds. I.e. a strategy using a short position-holding time period may find predictability in an asset that a long position-holding strategy finds random.

In this thesis we will use a popular method developed by Lo and MacKinlay (1988) to determine the predictability of an asset by investigating if it follows a random walk or not. This information will be used to determine which asset a certain algorithmic trader is most likely to trade in.

The test is based on the property that the variance of increments of a random walk process X_t is linear in the time interval, i.e. $\mathrm{Var}\left(\mathrm{X_t} - \mathrm{X_{t-1}}\right) = k \ \mathrm{Var}\left(\mathrm{X_t} - \mathrm{X_{t-k}}\right)$. The test is constructed as follows:

Let P_t be the asset price at time t and define $X_t \equiv \ln P_t$. The hypothesis is made that X_t follows a random walk process with i.i.d. normally distributed random terms:

$$H: \; X_t = \mu + X_{t-1} + \varepsilon_t, \; \varepsilon_t \text{ i.i.d } \mathrm{N}\left(0, \sigma_0^2\right)$$

Suppose we have $nq+1$ observations $X_0, X_1, \ldots, X_{nq}$ of X_t at equidistant time intervals and define the following estimators:

$$\hat{\mu} \equiv \frac{1}{nq}\sum_{k=1}^{nq}\left(X_k - X_{k-1}\right) = \frac{1}{nq}\left(X_{nq} - X_0\right)$$

$$\overline{\sigma}_a^2 \equiv \frac{1}{nq-1}\sum_{k=1}^{nq}\left(X_k - X_{k-1} - \hat{\mu}\right)^2$$

$$\overline{\sigma}_c^2 \equiv \frac{1}{m}\sum_{k=q}^{nq}\left(X_k - X_{k-q} - q\hat{\mu}\right)^2$$

$$m = q\left(nq - q + 1\right)\left(1 - \frac{q}{nq}\right)$$

Now define the following statistic:

$$\overline{M}_r\left(q\right) \equiv \frac{\overline{\sigma}_a^2}{\overline{\sigma}_c^2} - 1$$

It can be shown that under the hypothesis H the following is valid:

$$\sqrt{nq}\overline{M}_r\left(q\right) \sim \mathrm{AsN}\left(0, \frac{2\left(2q\text{ - }1\right)\left(q\text{ - }1\right)}{3q}\right)$$

Where $X \sim \mathrm{AsN}\left(\mu, \sigma^2\right)$ means that X is asymptotically normally distributed with mean μ and variance σ^2.

The test for market efficiency is made with the following test statistic:

$$z\left(q\right) \equiv \sqrt{nq}\overline{M}_r\left(q\right)\left(\frac{2\left(2q-1\right)\left(q-1\right)}{3q}\right)^{-\frac{1}{2}} \sim \mathrm{AsN}\left(0,1\right)$$

Lo and MacKinlay (1988) also developed a test statistic where no assumption of homoskedastic increments is made. However, since randomness will be measured

on a daily basis in this thesis we will assume homoskedasticity and use $z(q)$ as test statistic.

Note on further work

There are several other ways to measure the randomness of returns that might be interesting to compare with the above mentioned method.

Another popular method was proposed by Mech (1993) and Hou and Moskowitz (2005) where they model the return $r_t = X_t - X_{t-1}$ with an unrestricted model:

$r_t = \alpha + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \beta_3 r_{t-3} + \beta_4 r_{t-4} + \varepsilon_t$ and a restricted model $r_t = \alpha + \varepsilon_t$. Market efficiency is measured as the difference $1 - \frac{R^2_{restricted}}{R^2_{unrestricted}}$ where R^2 is the OLS R^2-coefficient for each model. The closer the difference is to 0 the smaller the influence of historical prices.

4.2 Measuring liquidity

As previously mentioned HFT depends on being able to quickly move in and out of a position. Not being able to immediately exit a position at current market prices imposes a market liquidity risk.

Market liquidity is often measured using the following three criteria (Bervas, 2006):

- **The tightness of the bid-ask spread.** The bid-ask spread gives an indication of the cost of immediate reversal of a position of standard amount.
- **Market depth.** Market depth is the amount of volume one can buy or sell without moving the best price (i.e. without slippage).
- **Market resilience.** Market resilience measures the speed prices revert to equilibrium price after a significant increase in order flow.

This thesis will focus on measuring the tightness of the bid-ask spread and the market depth.

4.2.1 Measuring tightness of the spread

The bid-ask spread will be measured continuously over the trading day and is calculated as:

$$\frac{AP - BP}{BP}$$

where AP is the best ask price available and BP is the best bid price. The averaged time weighted spread will be used to measure the tightness of the spread over the day. The spread is undefined when a two-way price is missing.

4.2.2 Measuring market depth

One of the most common measures of market depth is Kyle's Lambda λ (Kyle, 1985) in the following equation:

$$\Delta P_t = \alpha + \lambda NVOL_t + \varepsilon_t$$

where ΔP_t is the price change during period t, α is some constant, $NVOL_t$ is the net trade volume during period t and ε_t is a random error term. λ gives a measure of the market's ability to absorb transactions.

However, the detail of the data available for this thesis enables a more direct measure of the market depth since we have a complete image of the order book and the liquidity available at each price level.

The liquidity of the asset will be measured using the value weighted price. Let n_t be the number of price levels at time t and $v_{i,\,t}$ and $p_{i,\,t}$ be the volume and price at price level i with $i = 1$ meaning the top price level. The value weighted price $ValueWP\left(T\right)_t$ for the value V at time t is then calculated as:

$$ValueWP\left(V\right)_t \equiv \begin{cases} \text{undefined if } \sum_{i=1}^{n_t} p_{i,\,t} v_{i,\,t} < V \\ \dfrac{\sum_{i=1}^{n_t} w\left(i,V\right) p_{i,\,t}}{V} \quad \text{otherwise} \end{cases}$$

$$w\left(k,V\right) \equiv \begin{cases} 0 \text{ if } \sum_{j=1}^{k-1} p_{j,\,t} v_{j,\,t} \geq V \\ V - \sum_{j=1}^{k-1} p_{j,\,t} v_{j,\,t} \text{ if } \sum_{j=1}^{k} p_{j,\,t} v_{j,\,t} \geq V \\ p_k v_k \text{ otherwise} \end{cases}$$

We may calculate the value weighted price for both the ask and the bid side of the order book. Furthermore we may define the value weighted spread $ValueWAS$ as:

$$ValueWAS\left(V\right) \equiv \frac{ValueWP\left(V\right)_{ask} - ValueWP\left(V\right)_{bid}}{ValueWP\left(V\right)_{bid}}$$

The $ValueWAS$ will in comparison to the regular bid-ask spread be used as a measure of the market depth. Just like the bid-ask spread the value weighted spread will be calculated continuously and will be time weighted over each trading day.

Example

If the order book is:

Bid Volume	Bid Price	Ask Price	Ask Volume
1000	10	11	1000
10000	8	12	10000

We get:

$$\begin{aligned} ValueWP\left(20000\right)_{bid} &= \frac{10+8}{2} = 9 \\ ValueWP\left(20000\right)_{ask} &= 11 \cdot \frac{11}{20} + 12 \cdot \frac{9}{20} = 11.45 \\ ValueWAS\left(20000\right) &= \frac{11.45 - 9}{9} \approx 27\% \end{aligned}$$

4.3 Measuring volatility

The bigger the price movement of an asset is during a time period the larger profits can be made if successfully being able to predict the movement. A HFT depends on that the price movement during at short time period is larger than

the transactional costs. The volatility will therefore be used as one means of determining the suitability of an asset for HFT.

The volatility σ will be measured using historical stock prices. As before, let P_t be the asset price at time t and define $X_t \equiv \ln P_t$.

Suppose we have $n+1$ observations $X_{0,}X_{1,}...,X_n$ of X_t at equidistant time intervals. We use the sample standard deviation s as the estimator of the volatility:

$$s \equiv \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(X_k - X_{k-1} - \hat{\mu}\right)^2}$$

$$\hat{\mu} \equiv \frac{1}{n}\sum_{k=1}^{n}\left(X_k - X_{k-1}\right)$$

5 Measure of asymmetric information

Asymmetric information in a market leads to adverse selection and the possibility for more informed market participants to make money at the expense of less informed traders.

Information asymmetry can be detected in a number of ways. We will use a method similar to the one called Probability of Informed Trading.

5.1 Probability of Informed Trading

Easley, Kiefer, O'Hara and Paperman (1996) suggested this model which is used to determine the probability of informed trading in an asset. The model states that the probability that an information event that is only observable to some of the traders and that affects the value of the asset occurs is α. The probability that there is no such information event is $1-\alpha$. Given that there is an information event the probability that it affects the value of the asset negatively is δ. The probability that it affects the value positively is $1-\delta$.

Trade arises from both informed and uninformed traders. Regardless of which type of day arrivals of uninformed buyers and sellers are said to follow independent Poisson processes with arrival rate ε. Here a buyer means a trader entering a trade on the bid side and being the aggressive part. A seller is defined similarly. On days when an event has occurred informed traders also arrives. Their arrival also follow Poisson processes but with the arrival rate μ. The informed traders will place orders according to their information. In the case of positive information they will buy and respectively sell if negative information. Hence, in the case of positive information there would be more people buying than selling.

Easley, Kiefer, O'Hara and Paperman show that the probability that a trade that occurs at time t is informed is given by:

$$\mathrm{PI}(t) = \frac{(1 - \mathrm{P_n}(t))\mu}{(1 - \mathrm{P_n}(t))\mu + 2\varepsilon}$$

Where $\mathrm{P_n}(t)$ is the probability of a "no event day" at time t.

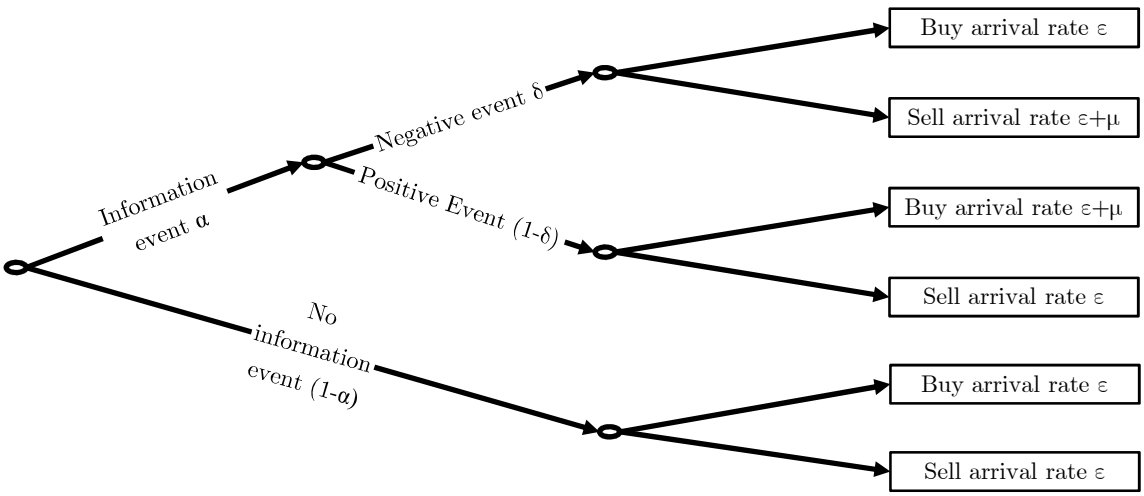

Figure 1: Tree diagram of the trading process described above.

5.2 Measuring information asymmetry

In this thesis we are not interested in determining the probability of that a trade entered at a specific time is informed on not but the level of informed trading over an entire day. As suggested by Easley et al. there will be asymmetric trading on days when an information event has occurred. I.e. the number of ask trades and bid trades will differ significantly. Hence, assuming the model described above, the natural way of measuring information asymmetry over a day is to compare the number of ask trades n_{ask} with the number of bid trades n_{bid}. In case of no informed trading we should have equal

trading, i.e. $n_{ask} \approx n_{bid}$. A natural estimator of the deviation from equal trading, and hence of information asymmetry, is given by $Assymmetry$:

$$Assymmtry = \frac{\left| \frac{n_{ask}}{n_{ask} + n_{bid}} - 0.5 \right|}{0.5}$$

Example

In an order book the number of trades where the ask side has been the aggressive part is 10. The number of trades where the bid side has been the aggressive part is 2. This would give:

$$Assymmtry = \frac{\left| \frac{10}{10 + 2} - 0.5 \right|}{0.5} \approx 67\%$$

6 Detecting HFT-traders

The previous chapters have been focusing on methods and measures of finding markets where HFT is most likely to occur. This chapter will focus on measures and methods of detecting market participants that follow a HFT strategy.

6.1 Measure of participant aggressiveness

Informed traders possess private information that isn't available to the general public and enables predictions of future price movements and spotting of market inefficiencies. This information could e.g. come from superior forecasts or superior news sources and can have a significant impact on the market. As a result informed traders, who often are HFT traders, are impatient and enter orders at or close to market price (Aldridge, 2009).

In this thesis we will measure a participant's aggressiveness as the ratio of the number of aggressive and passive trades he has made. The aggressive side of a trade is defined as the side that initiated the trade.

6.2 Measure of massive order flow and short order lifetime

One of the most revealing behaviors of HFT is the use of cancellations to extract information from the market. One example is the strategy "pinging" where the HFT trader enters an order and then immediately cancels it in order to discover hidden liquidity. Another strategy called "spoofing" consists of entering a large number of orders on one side of the order book to give the impression of liquidity and to trigger other algorithms while ultimately wanting to trade on the other side. The orders are then canceled and orders on the opposite side of the order book are entered (Cartea & Penalva, 2010).

This suggests the following methods of detecting HFT.

6.2.1 Measure of massive order flow

We will measure if a participant inserts updates or cancels a “large” amount of orders during a “short” period of time. The number of times this happens per order book will be counted.

6.2.2 Measure of short order lifetime

The median will be used to measure a market participant’s average order lifetime instead of the mean. This is to ensure a relevant measure of the average order lifetime also in the presence of non-canceled orders.

7 Data

Since we are trying to measure properties of HFT we need to use intra-day data collected at a high frequency.

The data used in this thesis consists of tick data for 12 assets traded at the Swedish exchange Burgundy over a time period of 28 days. Tick data means that the data includes all orders and trades entered over the time period. The assets have been selected on the basis of having the highest number of transactions over the chosen time period. The reason for this is to ensure the most accurate results possible. Also, HFT is most likely to appear in the most liquid assets (Aldridge, 2009).

Due to the great level of detail contained in the data it had to be anonymized before included in this thesis. This is also the reason why the names of the assets and the time period selected will not be stated in this report.

Only orders and trades entered during the continuous trading session will be considered.

The data for each market participant is collected at a market member level. There may be several different traders trading through one member and they may have different strategies and objectives. We are primarily looking HFT and are measuring properties typical of it. The behavior of HFT is so extreme that it should not be a problem discovering HFT strategies despite the "noise" from other traders.

8 Method

The scope of this thesis is to investigate some properties that are suggested by the literature to be characteristic of HFT and the assets they trade in. The relationship between these properties will be investigated. Therefore the main objective of this thesis is to measure these properties by the methods previously suggested and to check for correlations among them. An investigation will be made to see if the relationship varies between the order books and between the market participants.

The different properties are measured each day for each asset and market participant.

8.1 Estimating the price process

The methods of measuring the predictability and volatility of an asset rely in estimating the underlying price process. To get regularly spaced data the price process will be measured at the interval of 1 second. The decision to use the sampling frequency of 1 second is based on that it provides a good compromise between minimal loss of data and computational efficiency. The price of the asset will be estimated by the last trade price in that time interval.

8.2 Choosing q-value measurement of randomness of prices

Four different q-values were tested; 2, 5, 10 and 15 with similar results. For purpose of clarity only results for the q-value 10 will be shown.

8.3 Checking for significant correlations across order books

As previously mentioned we want to check for significant correlations among the suggested properties over the 12 different order books. More specifically we want to test the following hypothesis for each correlation:

H: Positive or negative correlation for all order books

I.e. we will not only test that the correlation is non-zero but that it is also either positive or negative for all order books. To test this we will use bootstrapping to determine a 95%-confidence interval for each correlation and order book. Since the standard estimator of the correlation has a skewed distribution the BC_a-method will be used (Englund, 2004). I.e. for every order book we will use bootstrapping to get confidence intervals for the correlation matrix. The confidence intervals are to check for significant correlation. This procedure is repeated for every order book to test the above hypothesis and look for significant correlations that are persistent across all order books. 1000 bootstraps per order book were used to determine the confidence intervals.

9 Results

The results from measuring the properties previously described can be found in the appendix. This chapter lists the results from investigating correlations between these properties. The results below include two different types of histograms for each market participant. The first type summarizes the correlation matrices for all order books in one picture. The second type shows the result of the hypothesis test of non-zero correlation. NR means not rejected, R means rejected and N/A means not available. The reason for the hypothesis test not being available could for instance be due to that the correlation could not be determined because the variance was zero (e.g. there were no zero liquidity events in an order book by a participant).

In addition to testing the hypothesis "positive or negative correlation for all order books" correlations which are significant over the majority of the order books (and all positive or all negative) will be highlighted.

One fact worth highlighting is that some of the market participants have been inactive (entered few orders) in some order books some days. Sometimes this is also the case for the entire market on some days and order books. This will of course make the results less reliable in these cases.

The rows and columns in the histograms correspond to following order book statistics:

- Row 1/Column 1: The number of trades in the order book.
- Row 2/Column 2: The annualized volatility in the order book.
- Row 3/Column 3: Z(10), measure of randomness of the price process in the order book.
- Row 4/Column 4: The total time the $TWAS$ has been undefined in the order book (i.e. the total time there has been no two-way price).
- Row 5/Column 5: The time weighted average spread ($TWAS$) in the order book.

- Row 6/Column 6: The total time the $TWVWAS(500000 \text{ SEK})$ has been undefined in the order book (i.e. the total time the total value on the ask and/or bid side has been less than 500 000 SEK).
- Row 7/Column 7: The time weighted and value weighted average spread for the value 500 000 SEK in the order book ($TWVWAS(500000 \text{ SEK})$).
- Row 8/Column 8: The absolute deviation from symmetric trading in the order book ($Assymmtry$).

Moreover, the rows and columns in the histograms correspond to following order book and participant specific statistics:

- Row 9/Column 9: Number of orders entered by the market participant in the order book.
- Row 10/Column 10: The median lifetime of the orders entered by the market participant in the order book. Orders that were immediately matched are excluded.
- Row 11/Column 11: The aggressive trade ratio of the market participant in the order book.
- Row 12/Column 12: The absolute deviation from symmetric trading for the market participant in the order book ($Assymmtry$).
- Row 13/Column 13: The number of liquidity bursts by the market participant in the order book. Note that this row/column only is shown for market participant 1. 2 and 3 since the other market participants never entered a liquidity burst.

9.1 Order book correlations

The sub-matrices consisting of the first eight rows and columns below pertain to statistics measured per order book. This section will focus on those statistics and correlations among them. The following sections will focus on the participant specific statistics.

9.1.1 Correlation: volatility – time without two-way spread

We see that there is significant positive correlation between the volatility and the time with no two-way spread for 11 of the order books. For order book 7 the correlation was not significant since the 95% confidence interval was -0.0004 to 0.5708. The hypothesis can thus be rejected. It is however of interest to point out that there is significant correlation for the majority of the order books. This is not a very surprising result since a tight spread is typical for a liquid order book and liquid order books should generally have more stable prices lower volatility. Another reason for the correlation is due to the way the volatility is measured and something called the bid-ask bounce. The bid-ask bounce refers to that the trade price will naturally "bounce" between the best bid and ask price depending on whether the ask or the bid side was the aggressive part of the trade. Since the trade prices are used to measure the volatility a large spread would generate a large bounce and thus generate a higher estimate of the volatility.

9.1.2 Correlation: volatility – percent undefined TWVWAS

There is significant positive correlation between volatility and the time the $TWVWAS(500000 \text{ SEK})$ has been undefined in 6 of the order books. In one order book the correlation was undefined due to that the $TWVWAS(500000 \text{ SEK})$ has was always well-defined. In six of the order books there is no significant correlation and the hypothesis is rejected. Since the undefined time of the TWVAS also is a measure of liquidity in the order book, it is not surprising that the result here is somewhat similar to the one above.

9.1.3 Correlation: volatility – TWVWAS

There is significant positive correlation between volatility and the $TWVWAS(500000 \text{ SEK})$ in six of the order books. In six of the order books there is no significant correlation and the hypothesis can be rejected. When comparing the correlation between volatility and the two different spread measures on sees that the correlation is significant in more order books when using the regular spread instead of the value weighted spread. One explanation for this is that while both are measures of the liquidity in the order book, the regular spread also captures the effect of the bid-ask bounce.

9.1.4 Correlation of liquidity measures

We also see that there is correlation between the different liquidity measures used. These are expected trivial results.

9.2 Participant 1

9.2.1 Histograms

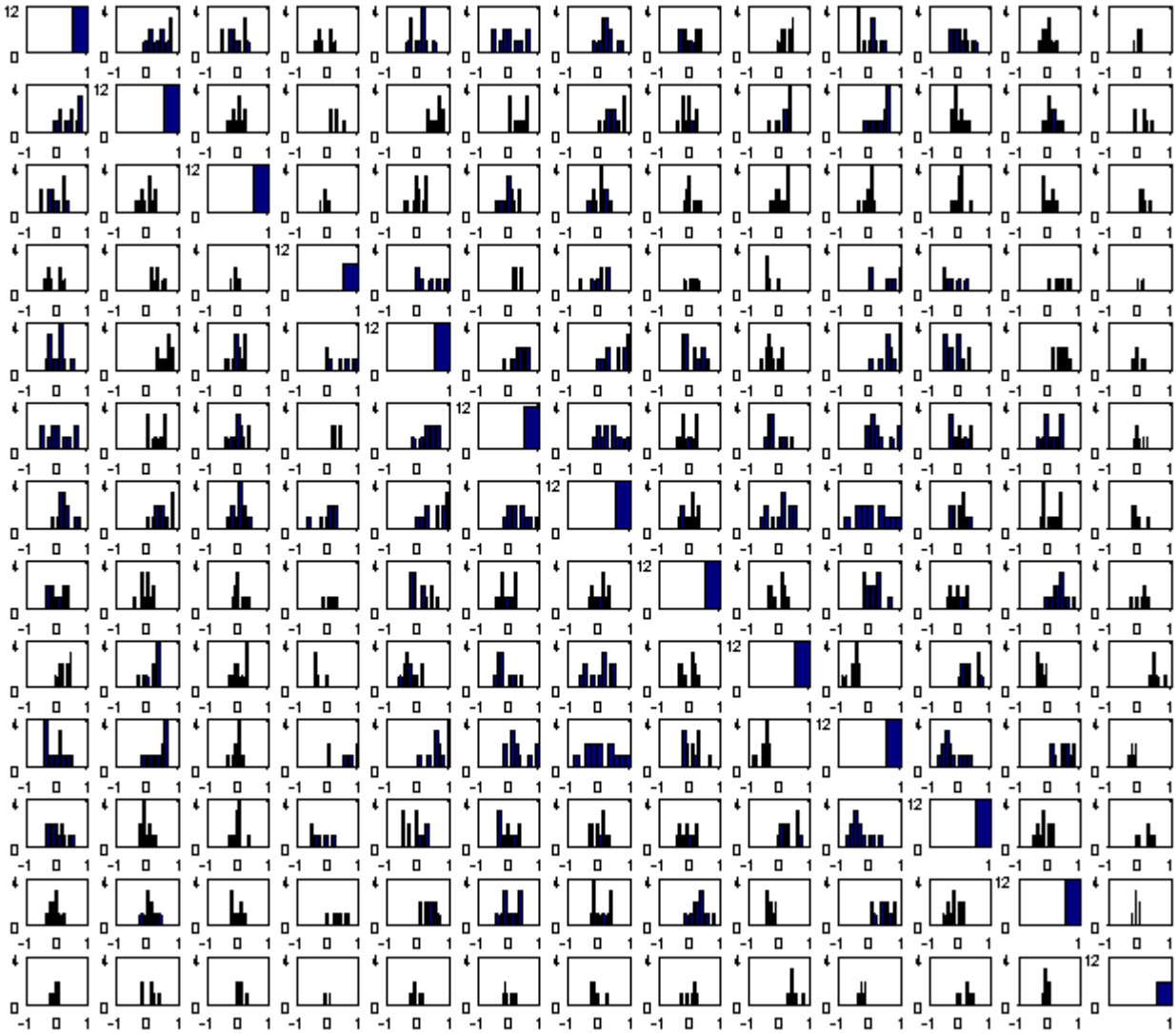

Figure 2: Summary of the correlation matrices for the 12 order books and market participant 1.

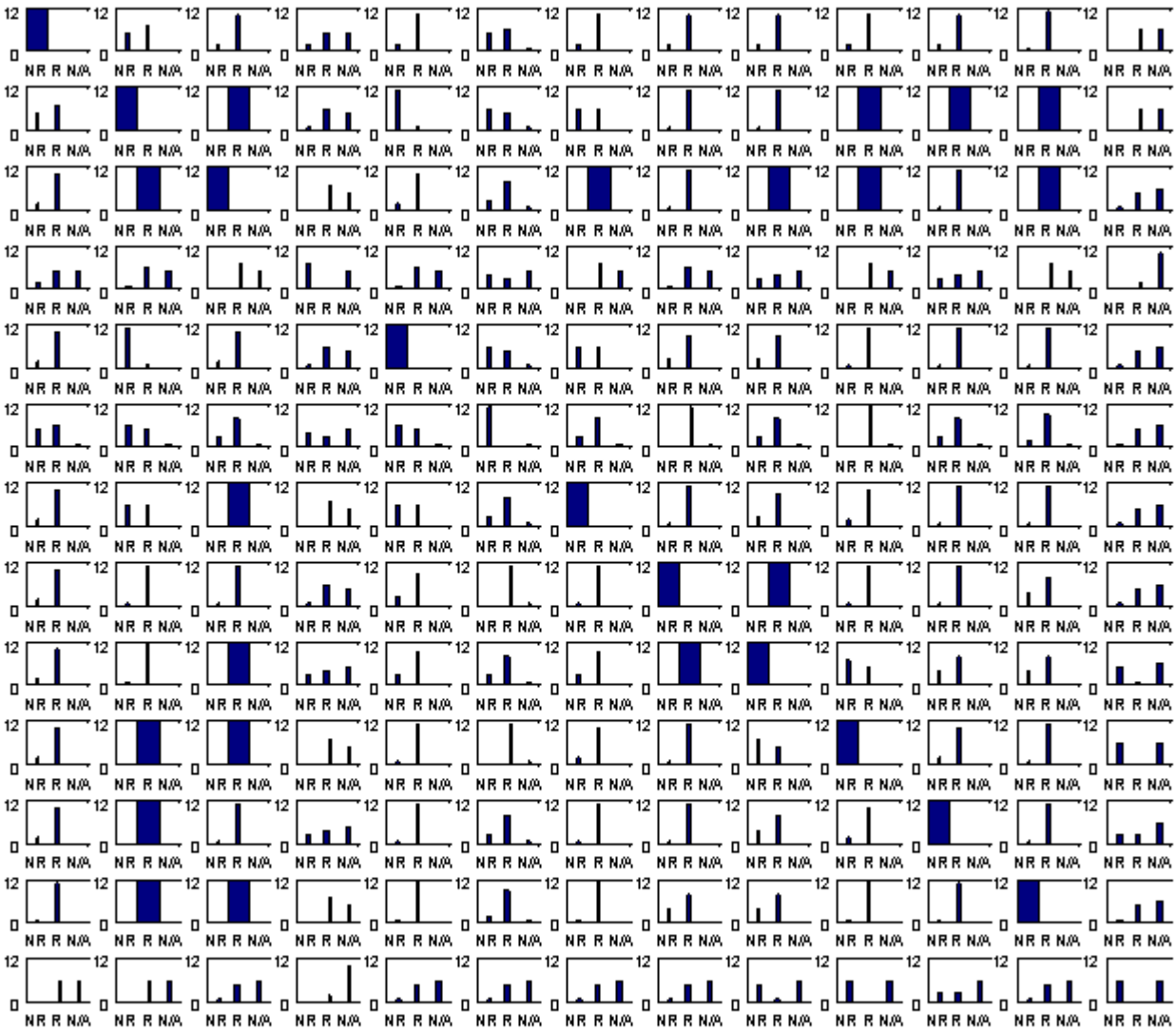

Figure 3: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 1.

9.2.2 Comments of histograms

9.2.2.1 Correlation: median order lifetime - number of liquidity bursts

We see that there was significant negative correlation between the median order lifetime and the number of liquidity bursts for six of the order books. For the other six order books the correlation could not be determined due to that the there were no zero liquidity events. The hypothesis can thus not be rejected for this correlation. This result is not surprising since entering large amounts of orders during a short period of time also requires cancelling large amounts of order during a short time period.

9.2.2.2 Correlation: number of orders – number of liquidity bursts

There was significant positive correlation between the number of orders entered and the number of liquidity bursts for five order books. For six order books the correlation could not be determined due to that there were no zero liquidity events. For one order book there was no significant correlation why the hypothesis is rejected. It is not surprising to find a significant positive correlation between the number of orders and the number of liquidity bursts since entering a number of liquidity bursts means entering a large number of orders during a short time period. In fact it is more surprising that there was one order book where the correlation was not significant. One explanation for this is that it is possible to enter a massive number of orders over a large time period, why it does not constitutes as liquidity bursts.

9.2.2.3 Correlation: number of orders – median order lifetime

There was significant negative correlation between the number of orders entered and the median order lifetime for eight order books. For four order books there was no significant correlation why the hypothesis is rejected. To find correlation between the number of orders entered and the median order life time was expected. Entering many orders over a day also require quick cancellations of many orders. Maintaining long order lifetime while continuously entering lots of orders means risking having lots of orders in the order book at some time.

9.3 Participant 2

9.3.1 Histograms

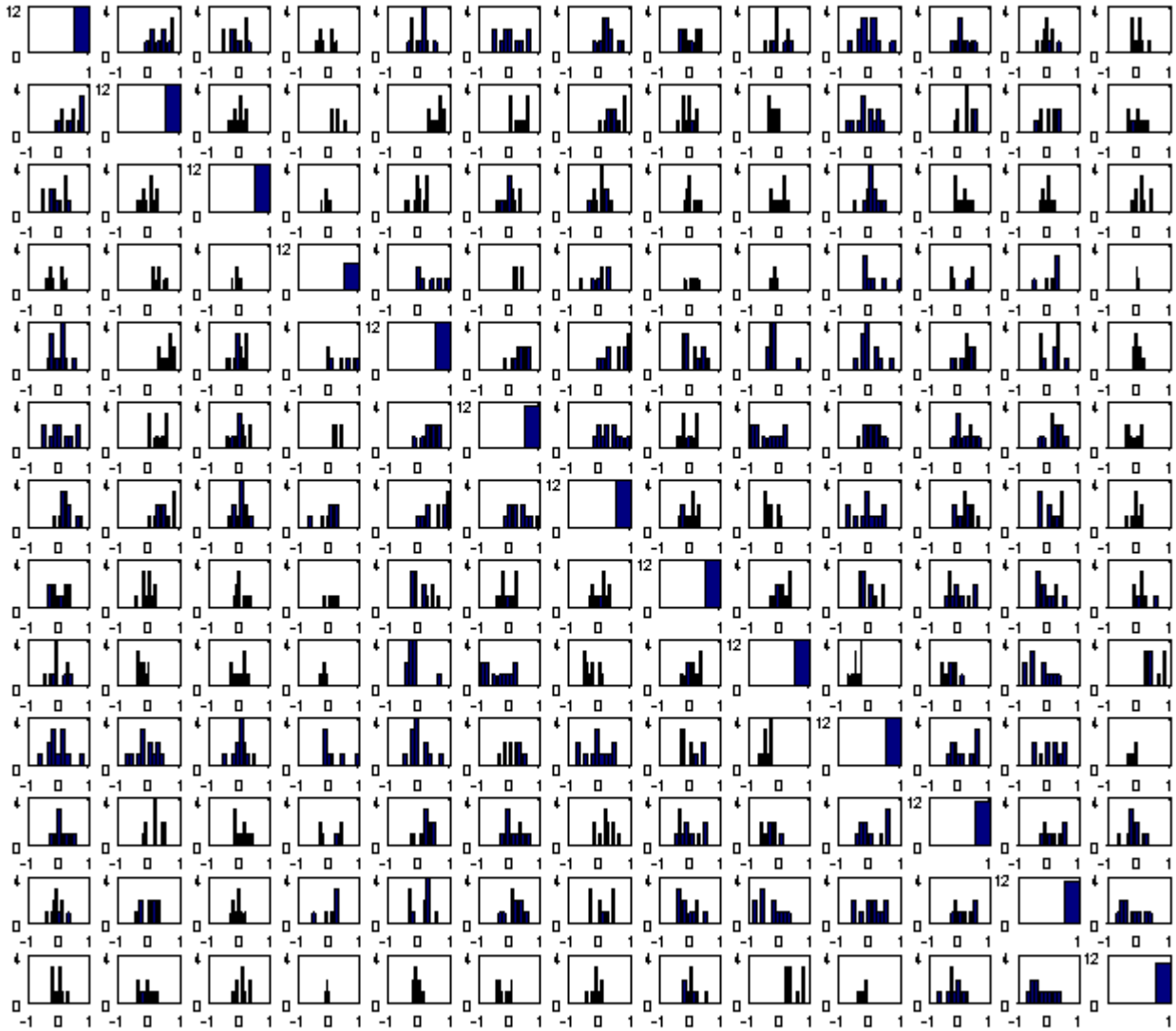

Figure 4: Summary of the correlation matrices for the 12 order books and market participant 2.

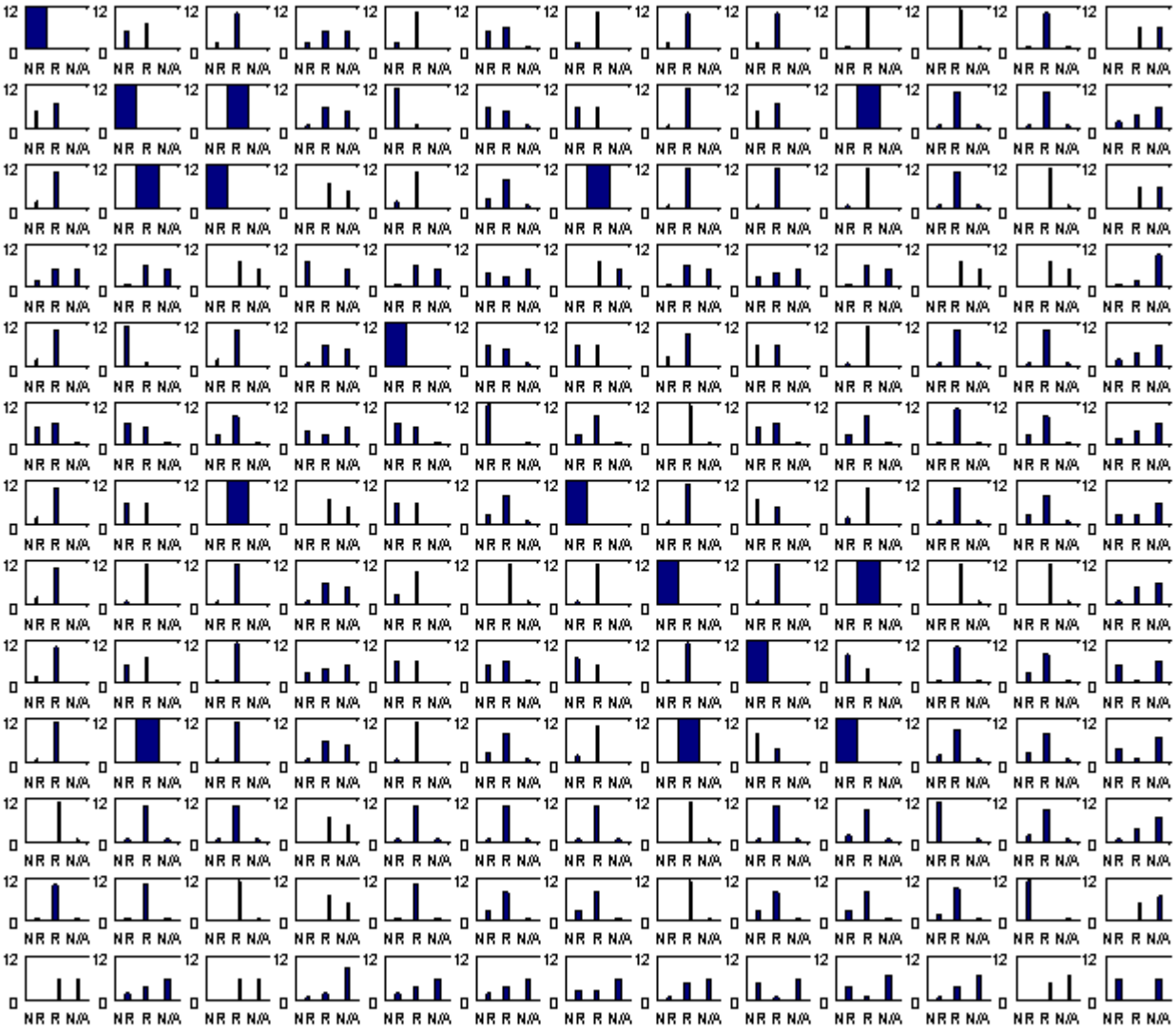

Figure 5: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 2.

9.3.2 Comments of histograms

9.3.2.1 Correlation: median order lifetime - number of liquidity bursts

There was significant negative correlation between the median order lifetime and the number of liquidity for four of the order books. For seven order books the correlation could not be determined due to that the there were no zero liquidity events. For one order book there was no significant correlation and the hypothesis can thus be rejected.

9.3.2.2 Correlation: number of orders – number of liquidity bursts

There was significant correlation between the number of orders entered and the number of liquidity bursts for five order books. For six order books the correlation could not be determined due to that there were no zero liquidity events. For one order book there was no significant correlation and because of that the hypothesis is rejected.

9.4 Participant 3

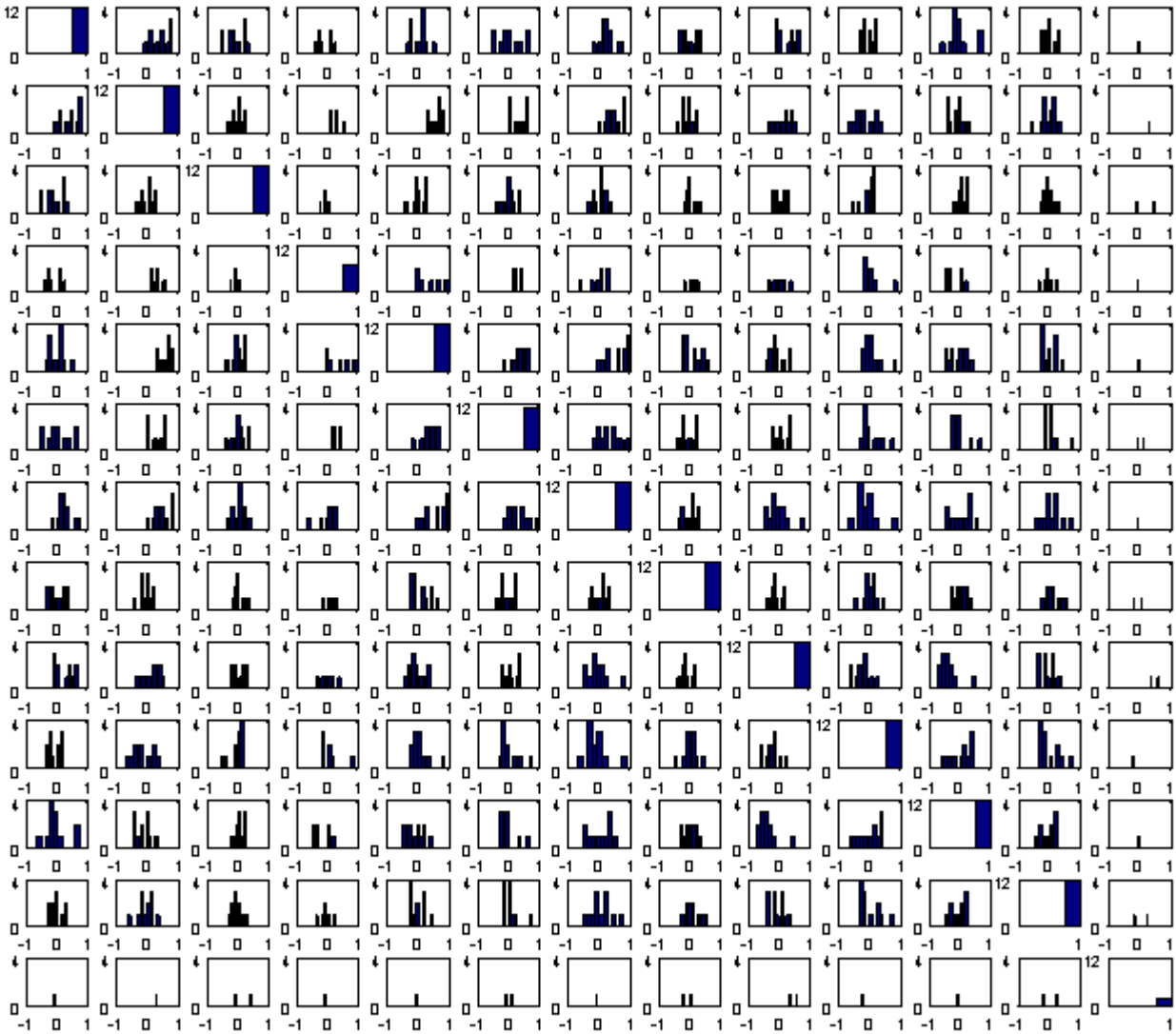

Figure 6: Summary of the correlation matrices for the 12 order books and market participant 3.

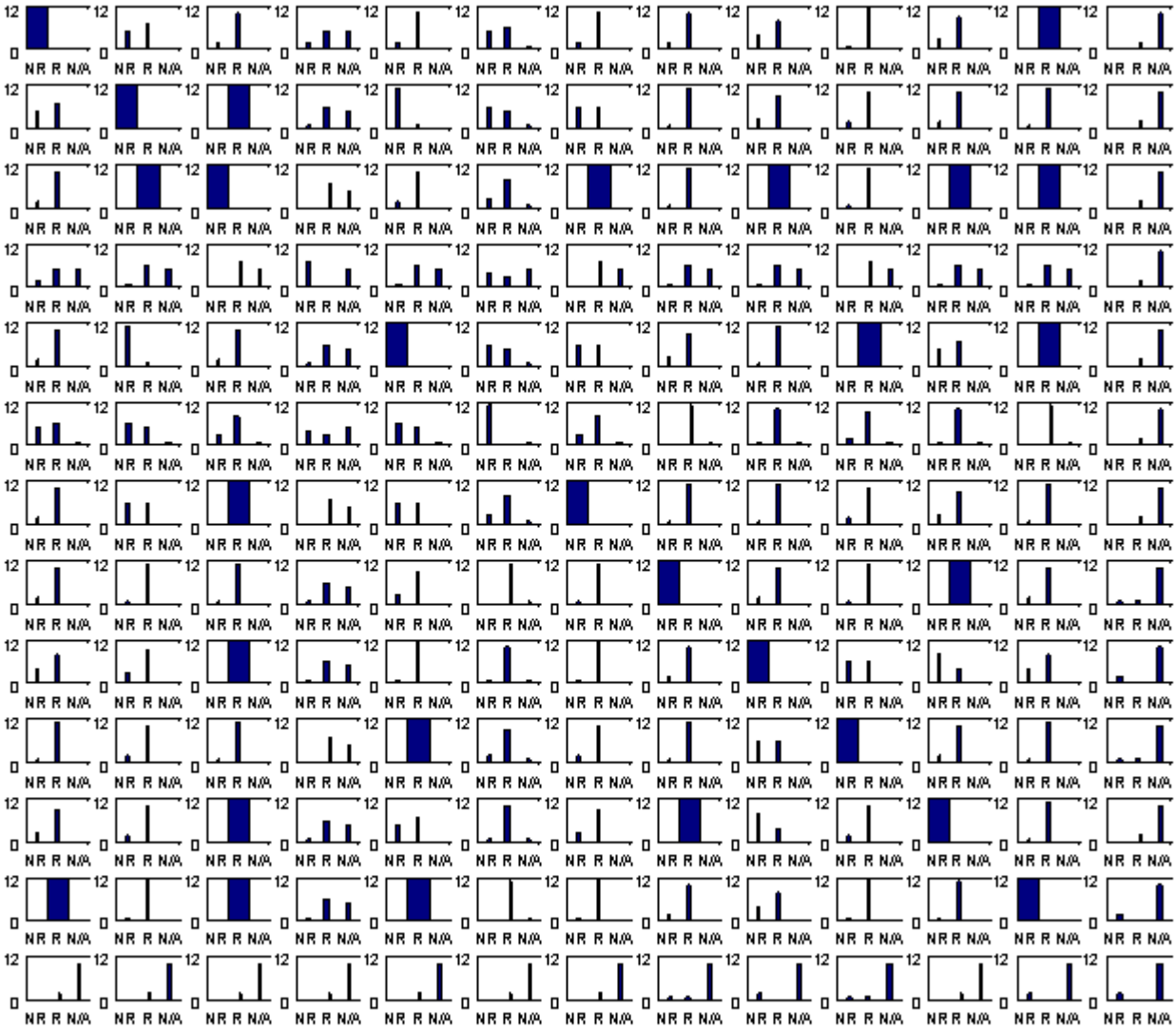

Figure 7: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 3.

9.4.1 Comments of histograms

9.4.1.1 Correlation: number of orders - number of liquidity bursts

There was significant negative correlation between the median order lifetime and the number of liquidity bursts for two of the order books. For ten of the order books the correlation could not be determined due to that the there were no zero liquidity events. The hypothesis can thus not be rejected.

9.5 Participant 4

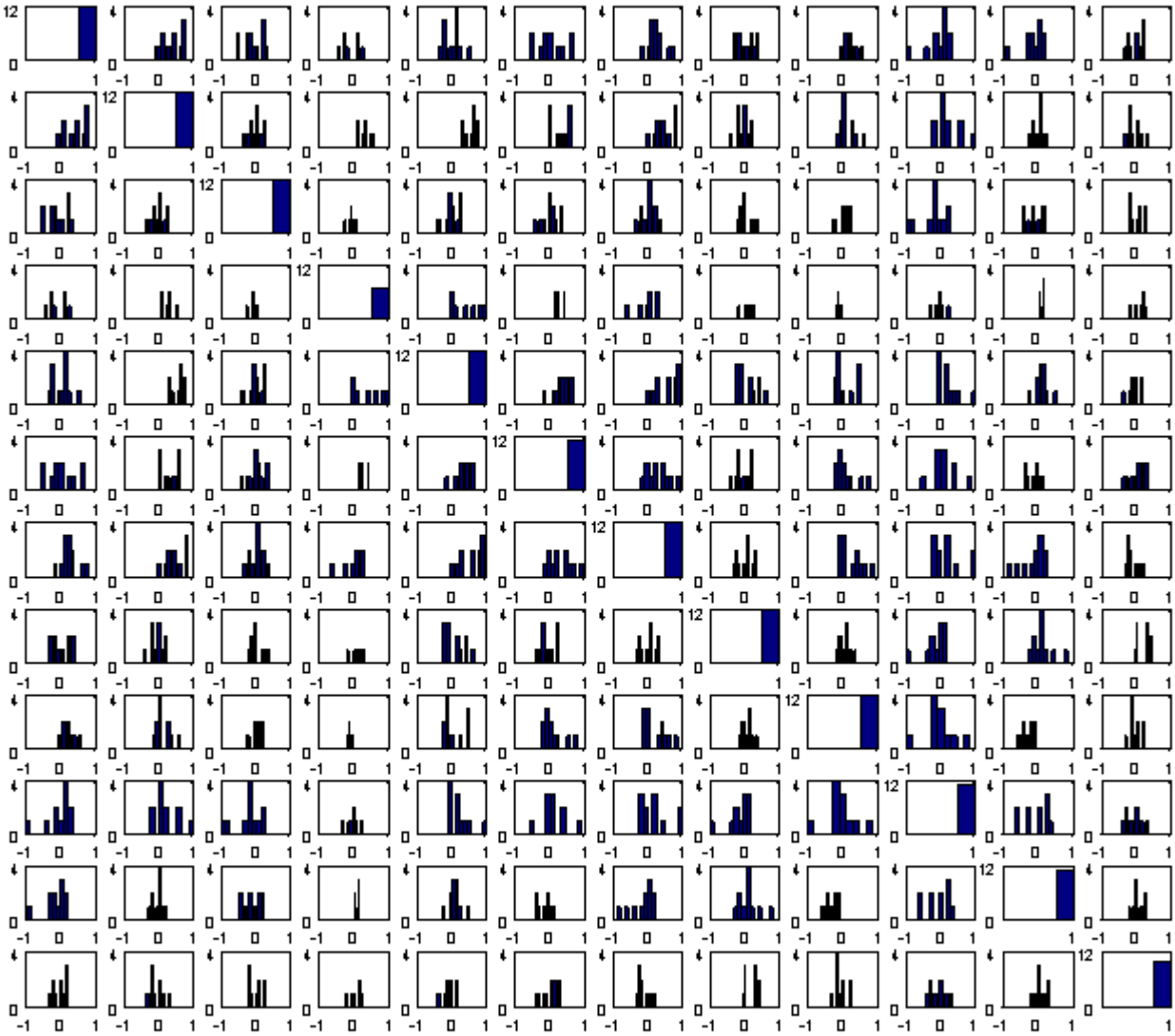

Figure 8: Summary of the correlation matrices for the 12 order books and market participant 4.

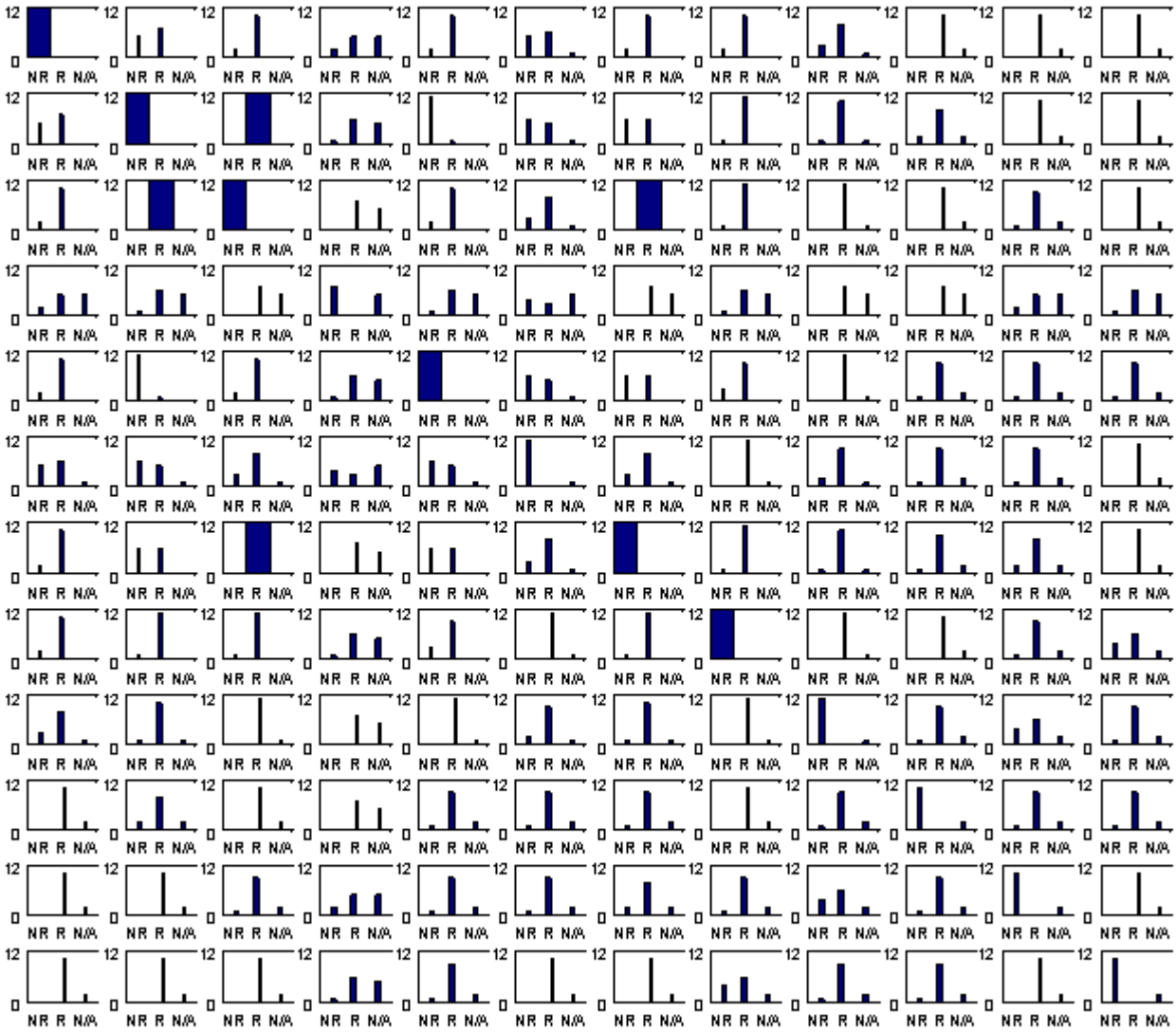

Figure 9: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 4.

9.5.1 Comments of histograms

There are no correlations for market participant 4 that are significant for a majority of the order books.

9.6 Participant 5

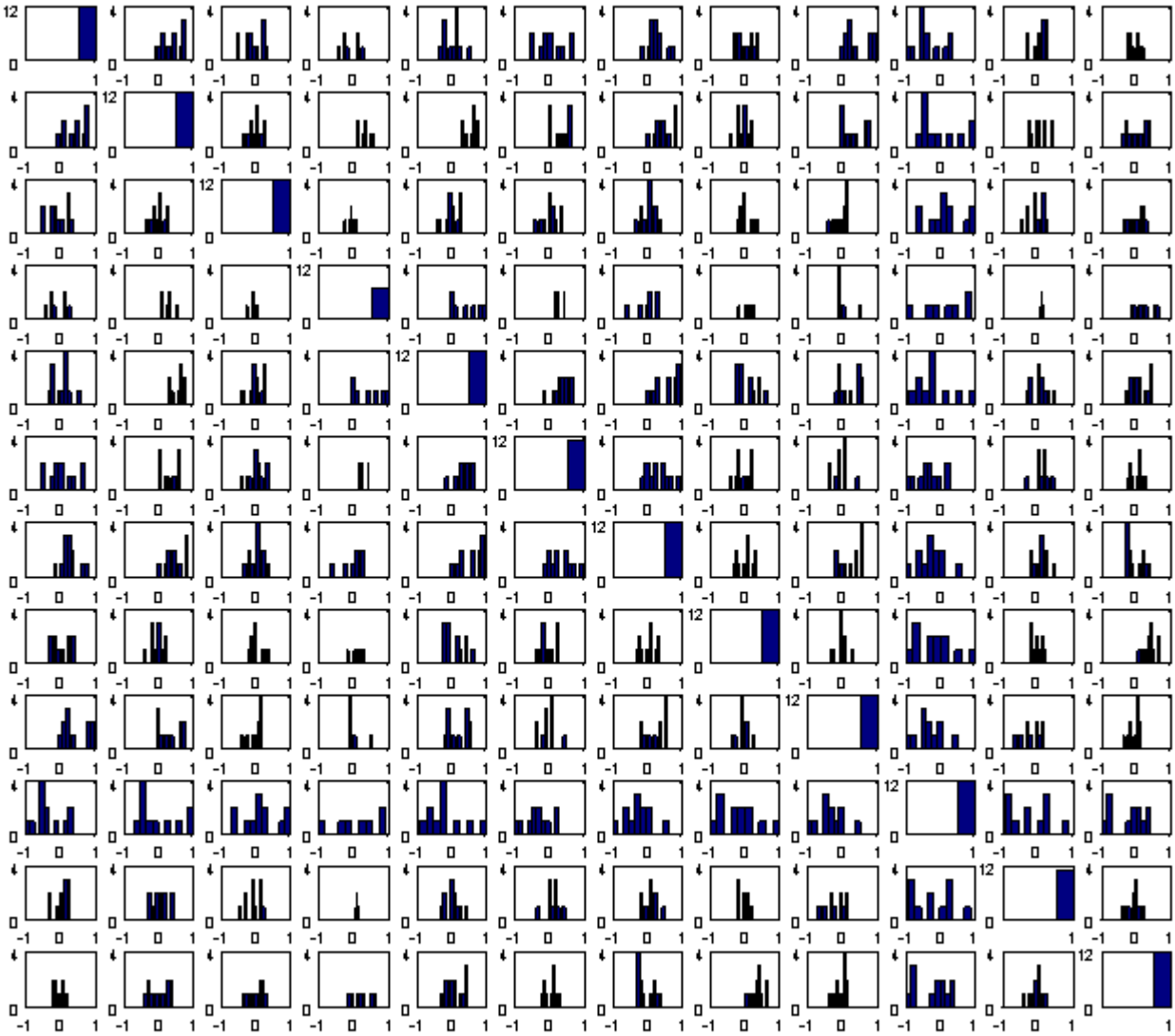

Figure 10: Summary of the correlation matrices for the 12 order books and market participant 5.

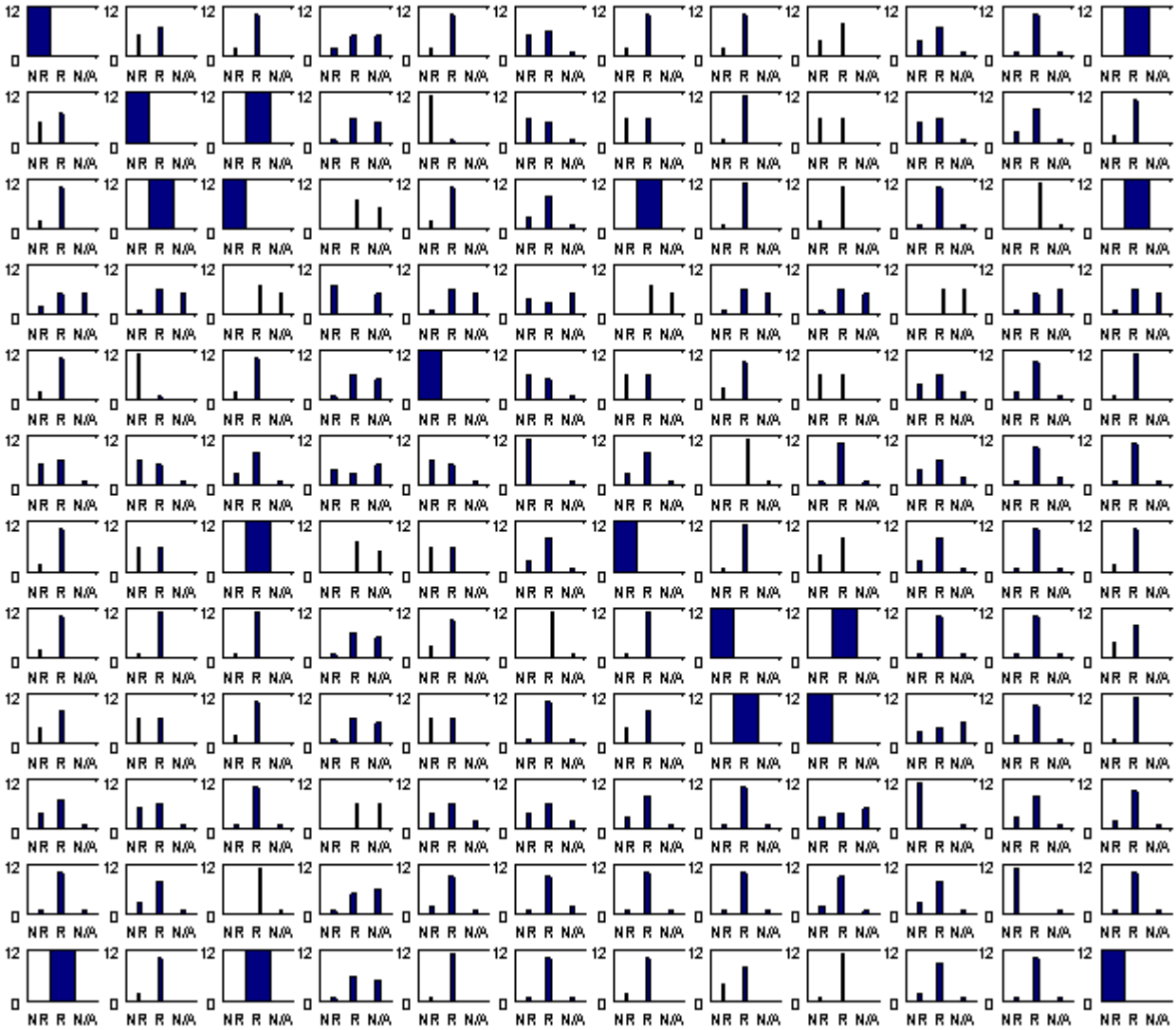

Figure 11: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 5.

9.6.1 Comments of histograms

There are no correlations for market participant 5 that are significant for a majority of the order books.

9.7 Participant 6

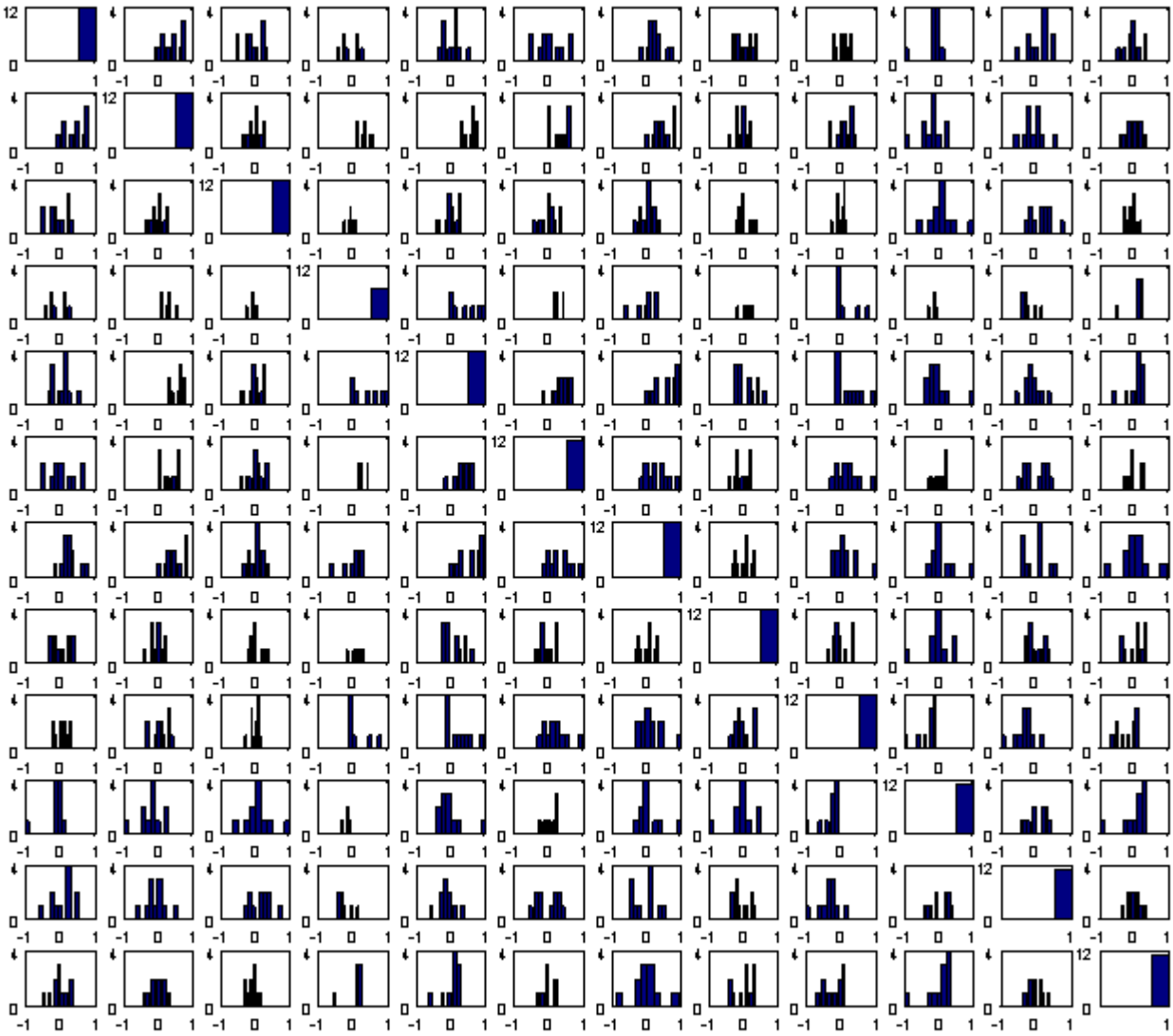

Figure 12: Summary of the correlation matrices for the 12 order books and market participant 6.

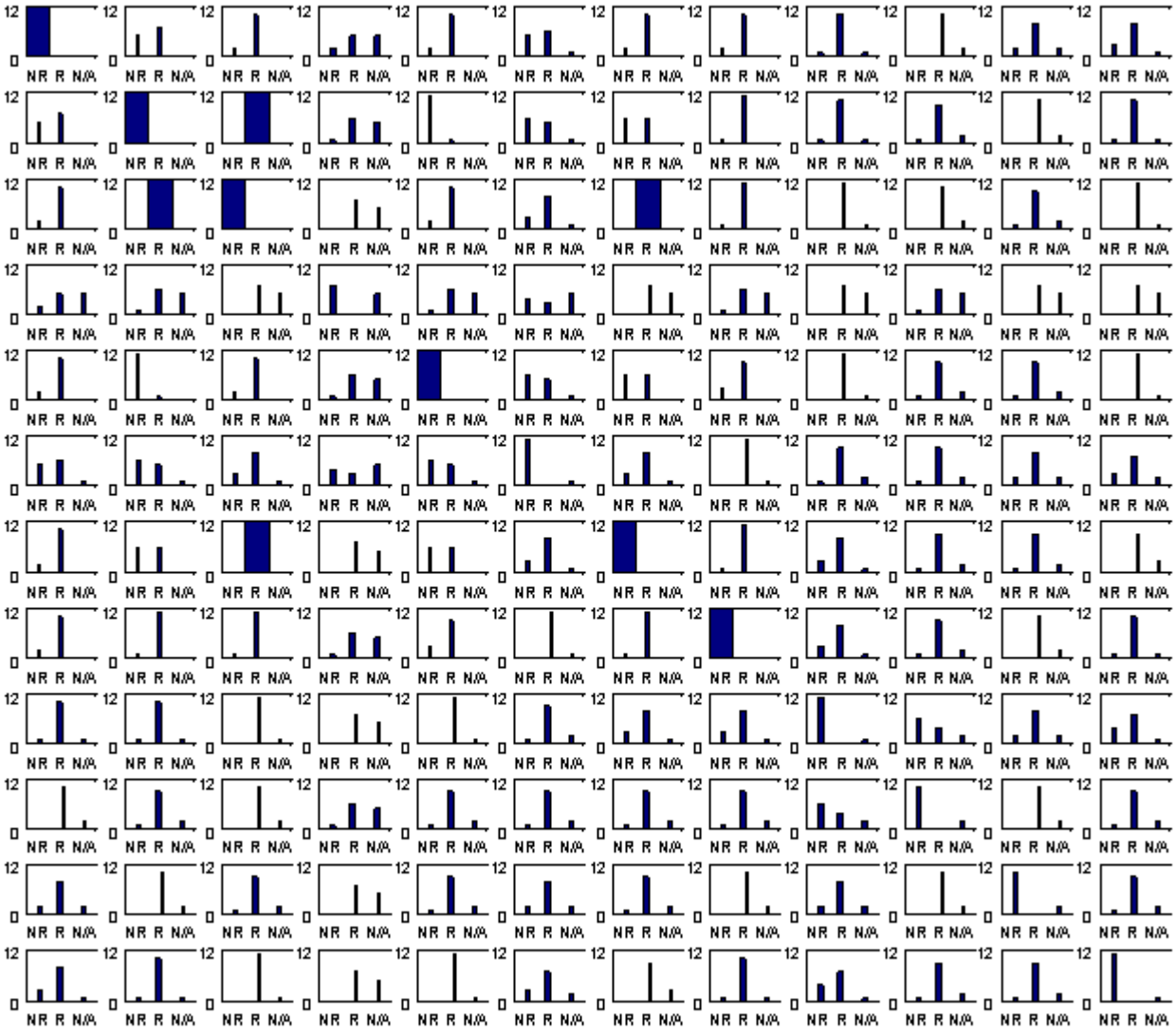

Figure 13: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 6.

9.7.1 Comments of histograms

9.7.1.1 Correlation: number of orders – median order lifetime

There was significant negative correlation between the number of orders entered and the median order life time for six of the order books. For two order books it was not possible to determine a confidence interval since there were so few days with orders. For four order books there was no significant correlation why the hypothesis is rejected.

9.8 Participant 7

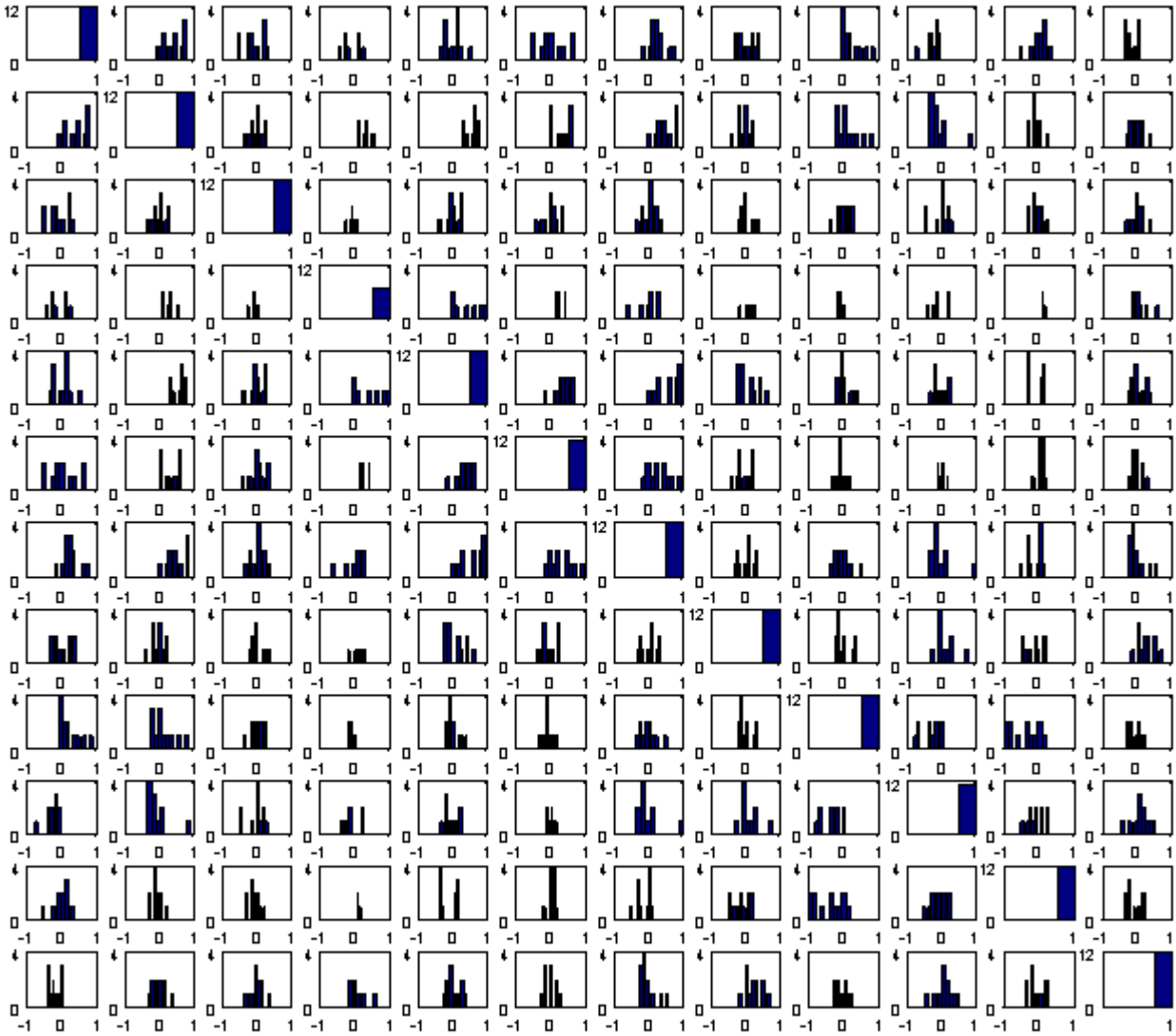

Figure 14: Summary of the correlation matrices for the 12 order books and market participant 7.

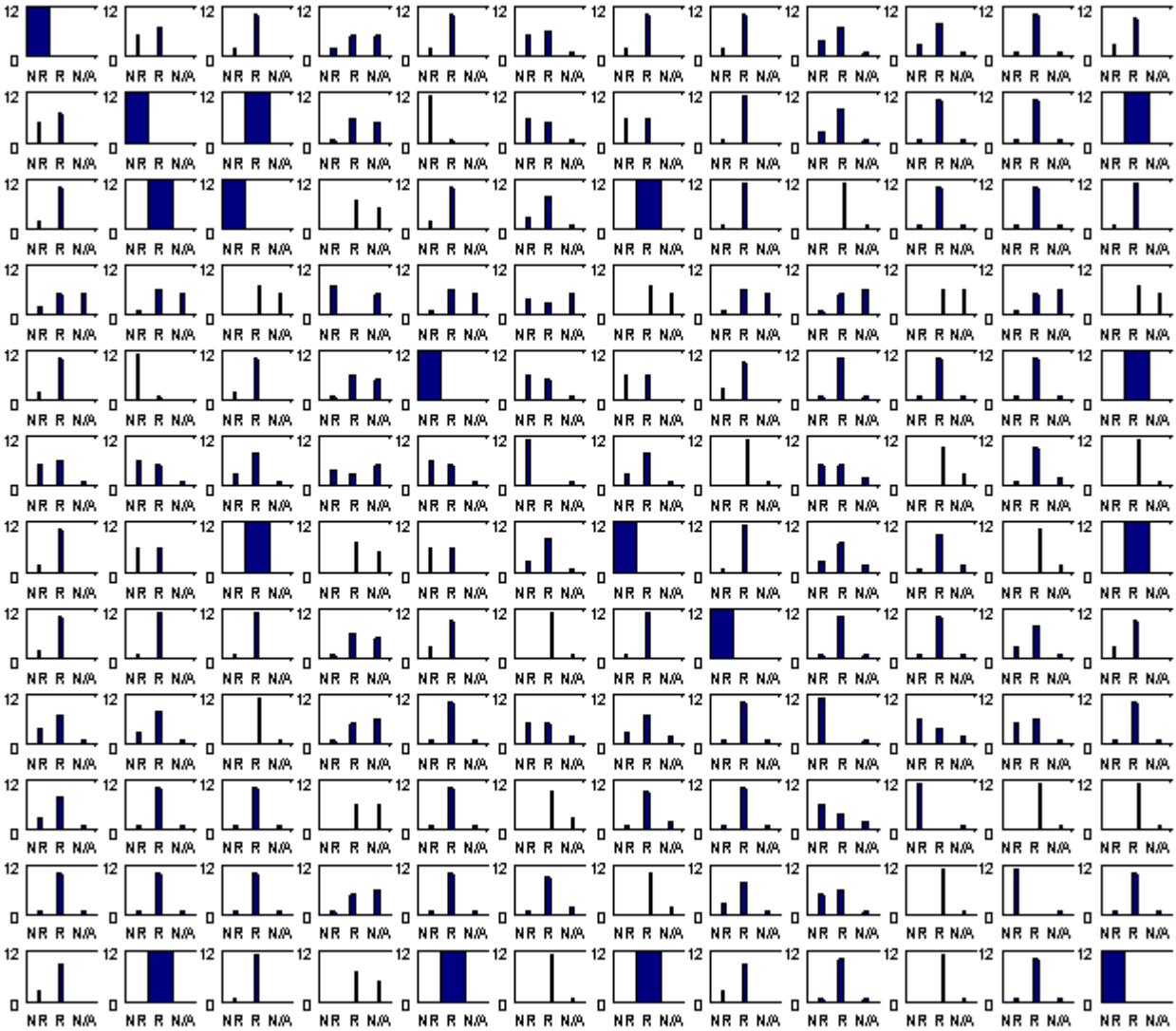

Figure 15: Summary of the hypothesis test "non-zero correlation" for the 12 order books and market participant 7.

9.8.1 Comments of histograms

9.8.1.1 Correlation: number of orders – median order lifetime

There was significant negative correlation between the number of orders entered and the median order life time for six of the order books. For two order books it was not possible to determine a confidence interval since there were so few days with orders. For four order books there was no significant correlation why the hypothesis is rejected.

10 Conclusion

In this thesis an investigation of different properties suggested by the literature to be typical of HFT was made. Methods to measure these properties were developed and described. The methods were run on historical data to determine if there were any significant correlations between them that were persistent across all order books.

Some correlations were found to be significant in a majority of the order books but none was found to be significant in all order books. In some cases this was due to that the correlation or confidence interval of it couldn't be estimated. In other cases it was due to the correlation being non-significant for some order books.

Most of the correlations that were significant in a majority of the order books were expected. One example is the correlation between the number of orders entered by a market participant and the number of liquidity events. One correlation that was not perhaps as obvious was the one between volatility and the different liquidity measures. This indicates that the volatility tend to be greater on days with less liquidity and wider spreads. Part of this result was deuced to the way the volatility is measured and the bid-ask bounce but not all since several different types of liquidity measures with similar.

The main result is however that there were few significant correlations between the measured properties when looking across all order books and market participants. This indicates that the trading behavior, despite different trading strategies, of market participants does not depend on the current properties of the market, at least not the ones measured.

10.1 Notes on further work

As noted above some of the market participants were quite inactive on some days in some order books. It would be interesting to rerun the tests on data where all market participants were active across all order books and over the

entire time period. It would also be of interest to use market participant data gathered on a trader level instead of on a market member level. The methods could also be run on several time periods to see if the results are persistent over time. Alternative methods could be used to measure the properties for comparison.

11 Appendix

This chapter lists the measured order book properties. In order to limit the number of tables in this chapter, only properties of the most active participant, participant 1, is listed. Most active refers to that participant 1 was the one who entered the most orders in all of the order books.

A summary of each property can be found in chapter 9.

11.1 Order book data

11.1.1 Order book 1

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	139	18.46%	1.4198	1.01%	0.35%	97.28%	0.61%	20.86%
2	308	24.63%	-1.342	0.09%	0.54%	87.45%	0.18%	11.69%
3	308	30.78%	0.5126	0.06%	0.53%	76.97%	1.43%	6.49%
4	388	33.34%	-0.156	1.07%	1.04%	87.81%	0.64%	2.30%
5	22	34.61%	0.0333	64.73%	8.04%	99.06%	0.89%	16.67%
6	393	28.53%	-1.212	58.30%	0.21%	81.91%	0.51%	21.52%
7	243	24.45%	-0.323	2.86%	0.18%	69.09%	0.15%	18.85%
8	161	44.16%	0.1397	2.14%	0.21%	75.19%	0.14%	54.04%
9	797	32.30%	-2.346	1.05%	0.17%	14.50%	1.14%	51.57%
10	320	27.66%	-0.258	1.03%	0.41%	64.64%	1.20%	5.30%
11	244	23.54%	-1.49	1.00%	0.14%	2.35%	0.39%	2.40%
12	164	28.00%	0.1829	0.04%	0.51%	26.33%	1.61%	13.41%
13	153	29.67%	0.0087	30.36%	0.23%	97.32%	2.45%	33.33%
14	139	33.64%	0.4542	0.95%	1.43%	99.34%	1.70%	0.72%
15	190	35.10%	-1.275	1.03%	0.13%	32.27%	0.31%	16.23%
16	339	35.44%	0.1607	0.02%	0.63%	90.40%	0.85%	8.55%
17	451	24.69%	-0.486	0.04%	0.43%	78.64%	1.22%	54.77%
18	352	24.16%	0.662	1.03%	0.11%	34.55%	0.43%	49.58%
19	355	26.63%	-0.135	0.99%	0.11%	35.83%	0.36%	62.25%
20	149	9.08%	0.0102	1.00%	0.10%	45.25%	0.13%	36.36%
21	311	18.61%	-0.552	0.02%	0.10%	15.49%	0.24%	2.89%
22	262	19.24%	-0.499	0.03%	0.15%	1.72%	0.38%	1.52%
23	736	20.90%	-3.83	0.96%	0.11%	3.89%	0.17%	45.11%
24	298	29.42%	-0.286	0.55%	0.11%	10.14%	0.17%	5.33%
25	346	25.13%	0.1948	0.13%	0.16%	5.69%	0.33%	9.83%
26	700	24.37%	0.6672	0.00%	0.10%	1.19%	0.14%	43.02%
27	345	23.40%	-0.318	0.03%	0.12%	2.82%	0.43%	17.03%
28	474	19.85%	-0.692	0.40%	0.12%	3.32%	0.20%	12.55%

11.1.2 Order book 2

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	1181	19.43%	-31.24	0.00%	0.07%	0.00%	0.23%	8.79%
2	1451	39.66%	0.3252	0.00%	0.10%	0.00%	0.36%	6.87%
3	2203	56.39%	-3.352	0.00%	0.12%	0.00%	0.53%	7.76%
4	1804	55.69%	0.3348	0.00%	0.14%	0.00%	0.63%	4.58%
5	1891	72.96%	-0.483	0.00%	0.23%	0.00%	0.78%	13.40%
6	1636	47.27%	-0.635	0.00%	0.11%	0.00%	0.38%	1.78%
7	1194	41.27%	-3.162	0.00%	0.11%	0.00%	0.40%	9.92%
8	789	44.82%	-0.798	0.00%	0.12%	0.00%	0.43%	12.29%
9	742	42.83%	-0.384	0.00%	0.20%	0.00%	0.55%	14.02%
10	919	40.66%	-2.194	0.00%	0.15%	0.37%	0.50%	12.57%
11	753	35.00%	1.6674	0.00%	0.12%	0.00%	0.41%	1.06%
12	1382	47.72%	-0.628	0.00%	0.17%	0.00%	0.56%	7.09%
13	1840	51.69%	-0.798	0.00%	0.14%	0.14%	0.59%	6.59%
14	1431	67.89%	-1.087	0.00%	0.20%	0.00%	0.66%	17.58%
15	924	42.22%	-1.159	0.00%	0.16%	0.13%	0.79%	9.62%
16	980	42.38%	0.379	0.00%	0.13%	0.48%	0.50%	1.73%
17	1222	27.45%	-1.192	0.00%	0.09%	0.00%	0.44%	19.97%
18	866	37.19%	-10.29	0.00%	0.13%	0.00%	0.30%	28.46%
19	1049	33.16%	1.7577	0.00%	0.10%	0.00%	0.24%	5.90%
20	331	20.51%	1.2002	0.00%	0.09%	0.00%	0.27%	0.90%
21	1314	36.93%	2.9935	0.00%	0.14%	0.47%	0.32%	18.64%
22	1336	35.46%	-1.192	0.00%	0.11%	0.00%	0.29%	7.19%
23	937	24.69%	-1.115	0.00%	0.09%	0.00%	0.24%	4.62%
24	1123	30.08%	1.752	0.00%	0.10%	0.00%	0.32%	4.72%
25	1382	36.48%	-0.711	0.00%	0.10%	0.00%	0.34%	8.16%
26	1004	43.41%	1.1323	0.00%	0.12%	0.00%	0.45%	22.91%
27	738	40.31%	1.4734	0.00%	0.14%	0.00%	0.54%	9.26%
28	800	27.57%	1.3016	0.00%	0.13%	0.00%	0.43%	27.02%

11.1.3 Order book 3

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	465	19.00%	-0.69	0.97%	0.05%	2.71%	0.09%	29.89%
2	839	17.72%	0.0731	0.32%	0.06%	2.09%	0.11%	4.02%
3	1219	19.40%	0.3866	0.04%	0.11%	7.15%	0.11%	2.21%
4	1294	28.61%	0.864	1.02%	0.06%	8.16%	0.12%	11.13%
5	662	38.16%	-1.442	1.09%	0.28%	85.57%	2.60%	2.26%
6	696	25.80%	-0.972	6.75%	0.06%	18.71%	0.11%	13.92%
7	308	21.71%	0.6002	0.33%	0.07%	10.16%	0.13%	11.69%
8	185	28.97%	-0.12	2.13%	0.08%	9.57%	0.14%	42.70%
9	280	32.22%	0.2016	1.03%	0.07%	5.40%	0.14%	3.57%
10	296	32.11%	2.6413	1.00%	0.08%	3.92%	0.16%	16.22%
11	117	16.88%	1.2282	0.98%	0.09%	6.41%	0.15%	22.31%
12	189	31.47%	0.1018	0.04%	0.11%	4.63%	0.17%	8.42%
13	301	30.42%	-0.261	0.98%	0.12%	18.97%	0.16%	35.55%
14	531	30.70%	0.0481	0.30%	0.10%	21.02%	0.18%	14.49%

15	195	27.74%	-0.048	1.03%	0.13%	2.67%	0.20%	8.72%
16	342	28.57%	-0.351	0.06%	0.10%	7.65%	0.20%	1.16%
17	244	18.33%	0.5991	1.01%	0.07%	2.63%	0.12%	52.65%
18	320	21.18%	1.4698	1.00%	0.05%	2.53%	0.11%	19.00%
19	135	20.13%	1.3881	0.86%	0.06%	2.37%	0.13%	12.59%
20	341	8.92%	-1.082	1.00%	0.05%	2.23%	0.10%	41.35%
21	688	23.71%	-0.998	0.93%	0.32%	13.43%	0.17%	22.66%
22	421	21.46%	0.9692	1.00%	0.07%	11.49%	0.22%	42.52%
23	567	20.84%	0.199	0.97%	0.06%	8.00%	0.27%	28.40%
24	542	24.61%	0.6332	0.96%	0.06%	6.58%	0.19%	11.43%
25	350	24.27%	0.4061	1.00%	0.07%	5.16%	0.19%	6.29%
26	587	27.60%	-0.039	0.69%	0.07%	1.60%	0.31%	5.96%
27	402	20.48%	1.3819	0.11%	0.07%	3.42%	0.17%	26.96%
28	323	21.28%	0.0802	0.41%	0.06%	2.22%	0.15%	48.61%

11.1.4 Order book 4

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	139	25.21%	-0.046	1.41%	1.49%	99.85%	0.24%	16.90%
2	768	60.39%	0.1386	0.03%	1.15%	87.56%	3.73%	9.11%
3	491	47.81%	0.2816	0.05%	1.03%	98.72%	4.41%	28.19%
4	227	48.49%	0.0323	1.07%	1.79%	100.00%		29.52%
5	135	67.42%	-0.368	72.17%	2.16%	100.00%		36.30%
6	203	35.86%	0.297	85.16%	0.11%	100.00%		31.03%
7	197	33.07%	0.5729	34.76%	0.17%	100.00%		1.01%
8	89	43.33%	-0.159	2.45%	1.01%	100.00%		1.12%
9	231	29.40%	0.2138	1.04%	1.14%	98.96%	3.14%	4.76%
10	216	31.02%	-0.825	0.79%	1.63%	100.00%		2.30%
11	235	30.92%	-1.203	0.99%	0.70%	100.00%		31.09%
12	231	45.95%	0.6155	0.03%	1.69%	99.74%	6.34%	18.30%
13	206	49.69%	-0.544	23.41%	1.44%	100.00%		1.92%
14	238	57.64%	0.0466	0.94%	2.39%	100.00%		26.14%
15	240	37.70%	2.0465	1.02%	0.88%	100.00%		12.03%
16	382	36.57%	0.0038	0.06%	1.11%	100.00%		20.52%
17	170	25.28%	0.0451	0.97%	0.96%	100.00%		4.71%
18	137	35.32%	0.0241	1.02%	1.04%	100.00%		9.49%
19	270	29.27%	0.3216	0.96%	0.20%	100.00%		22.79%
20	71	15.30%	0.0312	0.98%	0.34%	100.00%		15.49%
21	176	33.19%	0.0443	0.93%	0.29%	100.00%	3.31%	5.68%
22	569	30.45%	0.8042	0.99%	0.16%	44.02%	3.79%	47.02%
23	288	18.07%	1.3488	0.94%	0.15%	98.06%	3.87%	26.14%
24	428	46.42%	0.3035	1.00%	0.33%	77.10%	3.47%	31.98%
25	267	30.43%	0.6237	0.99%	0.24%	98.86%	4.96%	16.61%
26	414	36.67%	0.8638	1.00%	0.35%	100.00%		6.51%
27	283	30.52%	-0.183	0.06%	0.70%	100.00%		35.21%
28	475	33.43%	-0.414	0.39%	0.23%	99.51%	3.69%	44.54%

11.1.5 Order book 5

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	173	17.72%	-0.523	1.03%	0.20%	81.44%	2.11%	5.20%
2	564	27.33%	-0.425	0.86%	0.35%	53.19%	0.99%	11.86%
3	415	34.88%	-0.317	0.06%	0.42%	45.99%	1.92%	17.11%
4	304	36.65%	-1.382	1.06%	0.92%	94.86%	0.32%	2.63%
5	28	50.26%	0.1838	36.58%	2.59%	99.87%	0.40%	7.14%
6	311	37.78%	-0.55	2.82%	1.91%	94.97%	0.43%	11.25%
7	146	27.61%	-1.452	12.04%	1.42%	81.71%	1.67%	6.85%
8	130	26.23%	-0.04	1.54%	0.13%	97.14%	0.24%	33.85%
9	207	26.21%	0.2746	0.13%	0.18%	89.77%	0.38%	43.00%
10	295	28.61%	-0.302	0.13%	0.16%	99.78%	0.79%	20.68%
11	196	25.38%	-1.03	0.12%	0.16%	64.79%	0.54%	36.73%
12	214	35.49%	-0.051	0.03%	0.62%	90.27%	0.53%	17.76%
13	303	34.51%	-0.735	0.99%	0.43%	83.97%	0.89%	4.29%
14	268	46.28%	-2.444	0.03%	0.19%	72.71%	0.60%	7.46%
15	301	33.92%	0.0557	0.03%	0.19%	78.86%	0.62%	9.27%
16	212	29.26%	0.0656	0.06%	0.33%	90.98%	0.73%	5.66%
17	192	22.24%	0.9155	0.47%	0.35%	94.55%	0.42%	17.53%
18	283	26.06%	-1.659	0.06%	0.16%	66.17%	0.38%	6.71%
19	232	25.68%	0.0692	0.85%	0.13%	56.77%	0.42%	2.59%
20	137	14.83%	-0.208	0.79%	0.11%	30.79%	0.43%	3.65%
21	194	24.71%	-0.5	0.92%	0.16%	41.67%	0.44%	6.19%
22	295	25.38%	-1.054	0.86%	0.12%	15.29%	0.36%	16.22%
23	232	20.40%	-6E-04	0.43%	0.12%	7.94%	0.39%	17.24%
24	176	31.06%	0.2915	0.94%	0.19%	8.52%	0.73%	4.55%
25	270	24.82%	-0.296	0.09%	0.21%	58.64%	0.29%	23.99%
26	206	28.06%	0.1639	0.68%	0.15%	93.87%	0.24%	14.56%
27	134	21.98%	-1.12	0.07%	0.22%	75.69%	0.27%	28.47%
28	415	27.63%	0.0112	0.01%	0.11%	44.38%	0.41%	32.53%

11.1.6 Order book 6

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	138	25.32%	1.0354	1.58%	0.14%	100.00%	0.34%	7.25%
2	295	34.33%	0.4888	0.98%	0.15%	100.00%		50.51%
3	322	43.97%	-0.429	0.97%	0.15%	99.97%	0.34%	9.94%
4	243	43.19%	1.0908	1.01%	0.16%	97.99%	0.20%	1.23%
5	474	67.61%	-0.018	1.06%	0.91%	100.00%		25.47%
6	370	42.29%	-0.138	9.50%	0.14%	100.00%		40.00%
7	296	29.76%	0.5822	0.48%	0.18%	100.00%		34.46%
8	274	46.25%	-0.229	2.15%	0.13%	100.00%		9.49%
9	378	34.96%	0.8336	1.00%	0.17%	100.00%	1.11%	4.76%
10	235	41.32%	0.2593	0.13%	0.18%	100.00%	0.49%	37.82%
11	80	21.65%	-0.243	0.98%	0.16%	100.00%		12.50%
12	172	37.74%	0.3966	0.05%	0.16%	100.00%		24.42%

13	178	46.91%	-0.32	1.22%	0.19%	100.00%		11.11%
14	307	50.23%	0.5611	0.25%	0.17%	100.00%		22.48%
15	190	34.39%	0.1572	0.72%	0.19%	100.00%	0.75%	40.21%
16	251	35.97%	-0.746	0.08%	0.25%	99.48%	0.30%	27.49%
17	113	29.52%	0.0193	2.16%	0.13%	100.00%		20.35%
18	95	29.93%	-0.63	1.21%	0.16%	100.00%		32.63%
19	102	26.66%	-5.572	0.95%	0.16%	71.24%	0.31%	3.92%
20	78	14.82%	0.0294	0.96%	0.12%	33.03%	0.26%	25.64%
21	93	31.37%	-1.974	0.94%	0.16%	36.69%	0.32%	11.83%
22	155	26.66%	0.2189	0.98%	0.14%	22.07%	0.35%	22.58%
23	80	22.03%	-1.054	0.96%	0.13%	22.11%	0.28%	35.00%
24	97	27.59%	-0.003	0.95%	0.16%	32.28%	0.32%	30.61%
25	111	32.47%	-0.008	0.98%	0.13%	21.28%	0.29%	0.90%
26	137	29.58%	-1.743	0.70%	0.15%	26.97%	0.30%	0.73%
27	156	33.72%	-0.386	0.34%	0.13%	61.99%	0.27%	21.79%
28	92	37.36%	-0.226	0.41%	0.14%	25.82%	0.27%	0.00%

11.1.7 Order book 7

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	345	26.62%	-3.899	0.00%	0.22%	0.00%	0.24%	1.76%
2	630	40.06%	0.0291	0.00%	0.23%	0.00%	0.30%	45.40%
3	1100	54.21%	-3.326	0.00%	0.26%	0.00%	0.43%	18.84%
4	1221	60.99%	-3.658	0.00%	0.22%	0.00%	0.39%	1.48%
5	1813	88.89%	-2.731	0.00%	0.24%	0.00%	0.34%	14.65%
6	1609	68.45%	-8.5	0.00%	0.21%	0.00%	0.26%	12.95%
7	852	45.42%	-2.181	0.00%	0.22%	0.00%	0.27%	15.00%
8	486	53.95%	-1.139	0.00%	0.22%	0.00%	0.28%	25.51%
9	399	46.93%	-1.041	0.00%	0.28%	0.00%	0.42%	29.15%
10	819	52.83%	-3.238	0.00%	0.23%	0.00%	0.29%	4.80%
11	478	36.08%	-0.522	0.00%	0.22%	0.00%	0.26%	2.52%
12	795	50.40%	-3.246	0.00%	0.23%	0.00%	0.33%	10.24%
13	1244	30.60%	-4.151	0.00%	0.20%	0.00%	0.24%	16.49%
14	1228	65.55%	-3.899	0.00%	0.21%	0.00%	0.24%	4.14%
15	623	48.58%	-4.115	0.00%	0.21%	0.00%	0.31%	2.09%
16	970	46.37%	-2.368	0.00%	0.20%	0.00%	0.23%	15.11%
17	874	52.40%	-3.069	0.00%	0.20%	0.00%	0.22%	4.82%
18	946	43.60%	-4.121	0.00%	0.20%	0.00%	0.23%	8.99%
19	952	41.35%	-4.037	0.00%	0.19%	0.00%	0.23%	10.62%
20	369	25.36%	-1.253	0.00%	0.19%	0.00%	0.21%	9.78%
21	652	40.39%	-1.686	0.00%	0.20%	0.00%	0.24%	3.37%
22	674	41.28%	-1.319	0.00%	0.21%	0.00%	0.24%	13.73%
23	678	37.52%	-4.292	0.00%	0.20%	0.00%	0.23%	0.59%
24	1002	46.04%	-2.441	0.00%	0.21%	0.00%	0.24%	14.37%
25	986	44.52%	-3.098	0.00%	0.20%	0.00%	0.23%	14.11%
26	661	47.25%	-3.554	0.00%	0.21%	0.00%	0.23%	21.63%
27	864	42.68%	-1.217	0.00%	0.20%	0.00%	0.24%	8.35%
28	1020	89.89%	-3.244	0.00%	0.21%	0.00%	0.24%	12.12%

11.1.8 Order book 8

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	538	28.01%	-1.461	1.02%	0.12%	84.70%	0.44%	5.95%
2	850	46.62%	-0.005	1.02%	0.15%	90.39%	0.74%	26.12%
3	1413	58.17%	0.382	0.07%	0.27%	96.55%	1.11%	8.41%
4	1474	66.26%	-2.334	1.09%	0.13%	92.13%	0.77%	17.91%
5	594	87.08%	-0.8	52.18%	1.57%	100.00%		16.84%
6	248	63.65%	-0.518	10.14%	0.23%	84.42%	0.25%	5.22%
7	377	47.18%	-0.491	0.27%	0.17%	63.58%	0.23%	11.41%
8	199	57.45%	0.8167	2.15%	0.18%	73.20%	0.76%	25.63%
9	334	54.80%	0.5043	1.06%	0.16%	30.98%	0.40%	11.98%
10	450	47.65%	1.4927	1.10%	0.15%	31.85%	0.33%	12.44%
11	128	35.60%	-0.232	1.02%	0.20%	39.45%	0.37%	3.12%
12	342	55.19%	0.2699	0.06%	0.21%	23.34%	0.63%	3.51%
13	310	50.63%	1.6298	1.04%	0.20%	49.26%	0.46%	3.23%
14	444	72.57%	-0.158	0.95%	0.20%	59.89%	0.31%	4.04%
15	232	49.30%	-1.434	1.04%	0.19%	52.66%	0.32%	10.34%
16	389	44.13%	-0.431	0.07%	0.26%	45.11%	0.37%	13.33%
17	334	33.84%	0.5981	1.01%	0.20%	39.71%	0.23%	0.89%
18	419	44.79%	-1.478	1.02%	0.18%	33.55%	0.40%	30.31%
19	153	30.73%	-0.403	1.01%	0.15%	16.47%	0.31%	4.58%
20	80	20.23%	13.386	1.02%	0.14%	41.46%	0.25%	5.00%
21	181	34.78%	-0.768	0.99%	0.16%	26.10%	0.26%	4.35%
22	134	40.02%	-0.071	1.03%	0.18%	31.39%	0.39%	10.45%
23	149	24.84%	0.0308	1.02%	0.14%	11.77%	0.34%	11.41%
24	418	54.73%	-1.672	1.02%	0.15%	15.69%	0.30%	7.18%
25	250	43.40%	0.2055	1.02%	0.18%	18.52%	0.32%	20.80%
26	248	42.23%	0.3618	1.02%	0.17%	19.16%	0.48%	7.26%
27	206	37.96%	-0.037	1.02%	0.16%	16.42%	0.33%	22.33%
28	313	30.79%	1.3757	1.03%	0.15%	15.55%	0.25%	1.89%

11.1.9 Order book 9

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	588	23.19%	-14.92	0.00%	0.08%	0.00%	0.26%	4.42%
2	802	22.79%	-0.221	0.00%	0.10%	0.00%	0.40%	16.06%
3	1098	41.06%	-0.166	0.00%	0.11%	0.00%	0.44%	22.22%
4	916	44.29%	-1.132	0.00%	0.11%	0.00%	0.54%	0.33%
5	1014	65.84%	-1.408	0.00%	0.21%	4.54%	1.09%	2.37%
6	842	52.08%	-1.888	0.00%	0.15%	0.00%	0.55%	0.83%
7	826	31.33%	-0.7	0.00%	0.10%	0.00%	0.37%	25.12%
8	445	48.18%	-8.974	0.00%	0.10%	0.00%	0.47%	1.57%
9	624	37.41%	0.0671	0.00%	0.11%	0.00%	0.43%	26.77%
10	707	28.57%	-0.11	0.00%	0.11%	0.00%	0.47%	11.49%
11	670	30.01%	1.4533	0.00%	0.11%	0.00%	0.38%	3.58%
12	569	41.91%	-0.666	0.00%	0.15%	0.00%	0.53%	29.45%
13	1079	46.78%	0.2439	0.00%	0.13%	0.19%	0.67%	18.11%
14	983	52.11%	-0.935	0.00%	0.14%	0.00%	0.73%	11.65%

15	724	34.22%	-0.475	0.00%	0.13%	0.00%	0.49%	0.14%
16	1119	44.62%	0.5251	0.00%	0.11%	1.58%	0.36%	7.86%
17	1033	30.83%	-2.908	0.00%	0.09%	0.00%	0.25%	14.18%
18	1069	34.31%	0.4398	0.00%	0.09%	0.00%	0.21%	12.53%
19	1096	36.88%	-2.671	0.00%	0.09%	0.00%	0.21%	7.64%
20	362	18.40%	0.0903	0.00%	0.08%	0.00%	0.24%	8.89%
21	594	29.97%	-0.554	0.00%	0.09%	0.00%	0.27%	3.53%
22	525	26.11%	0.3714	0.00%	0.09%	0.00%	0.20%	17.89%
23	1058	26.50%	-1.124	0.00%	0.08%	0.00%	0.17%	25.62%
24	631	34.32%	-0.089	0.00%	0.13%	0.00%	0.35%	14.79%
25	671	30.84%	-0.106	0.00%	0.11%	0.00%	0.31%	12.04%
26	660	38.59%	0.4137	0.00%	0.11%	0.00%	0.32%	12.25%
27	558	30.48%	-0.125	0.00%	0.14%	0.00%	0.32%	14.74%
28	795	26.56%	0.1521	0.00%	0.10%	0.00%	0.33%	4.89%

11.1.10 Order book 10

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	304	14.33%	0.0516	0.00%	0.05%	1.86%	0.29%	12.58%
2	506	22.96%	-0.144	0.00%	0.06%	0.00%	0.52%	29.39%
3	536	22.42%	1.1084	0.00%	0.08%	2.68%	0.62%	8.58%
4	518	32.30%	0.4815	0.00%	0.09%	8.49%	0.57%	18.77%
5	552	43.56%	-0.482	0.00%	0.24%	22.25%	1.27%	0.71%
6	392	51.81%	-0.193	0.00%	0.17%	0.00%	1.04%	11.96%
7	724	18.74%	-0.006	0.00%	0.05%	0.34%	0.38%	26.73%
8	483	27.35%	0.5756	0.00%	0.07%	0.00%	0.60%	13.04%
9	665	29.01%	0.7243	0.00%	0.08%	0.14%	1.07%	5.26%
10	316	26.82%	0.1437	0.00%	0.08%	0.00%	0.75%	6.07%
11	376	18.88%	0.6327	0.00%	0.07%	0.98%	0.59%	3.72%
12	908	30.62%	2.3404	0.00%	0.08%	0.00%	0.91%	4.55%
13	489	31.79%	0.1066	0.00%	0.22%	0.00%	1.30%	14.05%
14	551	35.09%	1.021	0.00%	0.10%	0.20%	1.25%	10.92%
15	274	29.09%	-0.295	0.00%	0.08%	0.03%	0.68%	24.82%
16	422	41.36%	-5.502	0.00%	0.09%	1.68%	1.06%	10.69%
17	356	34.78%	-0.149	0.00%	0.09%	2.38%	0.99%	6.74%
18	415	26.09%	-0.739	0.00%	0.07%	0.88%	0.51%	1.20%
19	331	19.60%	0.3602	0.00%	0.07%	0.00%	0.32%	18.29%
20	206	11.12%	0.3523	0.00%	0.05%	0.92%	0.36%	10.68%
21	456	32.04%	-32.19	0.00%	0.10%	1.50%	0.62%	17.54%
22	281	18.90%	1.5649	0.00%	0.07%	0.12%	0.55%	6.01%
23	1101	15.81%	-0.338	0.00%	0.05%	0.00%	0.36%	47.79%
24	514	21.39%	-0.494	0.00%	0.08%	0.94%	0.76%	4.26%
25	395	23.46%	-0.195	0.00%	0.09%	8.33%	0.51%	25.57%
26	345	28.08%	2.1657	0.00%	0.09%	18.43%	0.70%	44.93%
27	207	19.11%	0.2452	0.00%	0.07%	2.46%	0.39%	2.42%
28	363	17.90%	-1.114	0.00%	0.07%	8.07%	0.53%	22.10%

11.1.11 Order book 11

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	289	13.36%	-0.824	0.00%	0.09%	0.00%	0.10%	4.23%
2	548	33.91%	-5.683	0.00%	0.10%	0.00%	0.11%	12.52%
3	1002	39.10%	0.466	0.00%	0.09%	0.00%	0.10%	14.49%
4	723	40.22%	-0.269	0.00%	0.10%	0.00%	0.13%	6.09%
5	1325	70.56%	0.2563	0.00%	0.13%	0.00%	0.19%	5.22%
6	1057	42.19%	5.49	0.00%	0.09%	0.00%	0.11%	6.59%
7	550	35.48%	-0.191	0.00%	0.10%	0.00%	0.12%	15.48%
8	292	39.88%	-0.232	0.00%	0.10%	0.00%	0.11%	19.86%
9	359	35.20%	0.1193	0.00%	0.14%	0.00%	0.19%	27.58%
10	404	38.85%	0.7998	0.00%	0.10%	0.00%	0.12%	5.91%
11	285	27.58%	0.6684	0.00%	0.10%	0.00%	0.11%	15.79%
12	686	53.10%	0.7581	0.00%	0.11%	0.00%	0.13%	12.70%
13	644	58.13%	0.9348	0.00%	0.11%	0.00%	0.13%	11.70%
14	795	53.10%	0.8157	0.00%	0.10%	0.00%	0.12%	2.24%
15	462	36.70%	0.6035	0.00%	0.11%	0.00%	0.13%	11.06%
16	746	39.93%	-3.5	0.00%	0.11%	0.00%	0.12%	5.63%
17	576	27.45%	-4.714	0.00%	0.09%	0.00%	0.11%	5.76%
18	660	30.94%	1.3527	0.00%	0.09%	0.00%	0.11%	1.82%
19	570	26.33%	0.586	0.00%	0.09%	0.00%	0.10%	3.16%
20	151	13.74%	0.0297	0.00%	0.09%	0.01%	0.10%	12.58%
21	553	29.14%	-0.049	0.00%	0.10%	0.00%	0.12%	5.97%
22	585	29.20%	0.8435	0.00%	0.10%	0.00%	0.11%	11.11%
23	462	21.06%	-0.193	0.00%	0.09%	0.00%	0.11%	0.22%
24	594	42.01%	-0.211	0.00%	0.09%	0.04%	0.10%	2.02%
25	573	33.65%	0.5368	0.00%	0.09%	0.00%	0.11%	5.63%
26	720	37.30%	0.5979	0.00%	0.10%	0.00%	0.11%	2.36%
27	514	28.84%	0.4819	0.00%	0.10%	0.00%	0.11%	26.07%
28	714	25.82%	0.5373	0.00%	0.09%	0.00%	0.10%	19.55%

11.1.12 Order book 12

Day	Trades	Volatility	Z(10)	TWAS Undef.	TWAS	TWVWAS Undef.	TWVWAS	Dev. Eq. Trading
1	71	16.63%	1.966	2.02%	0.57%	100.00%		23.94%
2	172	33.26%	-0.122	1.01%	2.24%	100.00%		15.12%
3	207	41.92%	-0.186	0.06%	2.20%	100.00%		23.67%
4	252	54.15%	-0.011	5.50%	2.23%	100.00%		69.17%
5	7	37.74%	0.0262	75.72%	8.64%	100.00%		100.00%
6	82	29.90%	-0.302	89.80%	0.14%	100.00%		19.51%
7	201	32.21%	-0.692	58.09%	0.36%	100.00%	1.08%	27.80%
8	25	28.95%	-0.005	2.13%	0.69%	100.00%		4.00%
9	92	27.12%	-0.707	6.41%	0.64%	100.00%		6.52%
10	189	36.27%	-0.66	1.02%	2.32%	100.00%		2.65%
11	218	25.76%	0.2866	1.01%	0.19%	99.86%	0.07%	5.36%
12	220	34.54%	1.0063	0.04%	0.41%	100.00%		0.91%
13	256	49.48%	-0.276	47.45%	0.81%	100.00%		7.03%
14	159	51.39%	0.4495	39.66%	0.53%	100.00%		31.33%

15	115	29.59%	-0.312	1.03%	0.86%	100.00%		45.76%
16	156	36.18%	0.0538	0.06%	0.59%	100.00%		20.51%
17	119	32.62%	0.0277	25.45%	0.52%	100.00%		1.67%
18	112	25.75%	-0.075	0.91%	0.24%	100.00%		28.70%
19	130	26.42%	-0.583	1.01%	0.55%	100.00%		35.88%
20	81	19.18%	0.0314	0.94%	0.25%	100.00%		25.93%
21	76	27.36%	0.0296	0.97%	0.15%	90.73%	0.22%	7.50%
22	167	26.08%	0.0229	1.02%	0.17%	48.62%	1.22%	28.14%
23	221	18.32%	-1.271	1.01%	0.16%	17.90%	0.22%	40.27%
24	155	38.06%	0.5709	1.01%	0.16%	24.80%	0.44%	43.23%
25	137	31.84%	0.1255	0.12%	0.23%	24.54%	0.79%	43.07%
26	95	30.19%	0.0302	1.03%	0.19%	28.93%	0.69%	21.21%
27	87	27.90%	0.002	1.00%	0.38%	90.48%	1.19%	14.94%
28	277	29.70%	-1.082	1.00%	0.16%	96.53%	1.09%	10.79%

11.2 Participant data

11.2.1 Participant 1

11.2.1.1 Order book 1

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	4391	1937.00	13.38%	22.29%	0
2	10365	1644.00	24.92%	16.72%	0
3	9721	1337.00	17.40%	5.52%	0
4	6432	945.50	32.76%	49.75%	0
5	34	7191432.50	0.00%	100.00%	0
6	7073	994.00	6.37%	34.75%	0
7	4695	1922.00	12.61%	4.35%	0
8	3602	1070.50	37.06%	11.19%	0
9	9810	1245.00	12.60%	62.60%	0
10	12083	821.00	18.91%	4.87%	0
11	7483	2036.00	16.55%	2.76%	0
12	13109	949.00	12.77%	5.32%	0
13	8975	2097.00	12.88%	25.15%	0
14	15269	666.00	9.74%	3.90%	0
15	15089	1064.00	16.08%	8.54%	0
16	11410	1172.50	11.34%	0.58%	0
17	7116	1277.00	5.53%	51.06%	0
18	6687	1431.00	10.53%	66.67%	0
19	6530	847.50	4.26%	59.66%	0
20	1570	7405.50	16.00%	14.67%	0
21	6530	2679.00	44.36%	33.83%	0
22	8205	3038.00	31.03%	14.18%	0
23	8471	600.00	6.21%	48.22%	0
24	30782	68.00	27.60%	8.44%	0
25	15906	566.50	19.61%	19.89%	0
26	14516	390.00	15.49%	28.28%	0
27	11803	266.00	45.63%	45.90%	0

28	11741	1055.00	33.63%	24.65%	0

11.2.1.2 Order book 2

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	7382	3320.00	43.84%	28.77%	0
2	19550	1140.50	68.35%	25.32%	4
3	26960	701.00	77.74%	5.65%	9
4	20568	795.00	74.83%	2.10%	10
5	49	6530258.00	42.86%	100.00%	0
6	24428	964.00	98.72%	20.51%	13
7	18333	1025.00	80.17%	25.86%	14
8	13889	580.00	78.64%	2.91%	13
9	17008	1761.50	87.64%	66.29%	2
10	21392	1220.50	81.18%	23.66%	11
11	15717	1108.00	85.44%	24.27%	19
12	25454	682.00	81.32%	35.41%	15
13	19033	999.00	85.68%	2.46%	9
14	28036	1076.50	85.92%	28.16%	3
15	14739	1215.00	75.45%	10.91%	2
16	16680	1519.00	81.11%	4.44%	2
17	13601	1172.00	47.62%	63.49%	3
18	16955	561.00	65.49%	28.17%	10
19	15089	799.00	76.92%	11.11%	16
20	4154	4902.00	59.46%	13.51%	0
21	21975	109.00	85.16%	18.75%	25
22	18507	434.00	83.06%	0.00%	17
23	8982	1162.50	51.92%	15.38%	7
24	13968	1013.00	74.07%	55.56%	6
25	17948	417.00	45.16%	13.98%	19
26	21387	416.00	52.53%	11.11%	12
27	18319	598.00	60.78%	68.63%	11
28	18760	425.00	65.96%	29.79%	35

11.2.1.3 Order book 3

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	19118	605.50	81.43%	11.43%	6
2	35717	238.00	88.46%	30.77%	24
3	39338	157.00	78.47%	2.78%	41
4	46014	206.50	66.67%	13.13%	49
5	28	7201319.00	100.00%	77.78%	0
6	39701	233.00	96.77%	48.39%	36
7	28610	485.00	71.43%	9.52%	23
8	27575	75.00	74.00%	8.00%	66
9	37881	245.00	61.82%	52.73%	37
10	46458	170.00	83.64%	23.64%	52
11	32169	220.00	88.00%	60.00%	32
12	47575	178.00	92.50%	20.00%	53

13	44291	143.00	74.74%	26.32%	79
14	54982	195.00	70.37%	22.22%	52
15	33758	198.00	65.85%	12.20%	47
16	49806	224.00	79.03%	35.48%	56
17	26749	605.00	76.67%	20.00%	22
18	31793	369.00	91.89%	80.18%	27
19	33483	269.00	78.38%	29.73%	49
20	9643	856.00	70.45%	59.09%	22
21	18533	337.00	84.75%	69.49%	33
22	16335	650.00	74.73%	45.05%	11
23	14641	1071.00	65.89%	8.53%	4
24	19453	910.00	52.00%	75.20%	7
25	18465	594.00	82.81%	21.88%	5
26	24456	529.00	62.24%	42.86%	8
27	15584	564.00	89.25%	1.08%	6
28	15844	546.00	80.56%	0.00%	2

11.2.1.4 Order book 4

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	990	8377.00	20.00%	4.00%	0
2	4944	2043.50	24.83%	6.38%	0
3	5634	1139.50	12.50%	39.06%	0
4	3601	896.00	18.72%	27.66%	0
5	37	7107002.00	0.00%	100.00%	0
6	2543	1532.00	7.08%	4.42%	0
7	6736	2099.50	12.50%	5.77%	0
8	3775	1047.00	12.50%	5.77%	0
9	7501	1170.00	10.21%	2.98%	0
10	8317	508.00	16.80%	5.74%	0
11	8511	1514.00	19.58%	18.88%	0
12	7666	1128.50	21.14%	4.88%	0
13	10407	524.00	20.85%	0.35%	0
14	7133	451.00	11.67%	22.50%	0
15	7202	2001.00	16.90%	5.63%	0
16	6520	2024.00	27.12%	3.73%	0
17	4055	2556.00	16.56%	11.66%	0
18	5026	1372.00	16.47%	1.18%	0
19	7489	2253.00	16.16%	21.95%	0
20	1637	7021.00	19.80%	8.91%	0
21	7203	1379.00	8.74%	6.01%	0
22	9838	1067.00	7.74%	33.58%	0
23	6724	1512.00	16.80%	9.38%	0
24	10499	946.00	21.13%	1.72%	0
25	10339	1758.00	21.70%	7.92%	0
26	11752	1599.50	18.09%	11.98%	0
27	6494	1610.50	14.89%	39.01%	0
28	12797	944.00	18.11%	30.51%	0

11.2.1.5 Order book 5

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	3162	5301.50	15.88%	25.88%	0
2	15093	1572.00	36.42%	12.59%	0
3	18236	1047.50	19.61%	7.57%	0
4	10055	1335.00	31.56%	1.23%	0
5	44	438843.00	0.00%	84.62%	0
6	12654	1001.50	8.84%	4.76%	0
7	11223	1239.00	12.03%	8.86%	0
8	6236	1999.50	23.60%	22.47%	0
9	19760	1499.00	14.51%	38.82%	0
10	19108	1744.00	18.44%	3.75%	0
11	12418	2388.00	17.01%	33.61%	0
12	19122	1271.00	15.44%	25.87%	0
13	23422	968.00	15.14%	4.00%	0
14	28620	1010.00	20.12%	1.18%	0
15	10850	2696.50	22.08%	28.08%	0
16	15555	1826.00	19.40%	7.76%	0
17	6486	2134.00	30.05%	3.29%	0
18	10817	2725.00	27.61%	38.04%	0
19	10434	1880.50	19.22%	3.20%	0
20	4261	3000.00	16.87%	3.61%	0
21	12048	2190.00	20.98%	16.96%	0
22	11456	2287.50	18.52%	22.56%	0
23	9865	2091.00	10.94%	7.81%	0
24	13507	1813.00	16.06%	2.59%	0
25	10976	2552.00	25.62%	24.56%	0
26	14853	1464.00	11.06%	13.27%	0
27	9331	2905.00	14.67%	30.67%	0
28	11808	1705.50	15.20%	18.50%	0

11.2.1.6 Order book 6

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	3478	180.50	44.44%	33.33%	5
2	8815	766.00	92.73%	83.64%	17
3	8390	1407.50	77.19%	15.79%	0
4	9644	999.00	55.56%	8.33%	6
5	10	7089198.50			0
6	6757	1198.00	63.64%	27.27%	0
7	6258	1440.50	54.05%	18.92%	3
8	6086	165.00	75.00%	14.29%	19
9	8177	1119.00	54.55%	21.21%	1
10	7165	2591.00	55.56%	55.56%	0
11	4792	2929.50	75.00%	0.00%	3
12	9411	1006.00	71.93%	5.26%	11
13	8799	1083.00	64.86%	67.57%	6
14	12105	1001.00	91.30%	13.04%	4

15	9424	421.00	74.07%	3.70%	26
16	6528	2000.00	44.44%	40.74%	3
17	4399	3009.00	40.00%	20.00%	0
18	5279	2678.00	77.78%	55.56%	0
19	4719	2291.00	40.00%	60.00%	4
20	1490	12520.50	63.64%	9.09%	0
21	4638	2543.50	90.91%	27.27%	6
22	4982	2888.00	11.11%	77.78%	0
23	2552	5715.00	83.33%	66.67%	0
24	5129	2911.00	75.00%	100.00%	4
25	5810	2949.00	76.92%	38.46%	9
26	5601	2892.00	52.38%	71.43%	2
27	4398	2901.50	29.17%	8.33%	2
28	4594	4565.00	88.24%	41.18%	1

11.2.1.7 Order book 7

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	398	6907.50	33.85%	38.46%	0
2	140	7890.50	61.54%	23.08%	0
3	132	8103.50	63.16%	5.26%	0
4	790	6197.50	24.78%	9.73%	0
5	3198	4295.50	35.51%	21.74%	0
6	3477	4681.00	31.84%	24.05%	0
7	485	4900.00	30.19%	24.53%	0
8	392	5244.00	43.55%	35.48%	0
9	139	4295.00	37.50%	41.67%	0
10	186	3301.50	33.33%	50.00%	0
11	633	5595.00	41.38%	37.93%	0
12	319	6100.00	56.25%	68.75%	0
13	2712	4513.50	48.16%	22.06%	0
14	1211	4700.00	54.37%	4.85%	0
15	2418	5320.00	62.26%	0.63%	0
16	8153	2001.00	64.55%	61.90%	0
17	10264	1624.50	44.21%	6.61%	0
18	10516	2000.00	57.77%	17.08%	0
19	7133	2006.00	51.54%	1.32%	0
20	2424	3610.50	41.10%	17.81%	0
21	8866	2005.00	34.88%	4.65%	0
22	8429	2000.00	38.58%	5.58%	0
23	6392	2000.00	50.78%	29.69%	0
24	8756	1624.50	50.81%	6.50%	0
25	8201	1994.00	41.28%	27.83%	0
26	11166	1599.00	46.56%	24.87%	0
27	8792	1590.00	40.20%	30.39%	0
28	7911	1604.00	25.10%	30.04%	0

11.2.1.8 Order book 8

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts

1	2671	2450.00	28.03%	29.41%	0
2	13660	977.00	32.94%	13.95%	0
3	12672	434.00	40.35%	18.99%	0
4	27988	557.00	52.25%	10.81%	0
5	16	7169102.50	0.00%	100.00%	0
6	34042	850.00	90.91%	81.82%	0
7	26578	1307.00	66.22%	27.03%	0
8	13866	350.00	33.33%	20.00%	0
9	28767	833.00	38.46%	38.46%	0
10	33889	1139.00	68.12%	1.45%	0
11	21313	1654.00	60.00%	0.00%	0
12	38317	538.00	55.56%	42.22%	0
13	31302	426.00	55.34%	6.80%	0
14	59153	141.00	66.96%	42.86%	0
15	33487	83.00	58.97%	28.21%	0
16	32676	265.00	51.76%	10.59%	0
17	20939	905.00	25.64%	33.33%	0
18	25433	314.00	24.19%	51.61%	0
19	17872	1162.00	57.14%	14.29%	0
20	5873	1953.00	50.00%	14.29%	0
21	24800	191.00	53.85%	12.82%	0
22	20115	822.00	65.22%	30.43%	0
23	10706	2481.50	55.56%	100.00%	0
24	29006	523.50	40.35%	40.35%	0
25	24921	547.00	57.50%	25.00%	0
26	27392	616.00	62.75%	41.18%	0
27	19403	1126.00	62.96%	18.52%	0
28	21647	858.00	41.67%	13.33%	0

11.2.1.9 Order book 9

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	6008	1210.50	74.29%	2.86%	0
2	15343	511.00	77.93%	83.45%	9
3	18174	599.50	66.45%	34.19%	8
4	15502	754.50	58.50%	7.48%	1
5	53	4396603.00	38.46%	84.62%	0
6	18282	487.00	96.00%	4.00%	1
7	12900	665.00	88.71%	74.19%	5
8	11072	117.00	62.50%	14.29%	28
9	18607	987.00	49.15%	45.76%	1
10	17100	1150.00	49.64%	82.48%	4
11	11089	1696.00	49.04%	17.31%	3
12	18843	745.00	72.22%	33.33%	16
13	18783	482.00	70.89%	30.38%	10
14	25635	511.00	62.16%	27.93%	8
15	12321	946.00	54.35%	26.09%	5
16	21681	356.00	46.79%	2.75%	15
17	14073	457.00	57.14%	18.37%	7
18	13480	569.00	25.00%	6.67%	0
19	13257	984.00	43.93%	21.50%	3
20	3068	3375.00	45.45%	22.73%	0

21	9218	623.00	52.94%	8.24%	4
22	7471	1362.00	70.18%	47.37%	2
23	9604	581.00	52.27%	18.18%	3
24	15151	68.00	41.38%	41.38%	37
25	10766	657.00	52.38%	14.29%	6
26	14258	246.00	29.23%	23.08%	30
27	11330	604.00	63.33%	30.00%	12
28	10829	1170.00	35.14%	43.24%	2

11.2.1.10 Order book 10

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	2339	948.00	47.37%	68.42%	2
2	8597	198.00	64.15%	1.89%	10
3	13430	64.00	78.26%	30.43%	11
4	13553	51.00	73.08%	38.46%	22
5	27	1376846.00	16.67%	100.00%	0
6	12597	153.00	83.33%	16.67%	11
7	10632	38.00	97.81%	92.70%	25
8	8157	36.00	86.21%	93.10%	20
9	11607	230.00	42.86%	28.57%	0
10	12209	138.00	92.86%	28.57%	19
11	8654	189.00	79.03%	12.90%	14
12	9053	265.00	83.64%	9.09%	8
13	11983	176.00	52.45%	38.46%	2
14	18751	47.00	55.26%	31.58%	15
15	11744	73.00	71.88%	12.50%	6
16	14420	43.00	55.00%	20.00%	28
17	8937	159.00	47.06%	5.88%	5
18	10940	104.00	61.54%	38.46%	12
19	8014	33.00	90.91%	68.18%	18
20	2684	197.50	100.00%	27.27%	5
21	8528	106.00	72.22%	16.67%	9
22	7211	291.00	68.42%	78.95%	1
23	8888	114.00	4.05%	97.30%	5
24	9844	51.00	82.61%	13.04%	14
25	9276	75.00	83.33%	66.67%	13
26	11123	121.00	90.32%	80.65%	10
27	7516	83.00	33.33%	55.56%	10
28	8121	194.00	80.56%	44.44%	1

11.2.1.11 Order book 11

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	1696	4309.50	46.15%	53.85%	0
2	8652	3434.50	39.02%	21.95%	0
3	23098	2695.00	39.41%	13.56%	0
4	13875	2720.00	45.81%	0.65%	0
5	640	1207.50	73.33%	53.33%	0
6	9028	2100.00	49.70%	5.39%	0

7	6363	2913.00	20.75%	32.08%	0
8	1369	2114.00	41.67%	66.67%	0
9	394	1995.00	66.67%	33.33%	0
10	910	1906.50	37.50%	25.00%	0
11	9993	3595.00	43.59%	7.69%	0
12	1378	2005.00	50.00%	50.00%	0
13	12796	1991.00	50.52%	19.59%	0
14	5411	2006.00	40.00%	6.67%	0
15	9557	2405.00	58.33%	16.67%	0
16	96897	805.00	72.70%	23.68%	0
17	90363	1190.00	67.56%	2.38%	0
18	100000	1186.00	77.21%	0.98%	0
19	78229	1200.00	80.36%	7.64%	0
20	24262	2800.00	63.21%	15.09%	0
21	92665	1194.00	64.56%	20.72%	0
22	94637	1196.00	68.18%	18.94%	1
23	73884	1201.00	58.33%	18.75%	2
24	97412	811.00	69.31%	12.41%	0
25	97670	816.00	72.98%	8.70%	2
26	100000	806.00	59.71%	17.03%	1
27	87962	1194.00	70.04%	23.47%	0
28	98792	824.00	67.81%	29.24%	0

11.2.1.12 Order book 12

Day	Orders	Med. Lifetime	Agg. Trade Ratio	Abs. Dev. Eq. Trading	Liq. Bursts
1	1716	2862.50	9.72%	41.67%	0
2	3201	2849.00	9.71%	10.86%	0
3	4363	2216.00	17.05%	14.29%	0
4	3859	755.00	9.48%	72.41%	0
5	18	4728594.00	0.00%	100.00%	0
6	1171	1557.00	11.43%	14.29%	0
7	2992	459.50	7.94%	19.58%	0
8	2439	401.00	4.35%	4.35%	0
9	3843	1161.00	16.67%	10.42%	0
10	4826	1327.50	17.55%	21.28%	0
11	4590	2090.00	10.09%	1.83%	0
12	6352	1342.00	24.40%	19.20%	0
13	5805	972.00	22.48%	26.36%	0
14	6534	753.00	18.46%	4.62%	0
15	4696	1662.00	6.86%	33.33%	0
16	6583	1411.00	20.11%	10.61%	0
17	2786	1741.00	18.84%	13.04%	0
18	4820	1843.50	8.93%	19.64%	0
19	3454	2559.50	24.83%	10.34%	0
20	1179	4926.00	4.82%	25.30%	0
21	5126	2403.00	26.92%	25.64%	0
22	7233	2318.00	14.94%	22.99%	0
23	3983	1567.00	22.46%	61.02%	0
24	8086	1063.50	15.25%	33.33%	0
25	7223	801.00	15.29%	30.59%	0
26	10255	1180.00	26.27%	5.08%	0

27	6299	1387.00	14.29%	31.87%	0
28	10203	1420.00	23.26%	4.65%	0

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