Pricing Contingent Convertibles using an Equity Derivatives Jump Diffusion Approach

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Abstract

This paper familiarizes the reader with contingent convertibles and their role in the current financial landscape. A contingent convertible is a security behaving like a bond in normal times, but that converts into equity or is written down in times of turbulence. The paper presents a few existing pricing approaches and introduces an extension to one of these, the equity derivatives approach, by letting the underlying asset follow a jump-diffusion process instead of a standard Geometrical Brownian Motion. The extension requires sophisticated computational techniques in order for the pricing to stay within reasonable time frames. Since market data is sparse and incomplete in this area, the validation of the model is not performed quantitatively, but instead supported by qualitative arguments.

Keywords: Contingent Convertible, CoCo, jump-diffusion, pricing, adaptive mesh model

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Contents

List of Figures

List of Tables

Chapter 1

Symbol Explanations

Symbol	Explanation
A	balance sheet asset side value
B	an integer number of diffusion steps
C_p	conversion price that states the implied purchase price for the converted shares
C_r	conversion rate that states how many shares each CoCo will be converted into
c_i	amount of coupon payment i
cs	credit spread
F	face value
F_C	face value of balance sheet CoCos
F_D	face value of balance sheet debt
H	trigger level for barrier options
h	magnitude of a diffusion move over one time step
K	option strike price
K_C	CoCo conversion threshold
K_D	debt default threshold
k	stochastic jump magnitude where $\log 1 + k \sim N(\gamma', \delta^2)$
$\bar{k}$	$\bar{k} = E(k) = e^{\gamma} - 1$
M	number of jumps possible for each time step (odd number as middle corresponds to no jump)
m	number of jumps possible in either direction for each time step where $m = \frac{M-1}{2}$
N	number of a time steps
p_x	probability that the event x takes place
q	continuous dividend yield
$q(j)$	probability of jumping j steps in one time step
R	recovery rate
r	(risk-free) rate of return
S	current price of the underlying stock
S_{trig}	share price when the trigger is hit
T	time to maturity

t_i	time to coupon payment i
Δt	length of a time step
V_t	stock return process
W_t	Standard Brownian Motion at time t
α	fraction that states how much of the face value of the CoCo that will be converted into shares or written down
β	distance between CoCo conversion threshold and book value of debt
γ	debt default parameter $0 \leq \gamma \leq 1$
γ	jump magnitude expected value parameter
γ'	jump magnitude expected value where $\gamma' = \gamma - \delta^2$
δ	jump magnitude standard deviation
η	magnitude of a jump move over one time step
λ	default intensity
λ_{trig}	trigger intensity
λ	jump intensity
μ	cumulant
Π	price of a contingent claim
σ	standard deviation

Table 1.1: Symbol explanations.

Chapter 2

Introduction

2.1 Scope

This report studies different aspects of Contingent Convertibles (CoCos), including their place in the financial landscape, the size of the market, some existing pricing approaches and an introduction of a new pricing approach. This introductory chapter explains the setting and market environment in which the need for CoCos has arisen. The third chapter goes in-depth on different CoCo properties, specifying the characteristics of the instrument. The fourth chapter outlines different existing pricing methods which are suggested in the literature of the topic, as well as a short analysis of the hedging aspects. The fifth chapter discusses an extension of one of the models and tries to resolve practical issues relating to mathematical tractability and computation. The sixth chapter presents the empirical results of the study of the extended pricing method as well as some real world applications. The seventh chapter concludes and summarizes.

2.2 Financial Landscape

2.2.1 Capital Structure

Banks, insurance companies and other financial institutions finance their operations by deposits and by issuing debt and equity. By current standards of seniority, the shareholders are the first to suffer losses whenever the company is in trouble. The company has no financial obligation in terms of dividends or coupons towards the shareholders and the company can not be defaulted by not being able to repay its equity capital. Towards its lenders and bondholders, the company is obliged to pay the stipulated coupons and amortizations, otherwise the company would be put in default. When the company has defaulted, its assets are divided among the stakeholders in a certain order, beginning with the lenders and bondholders. They receive back a part of their invested capital, called the recovery amount. The shareholders get the remaining assets after the debt holders have been paid off. Usually this amounts to zero or close to zero, i.e. the shares are worthless. To compensate for this extra exposure towards losing their invested money, the shareholders demand an overall larger return than the bondholders. From the perspective of the company, it is on the one hand better to use debt capital for financing since the cost of debt capital is lower than that of equity. On the other hand, the risk of default increases with rising debt, as there are more and bigger fixed payments. It is said that the equity capital is loss-absorbing whereas the debt capital is not.

2.2.2 Financial Institutions and Society

Financial institutions have for a long time been crucial to the increase in wealth and improvement in technology in the world[15]. They provide credit to companies in other industries and the credit allows these companies to exploit new ideas and to finance research and expansion projects. The financial institutions have become systemic for the whole society in the sense that if the financial industry breaks down, it will bring down many, if not all, other industries as well. The governing powers all over the world have registered this development and the general practice amongst governments is that whenever a systemically important financial institution is on the brink of collapse, it has to be saved. This creates situations with distorted incentives. The management of a bank or an insurance company tends to favor debt financing since they know they would most likely be supported by the government in a default situation. Furthermore, the shareholders, who also happen to have the voting rights in the company canalized through the annual shareholders' meeting, know that they run less risk of losing their capital even though the company favors debt financing. As a result, they are likely to induce the company to engage in more risky activities (which more probably will yield a higher return). To counter these incentives, various regulatory agencies over the world have set up capital requirements for banks and insurance companies. The regulatory agencies also understand that there are setbacks stemming from holding excessive and idle capital since it negatively affects economic growth.

2.2.3 Hybrid Debt

Ideal for both the company and the society would be a way of funding that behaves like debt in good times, but provides the loss-absorbing properties of equity capital in times of turbulence. Funding types that have a combined behavior of both debt and equity is not a new invention. The area of hybrid debt has been around for quite some time and the most prominent examples of hybrid debt are the convertible bonds. A convertible bond is an instrument issued by a company, which behaves similarly as a corporate bond, but with the option to convert the bond into equity at a predetermined price as a stipulated situation emerges. Due to this optionality, the convertible bond pays a smaller coupon than its respective corporate bond. On the other hand, the convertible bondholder can capitalize on a potential upside in the share price by exchanging her bond for shares. If the company is doing well the share price may have risen considerably and conversion of the bond is favorable. However, this type of instrument serves another purpose than saving the bank in troublesome times. No convertible bond holder would prefer converting into equity in bad times as the risks of losses are much more severe than when keeping the bond.

2.2.4 Bridging the Gap

When the smoke had cleared after the 2007-2009 financial crisis and when studies of what had gone wrong started to take place, recognized by many was the fact that the funding structure of systemically important banks and financial institutions was not fully functional. This lead to further regulation initiatives by the authorities and intensified the debate on new types of instruments designed to stabilize financial institutions in times of turbulence (so that the governments, and in the end the tax payers would not have to), while at the same time not putting too much a constraint on the industry in good times. Banks and insurance companies had already issued various types of hybrid debt instruments before the crisis, which were supposed to fulfill the above criteria. These hybrid instruments were not automatically transferring debt into equity but were relying on a management decision to commence the conversion. However, during this crisis the management of many financial institutions showed great aversion towards letting their hybrid investors down and relied on government intervention instead. This sparked off the idea about having instruments that forced conversion upon an event that could not directly be controlled from inside the company. The contingent convertible (CoCo) was introduced as an instrument that fulfilled all the above criteria. Given an event not controlled by the company itself (such as the share price falling below a certain threshold or the regulatory agency declaring the company insolvent), the bond would be converted into shares so as to strengthen the capital base of the company and at the same time ease the burden of the debt payments to stay away from defaulting. The CoCo would pay a higher coupon than its corresponding corporate bond since the downside risk is much greater due to its forced conversion property. The CoCo provides in this sense a first line of defense against bankruptcy. When the company runs into trouble, the CoCo is converted, which decreases the risk of default and injects capital automatically. In addition, the CoCos reorganize the incentives of the shareholders. The dilution effect of the CoCo will make the shareholders keen on keeping the CoCo from converting, hence confining the risky behavior of the management of the bank.

2.3 The CoCo Market

The CoCo market was, by overly optimistic expectations in early 2011, perceived to total some \$950bn. This was however adjusted sharply downwards when the Basel Committee announced in July the same year that CoCos would not be considered part of the core Tier 1 (T1) capital (a capital requirement measure for banks) in the Basel III capital requirement specifications. However, the market for CoCos have now started to grow in a more controlled way. Below is a table of public CoCo issues at the time of writing[1][2][3][4][5][6]:

[1]http://tinyurl.com/8yse2cn
[2]http://tinyurl.com/78xmrl3
[3]http://online.wsj.com/article/SB10001424052748704546704576150861690164484.html
[4]http://www.financialmirror.com/news-details.php?nid=22956
[5]http://tinyurl.com/84ban3t
[6]http://tinyurl.com/6pesm5k

Issuer	Date	Amount	Coupon	Conversion	Trigger	Comment
Lloyds	Dec 2009	£7bn	Hybrid bond +1.5-2.5%	Shares	Accounting	Exchanged for hybrid bonds
Rabobank	Mar 2010	€1.25bn	Libor +3.5%	Writedown	Accounting	Not publicly traded share, hence a writedown
Credit Suisse	Feb 2011	$2bn	7.875%	Shares	Accounting/ Regulatory	Swiss regulator decides on insolvency
Bank of Cyprus	Mar 2011	€1.3bn	6-6.5%	Shares	Accounting/ Regulatory	Tier 1 capital trigger or Central Bank of Cyprus decision. CoCoCo structure (conv. bond optionality on the CoCo)
ANZ	Sept 2011	$0.75bn	3.1-3.3%	Shares	Accounting	Named Convertible Preference Share
Rabobank	Nov 2011	$2bn	8.4%	Writedown	Accounting	Basel III compliant

Table 2.1: Public CoCo issues.

In addition, Allied Irish Banks (AIB) have a public issue in the pipe[7]. Non-public issues include the German insurer Allianz's €0.5bn sale to its Japanese counterpart Nippon Life[8], AIB's €1.6bn issue to the government of Ireland[9], Credit Suisse's $6.2bn issue to Qatar Holding and Olayan Group[10], and Barclay's who has CoCos as a part of their bonus system[11]. Further, positive attitude towards CoCos have been presented by among others the representatives of Swiss Re[12], UBS[13] and Danmarks Nationalbank[14] (the central bank of Denmark).

[7] http://www.reuters.com/article/2011/10/27/idUS243442+27-Oct-2011+RNS20111027
[8] http://www.reuters.com/article/2011/07/05/allianz-nipponlife-idUSL3E7I515E20110705
[9] http://www.reuters.com/article/2011/10/27/idUS243442+27-Oct-2011+RNS20111027
[10] http://online.wsj.com/article/SB10001424052748704546704576150861690164484.html
[11] http://tinyurl.com/4ynblaw
[12] http://tinyurl.com/6mfqqeb
[13] http://tinyurl.com/7ejkmqc
[14] http://tinyurl.com/7ksvw2w

Chapter 3

CoCo Properties

3.1 General Properties

When a CoCo is issued, it behaves as any normal corporate bond. It pays a periodical, predetermined coupon and has a maturity date on which the final coupon and the principal is paid to the holder. Generally the CoCo pays a higher coupon than a corporate bond. However, when the issuing company runs into trouble, the CoCo holders are going to be more worried than the bondholders. Given a certain event connected to the value of the assets of the company, called a trigger, the CoCo will automatically - depending on the CoCo type - either simply be written down or converted into shares in the underlying company. In the below figure, two simulated observations of an asset price process starting at 40 are depicted over one year.

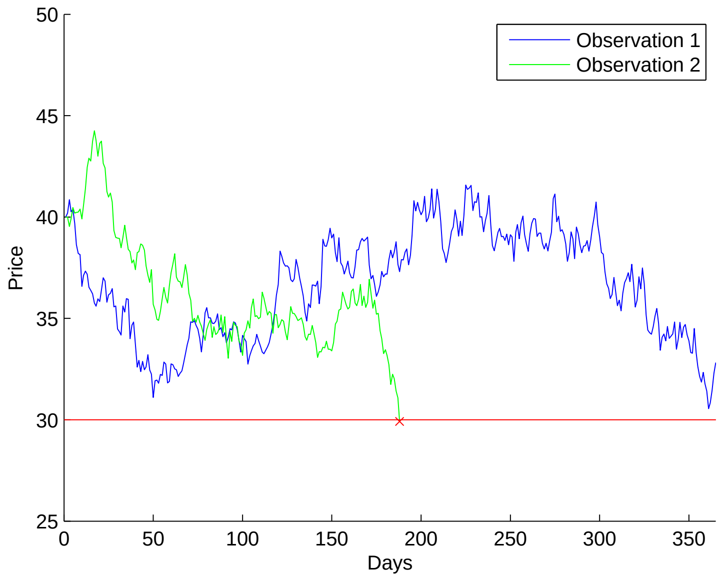

Figure 3.1: Two asset price observations and the trigger at 30.

For the first asset process observation in figure 3.1, the price does not fall below the trigger level at 30. In this case, a CoCo with one year maturity will make its periodical payments during the year and pay back its principal and the final coupon when the year has passed.

The CoCo behaves differently for the second observation. On the 188th day of the year, the asset price falls below the threshold at 30 and conversion takes place. All the coupon payments (including the one at maturity) from this day onwards are now canceled and will not be received by the holder. The face value will also be affected. In the case of a write-down of the CoCo, the face value will either not be paid at all or receive a haircut, losing a fraction of its value. In the case of a share converting CoCo, the face value will be exchanged for (i.e. converted into) a number of newly issued shares of the company. The number of shares that each CoCo is exchanged for is usually specified in advance. The more shares each CoCo will convert into, the higher value is assessed to the CoCo.

Related to the number of shares exchanged is the implicit price at which the shares are "bought". In practice, the conversion operation corresponds to a situation where the face value (or a part thereof) is paid to the CoCo holder in cash and for which she instantly buys shares at a predetermined price. This predetermined price is the price implied from the specified number of shares the CoCo is converted into. When the event is triggered, the market might have already known there was something not right with the company. Therefore the share price would have gone down, compared to the share price at the time of the CoCo issuance. The holder of a triggering CoCo with a low number of converted shares built in will buy the shares at a high implied price, and hence lose money if she sells sell them in the market at a lower price. On the other hand, if the number of shares exchanged implies a buy price same as the current share price, the potential loss for the CoCo holder is zero.

The market for CoCos is still in an introductory phase and there are no standards set for the design of the CoCos. Yet there are quite a few different properties that needs to be decided prior to the issuance.

3.2 Trigger Type

First of all, a decision on how the trigger will work needs to be taken. In general, there are three different approaches, out of which combinations are also possible. They are presented in detail in [16], together with the key features of a trigger:

- clear: the trigger must have the same meaning everywhere
- objective: the trigger should require no regulatory intervention and be well documented
- transparent: it should in all cases be obvious whether the trigger has been hit
- fixed: the trigger should ideally be constant
- public: the trigger should be made public to all stakeholders at the same time
- continuously observable: it should be possible to monitor the trigger continuously

3.2.1 Market Trigger

One approach is to use a market trigger. A market trigger could be for example the share price or the credit default swap (CDS) spread of the company. The upside with this approach is that the trigger will be clear, transparent, easy to monitor, and that the information is continuously publicly available. The pricing of the CoCo is also greatly simplified with this approach. The downside is the fact that the trigger is prone to flash crashes and market manipulation.

3.2.2 Accounting Trigger

Another approach is to use an accounting trigger, typically one that gives an indication of the solvency of the financial institution. The usual choice is the (core) T1 capital ratio. When the T1 ratio falls below a certain threshold it is a clear sign that the company is in trouble and that it needs extra capital, which is provided by the CoCo conversion. The upside with an accounting trigger is that it cannot easily be manipulated by outsiders and that it is released to the public periodically. The downside is the lack of clarity (accounting principles differ between countries) and transparency (the figure is released by the audit department of the company and the company itself might have undisclosed interests) and that it is not continuously updated which might result in speculation and uncertainty whether the trigger has been hit. From a pricing perspective, modeling an accounting trigger represents a hard-to-overcome obstacle.

3.2.3 Regulatory Trigger

The third approach is to use a regulatory trigger, which is favored by governments, since they usually have to pay for the financial institution's inability to keep themselves alive. Whenever a pre-specified regulatory unit considers the financial institution insolvent, the CoCo is triggered. The upside is that the trigger cannot be manipulated, neither by outsiders nor by the company itself and the information can be made publically available for everyone at the same time. However, it is neither objective, transparent, fixed nor continuously observable. Also modeling the behavior of a regulatory unit is impossible from a pricing perspective.

3.2.4 Approach in This Paper

This paper follows the market trigger approach with focus on the stock price, mainly due to its mathematical tractability. As described in detail in [16], quantitative modeling of the other trigger types is a research topic in itself and outside the scope of this paper.

3.3 Conversion Details

There are a few variations of properties for how the conversion takes place. These variations can significantly impact the valuation of the CoCo.

3.3.1 Funded or Unfunded

When the CoCos were introduced there was still an ongoing discussion about funded or unfunded CoCos. An unfunded CoCo requires capital injection at the trigger whereas a funded CoCo is paid upfront and no further capital will be required from the investor. The unfunded CoCos carry a lot of counterparty risk for the issuer. For this reason, they are usually not sold as public offerings but may play a role in the case that financial institutions sell CoCos to governments.

3.3.2 Conversion Type

The conversion type comes in two variations. The CoCo can either be converted into shares or simply written down by a certain amount. The writedown works like a "normal" haircut in that the face value and remaining coupons are reduced by a certain fraction, and no extra share capital is introduced. This approach makes sense in companies that are not publicly traded, or where new shareholders are undesired.

3.3.3 Conversion Fraction

The conversion fraction α is the fraction of the face value F that will be converted into shares. $\alpha = 0$ corresponds to a normal corporate bond since no part of the face value is converted. $\alpha = 1$ is called a "full" CoCo and is the most common choice up to date. Some participants suggest letting α depend on the severity of the troubles the financial institution is involved in. The factor should be set so as to convert as little as possible of the CoCos to make the institution back on track again. The upside of this approach is of course that the CoCo will take a smaller haircut (the expected loss on conversion will not be as large) but the downside is the fact that the confidence may not get restored from moving only as little as possible towards better solvency.

3.3.4 Conversion Rate and Conversion Price

The conversion rate C_r states how many shares each CoCo will be converted into and the conversion price C_p states what price the CoCo holder is implicitly paying for the shares. The relation between the two is as follows:

$$C_p = \frac{\alpha F}{C_r} \tag{3.1}$$

where F is the face value of the CoCo.

However, it is more common that the conversion price is determined by the issuer instead and the conversion ratio is implied from equation 3.1. The conversion price can be set in many different ways depending on what the purpose of the CoCo is. The most common examples include setting the conversion price equal to the share price at issue ($C_p = S$) or the share price at the (expected) trigger ($C_p = S_{\text{trig}}$). A conversion price equal to the trigger price is favorable for the CoCo holder as she will be paying a low price for her shares. With the conversion price equal to the share price on the issuing date, the current shareholders are better off because the dilution effect at conversion is lower. A third way is to go for $C_p = \max(S_{\text{trig}}, Q)$ for some positive constant Q. In this way, the current shareholders are protected against unlimited dilution by the floor at Q. There is also the option to set C_p to the average share price over a certain period in time, for example 30 days before conversion. This helps to deter share price manipulation incentives.

Chapter 4

Existing Pricing Approaches

4.1 Balance Sheet Pricing

Since CoCos depend heavily on balance sheet and accounting figures, one approach is to price them by pricing the balance sheet. In general, pricing the balance sheet is nothing new and was introduced as early as 1974 by Merton[12]. The basic idea is that the balance sheet is treated as if it was following a stochastic process, most often a Geometric Brownian Motion(GBM). Hilscher and Raviv[8] present a way of pricing CoCos by modeling the balance sheet. Assuming a bank has issued debt, CoCos and equity, they define the balance sheet asset value level for which the (senior) debt will default (the debt default threshold) as:

$$K_D = F_D(1 - \gamma)$$

where F_D is the face value of the debt and γ is a parameter with $0 \leq \gamma \leq 1$.

The asset value for which the CoCo will convert (the CoCo conversion threshold) looks as:

$$K_C = (1 + \beta)(F_C + F_D)$$

where F_C is the face value of the CoCo and β is a parameter that measures the distance between the conversion threshold and the book value of the debt.

The pricing of the CoCo is divided into three mutually exclusive events:

- 1. There is no default and the CoCo does not convert
- 2. There is no default and the CoCo does convert
- 3. There is a default and the CoCo does convert

The CoCo price can be estimated by using a combination of normal and binary barrier options with the balance sheet as underlying to match the payoff from the events. The no-default-no-conversion event can be modeled by long positions in binary down and out barrier options, and the two conversion events (with and without default) can be modeled as a spread position in two normal down and in barrier options. The price of the CoCo, P_{CoCo}, looks as follows:

$$P_{\text{CoCo}} = F_C \text{BDO}(K_C) + \alpha\Big(\text{DI}(K_C, F_D) - \text{DI}(K_D, F_D)\Big)$$

where $BDO(X)$ is a binary down and out barrier option with barrier X and $DI(X,Y)$ is a normal down and in barrier option with barrier X and strike Y. The first term corresponds to the payoff of F_C long positions in binary down and out options with barrier at the conversion threshold (corresponding to the no-default-no-conversion event), and the second term corresponds to the payoff of α long positions in normal down and in positions with barrier at the conversion threshold and α short positions in normal down and in positions with barrier at the debt default threshold (corresponding to the two conversion events).

Furthermore, it is assumed that the balance sheet asset process follows a GBM under the risk neutral probability measure:

$$\frac{dA}{A} = r_A dt + \sigma_A dW_t$$

By following this approach the price of the CoCo can be approximated using the closed form barrier options pricing formulas for an underlying following the GBM first presented by Merton in [11].

The model is based on all standard Black-Scholes assumptions (frictionless markets for the asset, its underlying and cash, fractional positions, no arbitrage, etc.). In addition, it assumes that trading in the underlying asset, the balance sheet, is possible. Furthermore, it assumes that all assets on the balance sheet have the same maturity T.

4.2 Credit Derivative Pricing

A second way of tackling the problem of CoCo pricing presented in [17] is to treat the CoCo as a credit instrument, and to use the reduced form approach to credit instrument pricing (also called intensity based credit modeling). The reduced form approach simplifies the Merton approach in modeling the balance sheet presented above by instead looking at default probability and loss given default for the involved company. In general, credit instruments are often quoted by their credit spread over the risk free interest rate. The higher risk the instrument inherits, or is perceived by the market to inherit, the higher the credit spread and the higher the yield to the holder will be. In the reduced form approach, the credit spread is linked to the probability of default and loss given default in a simple formula called the credit triangle:

$$\frac{cs}{1-R} = \lambda$$

or, since the sought after magnitude is the credit spread:

$$cs = \lambda(1-R)$$

where cs is the credit spread, R is the recovery rate and λ is the default intensity. λ is the intensity with which the company will default. In the event of default, the recovery rate is the percentage amount of invested money one can expect to get recovered. The default intensity is based on an underlying Poisson process, $V(t)$, which represents the number of events that occurred up to time t. The process starts at 0, has independent increments and a probability of an event in the timestep dt equal to λdt.

When applying this approach to CoCos, some tweaking of the variables is required. First of all, the default intensity, λ, has to be replaced by a trigger intensity, λ_{trig}, which reasonably has a higher value than the corresponding default intesity. The trigger intensity is linked to the probability of hitting the trigger, p_{trig}, in the following way (follows from the properties of the Poisson distribution):

$$p_{\text{trig}} = 1 - e^{-\lambda_{\text{trig}} T} \tag{4.1}$$

where T is the time to maturity for the CoCo. The probability of hitting the trigger can, however, be estimated from the probability of hitting the barrier in a barrier option setting. In this case, the barrier would correspond to the (expected) share price at conversion, S_{trig}.

Assuming a GBM for the underlying stock process under the risk neutral probability measure in the following equation, a closed form solution for p_{trig} is available (see [16] for details).

$$\frac{dS}{S} = (r - q)\, dt + \sigma dW_t \tag{4.2}$$

By inverting equation 4.1, the trigger intensity can now be expressed as a function of the stock process properties and T as:

$$\lambda_{\text{trig}} = -\frac{\log(1 - p_{\text{trig}})}{T}$$

Secondly, the recovery rate, R_{CoCo}, will be determined as a function of the conversion price, C_p, and the share price at conversion, S_{trig}. When conversion takes place, the CoCo holder will receive C_r shares per CoCo. These shares are currently trading at S_{trig}. The loss for the CoCo holder springing from the conversion is:

$$L_{\text{CoCo}} = F - C_r S_{\text{trig}}$$

However, inserting equation 3.1 yields:

$$L_{\text{CoCo}} = F - C_r S_{\text{trig}} = F\left(1 - \frac{C_r S_{\text{trig}}}{F}\right) = F\left(1 - \frac{\alpha S_{\text{trig}}}{C_p}\right) \tag{4.3}$$

Equation 4.3 implicitly tells us that the recovery rate for the CoCo is $\frac{\alpha S_{\text{trig}}}{C_p}$ since this is the part subtracted when computing the loss. Hence:

$$R_{\text{CoCo}} = \frac{\alpha S_{\text{trig}}}{C_p}$$

and the equation for the credit spread of the CoCo finally becomes:

$$cs_{\text{CoCo}} = \lambda_{\text{trig}}(1 - R_{\text{CoCo}}) = -\frac{\log(1 - p_{\text{trig}})}{T}\left(1 - \frac{\alpha S_{\text{trig}}}{C_p}\right)$$

This approach, as with the balance sheet approach, is based on all standard Black-Scholes assumptions. In addition, it is based on the assumption of a flat risk free interest rate curve on which the credit spread can be applied everywhere. Further it assumes a Poisson process for the trigger event.

4.3 Equity Derivatives Pricing

A third method of pricing CoCos presented in [17] is the equity derivatives approach. The idea is to replicate the CoCo by using well-known instruments that can mimic the payoff structure and the other properties of the CoCo. One CoCo contract can be approximated by taking a long position in a corporate bond (CB) of the issuer, a long position in a down and in forward contract (DIF) and short positions in i binary down and in options (BDI) (more specifically these are called down and in cash at expiration or nothing binary barrier options, see [5] for details), all with the stock as underlying and where i corresponds to the coupon payments that are to be delivered over the life of the CoCo. The approximation looks as follows:

$$\text{CoCo} \approx \text{CB} + \text{DIF} - \sum_i \text{BDI}_i \tag{4.4}$$

Before conversion takes place, the CoCo behaves like a long position in a corporate bond with a face value F and periodical payments c_i at time points t_i. When conversion takes place, the CoCo holder will not receive these periodic payments any more. This is handled by the down and in binary options with the barrier situated at the (expected) share price when conversion takes place, S_{trig}, and maturities at t_i. Selling short c_i down and in binary options and doing this for all i will cancel out the cash flows that were originally expected from the corporate bond.

The CoCo holder is exposed to acquiring C_r shares of the company in the event of conversion as long as the CoCo is alive. At first sight, this may look easy to replicate with a down and in call option with the barrier situated on the (expected) share price at conversion, S_{trig}. However, this is not the case since the down and in call option is only exposed towards the underlying share on the upside. When the CoCo is converted into shares, the holder is exposed to further share price deterioration as well. As a long position in a normal forward contract on the share can be replicated by a long position in a normal call option and a short position in a normal put option with the same strike (called a synthetic forward), a long position in a down and in forward can be created in a similar way by taking a long position in a down and in call option and a short position in a down and in put option with the same strike and barrier. The holder of the down and in forward will receive a forward contract on the share when the barrier is touched. This is not the same as receiving shares, since there may be a major difference in the fact that the holder will not receive the dividends of the underlying stock over the duration of the forward contract. However, it is reasonable to believe that the company will be restrictive with its dividend payments when they are in a situation in which the CoCos have been triggered, which renders the down and in forward a reasonable approximation of receiving shares.

By assuming a GBM for the underlying stock process as in equation 4.2, a closed form solution for the price of the CoCo can be derived. As the credit derivative approach, the equity derivative approach relies on the barrier option pricing formulas presented in [11]. The present value of the long position in the corporate bond can be computed in the following way:

$$P_{\text{CB}} = Fe^{-rT} + \sum_{i=1}^{k} c_i e^{-rt_i} \tag{4.5}$$

Note that the bond cash flows are discounted with the risk-free interest rate r. This makes sense since the extra risk that the corporate bond inherits over the risk-free asset is already taken care of in the down and in forward and binary options.

The formula for the present value of the long position in the down and in forward looks like follows:

$$\begin{aligned} P_{\mathrm{DIF}} = C_r\Bigg(\Bigg(&S e^{-qT}\left(\frac{S_{\text{trig}}}{S}\right)^{2\mu} N\left(\frac{\log\left(\frac{S_{\text{trig}}}{S}\right)}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T}\right) - \\ &Ke^{-rT}\left(\frac{S_{\text{trig}}}{S}\right)^{2\mu-2} N\left(\frac{\log\left(\frac{S_{\text{trig}}}{S}\right)}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} - \sigma\sqrt{T}\right)\Bigg) - \\ &\Bigg(Ke^{-rT}N\left(-\frac{\log\left(\frac{S}{S_{\text{trig}}}\right)}{\sigma\sqrt{T}} - \mu\sigma\sqrt{T} + \sigma\sqrt{T}\right) - \\ &S e^{-qT}N\left(-\frac{\log\left(\frac{S}{S_{\text{trig}}}\right)}{\sigma\sqrt{T}} - \mu\sigma\sqrt{T}\right)\Bigg)\Bigg) \end{aligned} \tag{4.6}$$

where

$$\mu = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

and where $N(x)$ is the cumulative distribution function for the normal distribution. Equation 4.6 corresponds to C_r long positions in down and in call options and C_r short positions in down and in put options, both with strike S_{trig}.

The formula for the present value of the short positions in the binary down and in options looks like follows:

$$\begin{aligned} P_{-\sum_i \mathrm{BDI}_i} = -\alpha\sum_{i=1}^{k} c_i e^{-rt_i}\Bigg(&N\left(-\frac{\log\left(\frac{S}{S_{\text{trig}}}\right)}{\sigma\sqrt{t_i}} - \lambda\sigma\sqrt{t_i} + \sigma\sqrt{t_i}\right) + \\ &\left(\frac{S_{\text{trig}}}{S}\right)^{2\lambda-2} N\left(\frac{\log\left(\frac{S_{\text{trig}}}{S}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} - \sigma\sqrt{t_i}\right)\Bigg) \end{aligned} \tag{4.7}$$

Equation 4.7 corresponds to αc_i short positions in the binary down and in option maturing at t_i with strike S_{trig} for all i.

This approach relies on all standard Black-Scholes assumptions for all the barrier and binary barrier options.

4.3.1 Pricing Example

In this section a pricing example for the equity derivatives approach is worked out. We want to price a full ($\alpha = 1$) CoCo with face value \$100, annual coupon of 4% paid semi-annually and with a remaining time to maturity of 3 years. The current stock price is at \$7, the market trigger at \$3 and the conversion price at \$4. Furthermore, the risk-free rate of return is 3%, the stock volatility is 40% and the stock pays a continuous dividend yield of 2% per year. Inserting the above properties in the formulas presented in section 4.3, yields the following price of the CoCo:

$$P_{\text{CoCo}} = P_{\text{CB}} + P_{\text{DIF}} + P_{-\sum_i \text{BDI}_i} = 102.7831 + (-6.8648) + (-1.7336) = 94.1848$$

Below are tables on how the above CoCo price changes with changes in the parameters:

Table 4.1: CoCo price sensitivities.

(a) Share Price S

S	CoCo Price
3	70.6323
4	79.6345
5	86.3449
6	90.9752
7	94.1848
8	96.4408

(b) Time to maturity T

T	CoCo Price
0.5	100.3768
1	99.6749
2	96.8008
3	94.1848
4	92.0561
5	90.2769

(c) Volatility σ

σ	CoCo Price
0.1	102.7831
0.2	102.3266
0.3	99.0159
0.4	94.1848
0.5	89.6518
0.6	85.8891

(d) Conversion price C_p

C_p	CoCo Price
2	115.8428
3	101.4041
4	94.1848
5	89.8531
6	86.9654
7	84.9027

(e) Trigger price S_{trig}

S_{trig}	CoCo Price
1	101.9835
2	96.9412
3	94.1848
4	99.8504
5	114.5944
6	136.8580

(f) Conversion fraction α

α	CoCo Price
0	102.7831
0.20	101.0634
0.40	99.3438
0.60	97.6241
0.80	95.9044
1	94.1848

As can be seen in table 4.1(a) the CoCo price increases with increasing share price, i.e. the delta of the CoCo is positive. In this CoCo example, there is some loss to be taken by the CoCo holder at conversion since the conversion price is higher than the trigger price. The CoCo will be worth more the lower the risk of conversion. Naturally this risk decreases for increasing share price.

In table 4.1(b), the CoCo price for different Ts are shown. The CoCo theta is negative, that is, the price decreases with increasing T. Naturally, a longer time period leaves more room for the negative event to happen. It is more likely that the company runs into trouble in a time scope of five years rather than in only one year, ceteris paribus.

The vega sensitivity, the sensitivity to changes in σ, is negative as seen in table 4.1(c). A higher volatility implies a higher risk of hitting the trigger. For the very low volatility of 10%, the CoCo price coincides with the corporate bond price down to the fourth decimal, suggesting an extremely low conversion probability.

Table 4.1(d) shows the sensitivity towards the conversion price C_p. The higher the conversion price, the higher the loss for the CoCo holder at conversion will be, as she will be receiving less shares per CoCo. For a conversion price of \$2, which is lower than the trigger price, conversion is a positive event for the holder, i.e. she will make a profit. Hence, the price is higher than for only the corporate bond.

The CoCo prices for different trigger prices S_{trig} are shown in table 4.1(e). At first glance, this table looks very unintuitive. For a high trigger price (rendering a high probability of conversion), the CoCo price is increasing with increasing trigger price. This is however a secondary effect from the fact that the conversion price is not scaled up at the same time. For S_{trig} = \$5 or \$6 the trigger price is higher than the conversion price, rendering the conversion a positive event for the holder. This illustrates the very important influence the combination of the trigger price and the conversion price has on the CoCo price.

As can be seen in table 4.1(f), the CoCo price is linearly decreasing with increasing α. The losses at conversion are larger with a higher α.

4.4 Hedging

Hedging CoCo positions is indeed an intricate task. As seen in section 4.3.1, the CoCo has a positive delta. Delta hedging is therefore performed by short selling the underlying share. For long maturities, this presents no particular problem as the shares are shorted on a more or less one-to-one basis with respect to the CoCo. For short maturities, delta hedging is more tricky. Since there is a discontinuity in the instrument payoff at the trigger, there is also a discontinuity in the delta at that same point. This discontinuity is more outspoken for short maturities. With a share price close to the barrier, delta increases beyond one. This means that the CoCo holder, while trying to hedge her perceived soon-to-be long position in the underlying shares, will short sell more than one share per CoCo. This will likely lower the share price, which in turn increases the delta of the CoCo, inducing the holder to short sell even more shares. Given enough delta hedging CoCo investors, this may itself trigger the CoCo conversion. This negative feedback loop is called a death spiral and it is the most criticized property of the CoCo's among its opponents. In the case the trigger is not hit, a jump upwards in the share price is probable as the "dangerous situation" is over, leaving the now over-short sold CoCo hedger with losses on the hedge. These behaviors of the instrument towards its underlying is called negative convexity. For more details and in-depth analysis on the hedging issue, see [16].

Chapter 5

A New Approach

5.1 Critique

The general critique against the usual pricing methods within mathematical finance is that they are based on models which assume that log returns of assets are independent and identically normally distributed. Numerous studies have shown that the normal approximation is too simplistic and that it does not capture the events that usually have the most impact on the price of the asset. This is of course also true when pricing CoCos and even more so than for vanilla stock options since the behavior close to the barrier may be even less well-approximated by a normal distribution than that for vanilla options. There is a reasonable risk that large jumps in the share price will occur when the stock is trading close to the trigger level. Barrier options in general and CoCos in particular hence introduce an additional property which moves them even further away from the world of vanilla options pricing, for which the normal distribution is not a good approximation in the first place. In addition, a large fraction of the price of a CoCo depends on the behavior around the trigger. An investor with long positions in CoCos is especially interested in determining a fair value of the instrument when the trigger is within reach. Therefore the GBM is simply not good enough as an approximation for modeling an asset process related to CoCos.

A second shortcoming, specifically related to the balance sheet pricing approach, is the fact that the underlying is not a tradeable asset. The closed form analytical formulas for barrier options are derived in a risk-neutral world where a position in an option can be fully replicated by taking positions in the asset underlying the option and some risk-free asset. In this case, there is no way of investing in the underlying asset (the asset side of the balance sheet), meaning that the theoretical foundation of the pricing approach is weak. One would argue that investing in the shares of the company works as a proxy replication of taking a position in the assets. The theoretical territory in which this approach is performed is however quite unclear.

The credit derivative approach suffers from the shortcoming that it moves the pricing away from the underlying stock behavior and focuses more on the similarities with bonds. However, the stock process is the main driver of the CoCo valuation, particularly for market based triggers.

Hence, the rest of this paper will focus on the equity derivatives approach.

5.2 Introducing Jump-Diffusion

Already in 1976, Merton[13] suggested a development of the GBM approach by allowing for discrete jumps to take place in addition to the existing diffusion process. The resulting jump-diffusion is presented to the point in [18]. Under the risk neutral probability measure, the underlying process looks as follows[2]:

$$\frac{dS}{S} = \left(r - q - \lambda\bar{k}\right) dt + \sigma dW + kdq$$

The jumps occurrence are assumed to follow a Poisson process with parameter λ, and the jump magnitudes are assumed to follow a log-normal distribution. For European vanilla options with an underlying following the above process, closed form analytical formulas are available. Unfortunately, this is not the case for barrier options.

5.2.1 Barrier Options under Jump-Diffusion

There are two main approaches to option pricing when closed form analytical formulas are not available, tree modeling and simulation. Hillard and Schwartz[7] describe an approach for pricing normal American options by constructing a bivariate tree (one dimension for the diffusion and one for the jumps). The diffusion is modeled by a standard binomial tree and the jumps are added in every time step. Influenced by [7], Albert, Fink and Fink[1] further extend the model to better incorporate barrier options. This approach is presented below.

5.2.2 Trinomial Hillard and Schwartz

In a first extension step, a trinomial tree is used in the diffusion dimension and the jumps are added. By noting that the return process, V_t, of the underlying can be written in the following way:

$$\begin{aligned} V_t &= \log\left(\frac{S_t}{S_0}\right) = X_t + Y_t \\ X_t &= \left(r - q - \lambda\bar{k} - \frac{\sigma^2}{2}\right) t + \sigma W_t \\ Y_t &= \sum_{i=0}^{n(t)} \log(1 + k_i) \end{aligned}$$

where X_t corresponds to the diffusion dimension, Y_t corresponds to the jump dimension, $n(t)$ is the Poisson distributed number of jumps up to time t and k_i is the magnitude of the ith jump, the two dimensions can be separated. This approach greatly simplifies the setting up of the tree. With the diffusion and jump dimensions separated into two different terms (implicitly stating their independence), the moment matching equations for determining the move probabilities and magnitudes can also be solved separately.

One time step in the tree looks like follows for the stock return process:

$$V^{up}_{t+\Delta t,j} = V_t + h + j\eta, \qquad j = 0, \pm 1, \pm 2, \ldots, \pm m \tag{5.2a}$$

$$V^{0}_{t+\Delta t,j} = V_t + j\eta, \qquad j = 0, \pm 1, \pm 2, \ldots, \pm m \tag{5.2b}$$

$$V^{down}_{t+\Delta t,j} = V_t - h + j\eta, \qquad j = 0, \pm 1, \pm 2, \ldots, \pm m \tag{5.2c}$$

where h is the magnitude of the diffusion move and η is the magnitude of the jump move. The three equations correspond to an up, down and no move for the diffusion and there are M states in each group after the jump for each of these moves. There are m jumps in each direction (up and down) and one state corresponding to no jump. In [7], studies with $M = 5, 7, 9$ are examined and a reasonable combination of speed and accuracy is provided when $M = 7$. That is also followed in [1]. This means $m = 3$ and hence three up and three down jumps. Grouping the nodes by diffusion (i.e. all nodes with a certain number of up diffusion moves belong to the same group) there will be $1 + 2i$ groups and $1 + 6i$ nodes per group in the ith time step. In each time step, the number of attainable diffusion states increases by 2 and the number of attainable jump states per group increases by 6. Below is a figure illustrating the tree setup for two time steps. For visual reasons the diffusion magnitude h is larger than the jump magnitude η, however this may not always be the case.

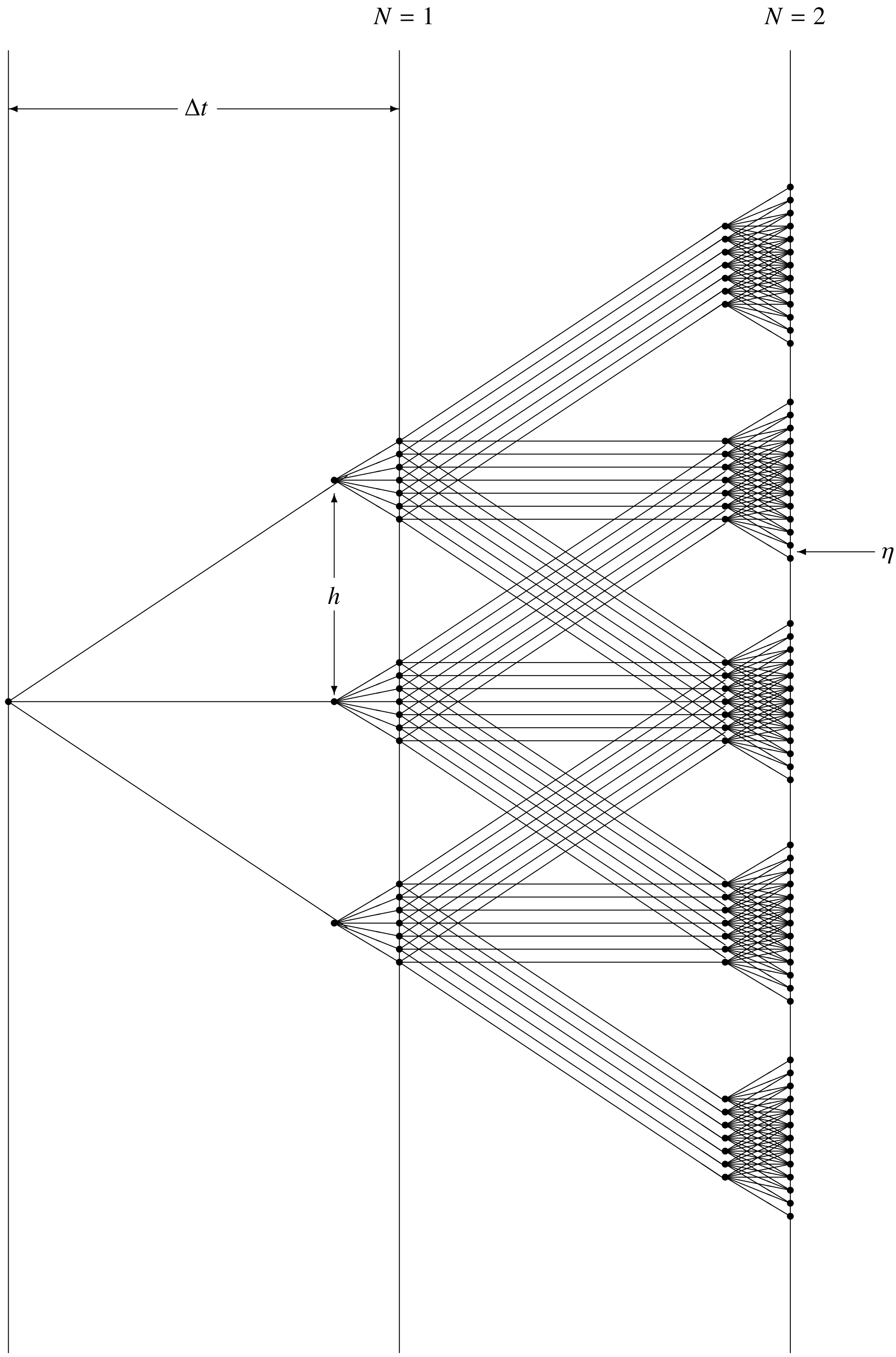

Figure 5.1: Sketch of the two first steps in the Hillard Schwartz trinomial tree.

By deciding on a reasonable number of time steps N, Δt is determined through the relation $\Delta t = \frac{T}{N}$ and a negative diffusion probability can be avoided by keeping a reasonable proportion between Δt and h. This is carried out by making sure h fulfills the relation:

$$h = \sqrt{3\sigma^2 \Delta t} \tag{5.3}$$

The jump magnitude is set to be:

$$\eta = \sqrt{(\gamma')^2 + \delta^2}$$

The option price is worked out by backward recursion in the following way:

$$\Pi(V_i, t) = e^{-r\Delta t} \sum_{j=-m}^{m} q(j)\Big(p_u \Pi(V_i + h + j\eta, t + \Delta t) + \\ p_m \Pi(V_i + j\eta, t + \Delta t) + p_d \Pi(V_i - h + j\eta, t + \Delta t)\Big)$$

The diffusion probabilities p_u, p_m, p_d are determined by matching the first two moments of the diffusion dimension of the tree with the first two moments of the diffusion dimension of the underlying process. This is done by solving the following system of equations:

$$E\Big(\log\left(S\left(t+\Delta t\right)\right) - \log\left(S\left(t\right)\right)\Big) = \left(r - q - \lambda\bar{k} - \frac{\sigma^2}{2}\right)\Delta t = p_u h + p_m 0 + p_d(-h)$$
$$E\Big(\Big(\log\left(S\left(t+\Delta t\right)\right) - \log\left(S\left(t\right)\right)\Big)^2\Big) = \sigma^2 \Delta t = p_u h^2 + p_m 0 + p_d(-h)^2$$
$$p_u + p_m + p_d = 1$$

Solving this system generates the following expressions:

$$p_u = \frac{\left(r - q - \lambda\bar{k} - \frac{\sigma^2}{2}\right)\Delta t}{2h} + \frac{\sigma^2 \Delta t}{2h^2} \tag{5.5a}$$

$$p_d = \frac{\sigma^2 \Delta t}{2h^2} - \frac{\left(r - q - \lambda\bar{k} - \frac{\sigma^2}{2}\right)\Delta t}{2h} \tag{5.5b}$$

$$p_m = 1 - p_u - p_m = 1 - \frac{\sigma^2 \Delta t}{h^2} \tag{5.5c}$$

For the jump probabilities, $q(j)$, a moment matching condition is again set up. The setup permits $2m$ moments to be matched and the goal is to match the moments of the jump dimension in the tree with the moments of the jump dimension of the underlying process. This can be done by solving the following system of equations:

$$\sum_{j=-m}^{m} (j\eta)^{i-1} q(j) = E\Big(\sum_{j=0}^{n\Delta t} \log(1 + k_j)\Big)^{i-1} \tag{5.6}$$

However, the right hand side of equation 5.6 is hard to compute. The cumulants (see appendix A for a definition of the relevant cumulants) $\mu'(\Delta t)$ are used to approximate the right hand side of equation 5.6 in [7] in the following way:

$$\sum_{j=-m}^{m} (j\eta)^{i-1} q(j) = \mu'_{i-1}(\Delta t) \tag{5.7}$$

In addition, the sum of all qs needs to be equal to 1. This generates the following system of equations for $M = 7$:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 9 & 4 & 1 & 0 & 1 & 4 & 9 \\ -27 & -8 & -1 & 0 & 1 & 8 & 27 \\ 81 & 16 & 1 & 0 & 1 & 16 & 81 \\ -243 & -32 & 1 & 0 & 1 & 32 & 243 \\ 729 & 64 & 1 & 0 & 1 & 64 & 729 \end{pmatrix} \begin{pmatrix} q(1) \\ q(2) \\ q(3) \\ q(4) \\ q(5) \\ q(6) \\ q(7) \end{pmatrix} = \begin{pmatrix} \mu_0 \\ \mu_1/\eta \\ \mu_2/\eta^2 \\ \mu_3/\eta^3 \\ \mu_4/\eta^4 \\ \mu_5/\eta^5 \\ \mu_6/\eta^6 \end{pmatrix}$$

where $q(1)$ is the probability of 3 jumps downwards, $q(2)$ the probability of 2 jumps downwards and so on.

5.2.3 The Adaptive Mesh Model

When applying tree modeling in option pricing, the specification is mainly prone to two types of errors. First, there is the distribution error. The tree approximates continuous probability distribution of the underlying with a discrete approximation. Second, there is nonlinearity error stemming from the nonlinearity of the option payoff function. This nonlinearity error is especially pronounced around the strike price for vanilla options and around the barrier for barrier options. The second error can be countered by increasing the number of steps in the tree. However, this greatly increases the demand for computational power. Instead, another method introduced by Figlewski and Gao [4], called the adaptive mesh model (AMM), can help to reduce the nonlinearity error. By increasing the number of steps only where it is needed the most, the nonlinearity error can be brought down to reasonable levels while the computational effort stays within manageable limits. The AMM technique differs for various types of options. For vanilla options, a finer mesh is introduced for time steps close to exercise and when the price of the underlying is close to the strike. For barrier options, the finer mesh is instead introduced around the barrier and for all time steps. In the base tree, a more granular branch structure is introduced that flows into the base tree at certain anchor points. The fine mesh information is branched out in the base tree and incorporated into the pricing recursion. The behavior can be micromanaged in the part of the tree that requires most attention from a pricing perspective, without having the tree containing superfluous information in the remaining parts, which are less sensible.

5.2.4 The Ritchken Technique

By applying a technique presented by Ritchken[14], the nonlinearity error for barrier options can be reduced further. He notes that the best convergence is attained when the log-distance between the barrier and the stock price is equal to an integer number of diffusion steps. This makes a branch level in the tree fall directly on the barrier. The barrier in the tree does not have to be "approximated" by the nearest branch level but is represented exactly instead, to reduce the nonlinearity error. Using this technique, N is not determined exogenously but rather set within the model. h is determined from specifying an integer B number of diffusion moves between the barrier and the initial stock price. Combining this with equation 5.3, Δt can be implicitly determined, which in turn determines N in the following way:

$$N = \frac{3T\sigma^2 B^2}{(\log(S) - \log(S_{trig}))^2} \tag{5.8}$$

5.2.5 The Albert, Fink, Fink Approach

In [1], Albert, Fink and Fink (AFF) combine the AMM with the Ritchken technique and the trinomial jump-diffusion tree developed by Hillard and Schwartz into a method that is able to incorporate jumps into the diffusion, which still can price down and out barrier options accurately.

The tree is divided into the base tree and a fine mesh. Each time step of the base tree is, when the share price is close to the barrier, divided into finer steps that fit into the base tree at certain anchor groups. For the finer mesh, the diffusion magnitude is $h/2$, whereas the jump magnitude is the same as in the base tree. However, the amount of jump states has to be different in the fine mesh. With $M = 7$, the base tree adds 6 new jump states for each base time step. Hence, the fine mesh has to add 6 in total after its intermediate steps. For maximum information preservation the following equation has to be fulfilled:

$$M_{\text{base}} = n_{\text{fine}}(M_{\text{fine}} - 1) + 1$$

where n_{fine} is the number of fine time steps for each base time step. AFF choose $n_{\text{fine}} = 3$ which gives $M_{\text{fine}} = 3$.

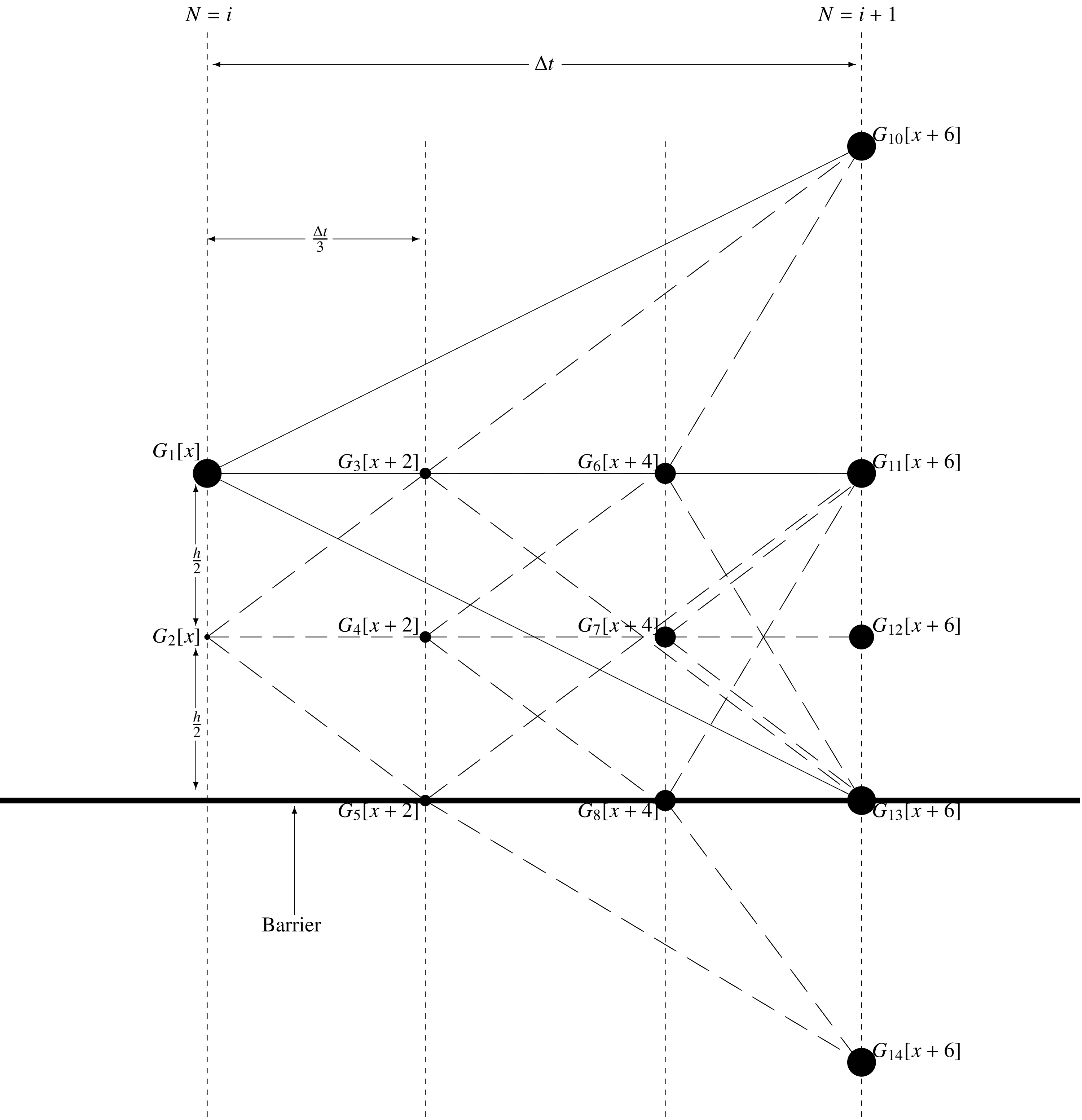

Figure 5.2: Sketch of an AMM step with Hillard Schwartz trinomial base tree.

In figure 5.2, one time step of the AFF setup is outlined. Each dot corresponds to a group of nodes. The number of nodes in each group depends on the time step i (in brackets [] with $x = 1 + 6i$). The full lines correspond to the base tree (connecting node groups G_1, G_{10}, G_{11} and G_{13}), and the dashed lines correspond to the fine mesh. The base tree is constructed by introducing its starting point (group G_1) above the barrier at twice the log-distance between the current share price and the barrier. The current share price, for which the option price is interesting, is now the starting point in the finer mesh (group G_2). The diffusion magnitude, h, is set to the log-distance between the base tree starting point and the barrier.

As mentioned above, the number of jump states for the fine mesh is 3. However, the number of jump states has to be $M_{G_3} = M_{G_5} = 5$ for the groups G_3 and G_5 since they connect directly to the base tree at G_{10}, G_{11} and G_{13}. In spite of the fact that the option is seemingly void for the groups G_5, G_8 and G_{13}, this is not the case as there are states in these groups with jumps upwards, hence not on or below the barrier. The support group, G_{14} (only needed in the first base time step), shares the same visual deception. It may be the case that the jump magnitude is be larger than the diffusion magnitude. Hence, a diffusion move downwards from the barrier in combination with a jump move upwards would end up above the barrier. Even though it looks like the group G_{14} is below the barrier, it may be the case that states in the group would still be above it.

The probabilities in the fine mesh are not the same as in the base tree. For the groups G_3, G_5, G_6 and G_8, the diffusion probabilities are determined by replacing Δt with $\frac{2}{3}\Delta t$ for G_3 and G_5 and with $\frac{1}{3}\Delta t$ for G_6 and G_8 in equations 5.5. For G_2, G_4 and G_7, Δt is replaced by $\frac{1}{3}\Delta t$ and h by $\frac{h}{2}$ in the same equations. The jump probabilities are computed by solving equations 5.7 with $m = 2$ for $\frac{2}{3}\Delta t$ corresponding to the points G_3 and G_5, and $m = 1$ for $\frac{1}{3}\Delta t$ corresponding to the rest of the fine mesh.

A final note to be mentioned is that the AMM applied in the AFF approach is only required when the stock price is close to the barrier. With larger distance between the two, the diffusion magnitude h is larger and the number of time steps in the tree N are smaller. In these areas it is sufficient to apply the trinomial Hillard and Schwartz approach with Ritchken technique, but with an integer number of diffusion move steps B (instead of only one), to produce a reasonable number of time steps N.

5.2.6 Application on CoCos

The AFF method is set up for down and out options. In the CoCo case, down and in options needs to be priced. Similar to the put-call-parity for vanilla options, there is a in-out-parity for barrier options:

$$P = P_{\mathrm{DI}} + P_{\mathrm{DO}} \tag{5.9}$$

which states that the price of the vanilla option is the sum of the price of the down and in option and that of the down and out option. Equation 5.9 holds for both call and put options as well as for down and up barrier ones. With the closed form analytical formula for vanilla options under a jump-diffusion process with normally distributed jumps developed by Merton[11] and presented by Hull[9]:

$$P_{\mathrm{JD}} = \sum_{n=0}^{\infty} \frac{e^{\lambda(1+\bar{k})T}(\lambda(1+\bar{k})T)^n}{n!} C_{BS}\left(S, K, T, r - \lambda\bar{k} + \frac{n\gamma}{T}, q, \sigma^2 + \frac{n\delta^2}{T}\right) \tag{5.10}$$

where C_{BS} is the Black-Scholes price, the price for the vanilla option can be determined. The two options in the down and in forward package can be determined by combining equations 5.9 and 5.10 with the down and out barrier options priced by the AFF approach. For the binary barrier options, it is worth mentioning that the option price reflects the probability of hitting the barrier. Accordingly, the payoff of the down and in option is mutually exclusive to that of the down and out option. Holding both options renders a guaranteed \$1 at maturity. The price of the binary down and in option is therefore \$1 discounted with the risk-free rate minus the price of the binary down and out option determined by the AFF approach.

The approximation of a CoCo contract in equation 4.4 can be then rewritten in the following way:

$$\mathrm{CoCo} \approx \mathrm{CB} + \mathrm{DIF} - \sum_i \mathrm{BDI}_i = \mathrm{CB} + \Big((\mathrm{C} - \mathrm{DOC}) - (\mathrm{P} - \mathrm{DOP})\Big) - \sum_i (1 \times e^{-rt_i} - \mathrm{BDO}_i)$$

where C and P are vanilla call and put, DOC and DOP are down and out call and put and BDO_i are binary down and out options.

When pricing, P_{CB} is determined in equation 4.5, P_C and P_P in equation 5.10, and P_{DOC}, P_{DOP} and P_{BDO_i} for all i are computed by the AFF approach.

Chapter 6

Empirical Results and Discussions

6.1 Convergence

6.1.1 Hillard and Schwartz Approach

Pricing of a CoCo is feasible by setting up a Hillard and Schwartz tree for each barrier option included in the CoCo structure (two for the down and in forward and i for the binary down and in options). By setting the jump intensity, λ, to zero, a convergence study between the AFF approach versus the analytical value of the CoCo can be made.

The following figure shows the error development for the put option included in the synthetic down and in forward for the CoCo setup presented in section 4.3.1:

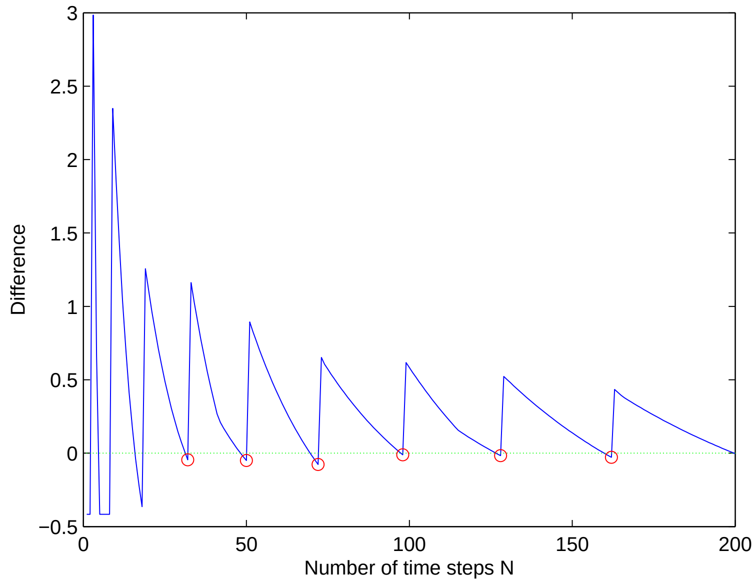

Figure 6.1: Difference between tree and analytical option prices for the down and in put option.

As one can see, the convergence of the Hillard and Schwartz approach shows the sawtooth behavior which is well-known in barrier option pricing with trees (see [3] for examples and more details). The points in figure 6.1 highlighted with red circles are the ones where one diffusion level falls on the barrier (i.e. where there is an integer number of diffusion steps between the starting point and the barrier). Finding these points is exactly the scope of the Ritchken technique.

6.1.2 The Ritchken Technique

For a single option, the Ritchken procedure starts with specifying a minimum number of allowed time steps $N_{\min}$, followed by finding the smallest N larger than $N_{\min}$ that makes the log-distance between S and S_{trig} an integer B number of diffusion moves h. The following problem is to be solved for the Hillard Schwartz setup:

$$\min_B N = \frac{3T\sigma^2 B^2}{(\log(S) - \log(S_{\text{trig}}))^2}$$
$$\text{s.t. } N \geq N_{\min}$$

Although finding the Ritchken optimal points for multiple options in the same setup may be difficult, the computation is greatly simplified in the CoCo case as S, S_{trig} and σ are the same for all options (they share the same underlying and barrier). In addition, the down and in call and down and in put from the synthetic forward as well as the binary barrier option for the final coupon payment (at maturity) share the same T. The figure below shows the convergence of these three options:

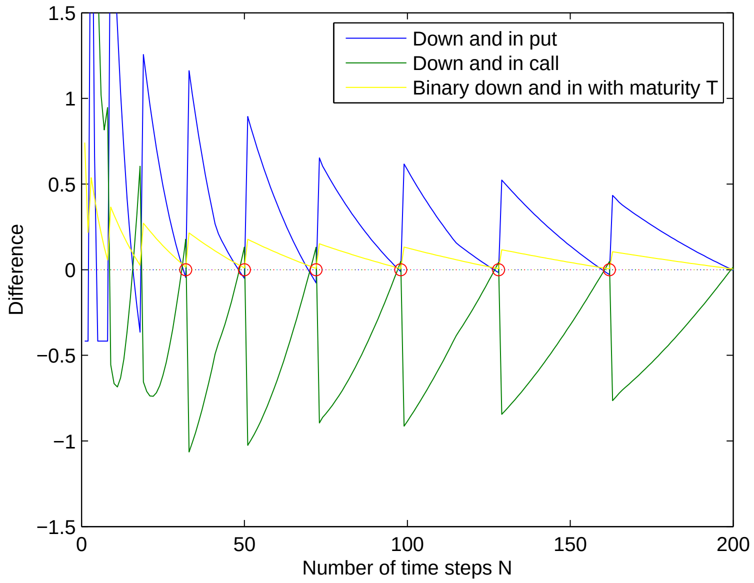

Figure 6.2: Difference between tree and analytical option prices for the down and in put, call and maturity T binary options.

Figure 6.2 shows that the three options have their Ritchken optimal points for the same Ns. However, the rest of the coupon-matching binary barrier options has different time to maturities, t_i, suggesting that different Ritchken optimal Ns are required. A feasible method that makes the time points of the different barrier options coincide, is to scale the time step proportionally to the time to maturity. In the example from section 4.3.1, the factor needed for binary barrier option i will be $\frac{0.5i}{3}$. For the option matching the second-to-last coupon, the relation will be $N_5 = \frac{0.5 \times 5}{3} N_6 = \frac{2.5}{3} N_6$ where N_6 is the number of steps required for the final barrier option as well as for the down and in forward options. The figure below shows the convergence for the second to last binary barrier option:

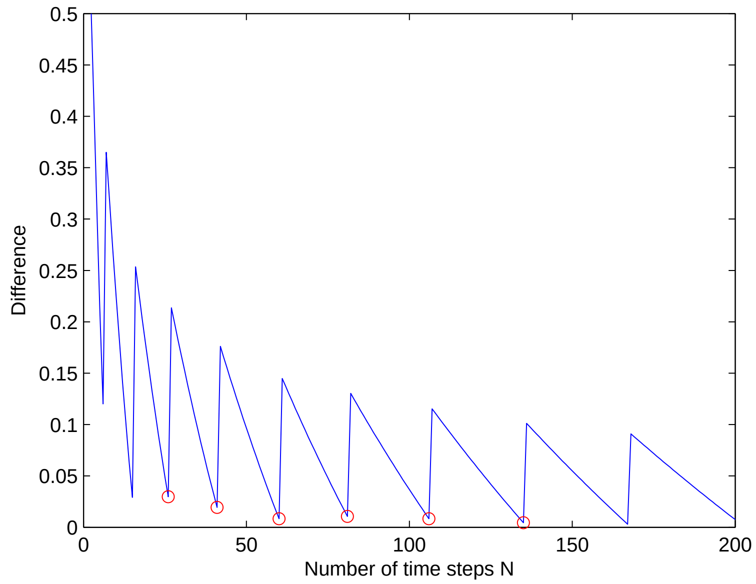

Figure 6.3: Difference between tree and analytical option prices for the second to last binary down and in option.

The optimal point at $N = 162$ in figure 6.2 corresponds to the optimal point $N = \frac{2.5}{3} \times 162 = 135$ in figure 6.3.

A second remark to the two figures above is that the Ritchken optimal points may occasionally diverge with increasing N. However, this is only an effect of the fact that N can only be an integer. In general, the Ritchken optimal points converge for increasing N.

The total CoCo price convergence is shown in the following figure:

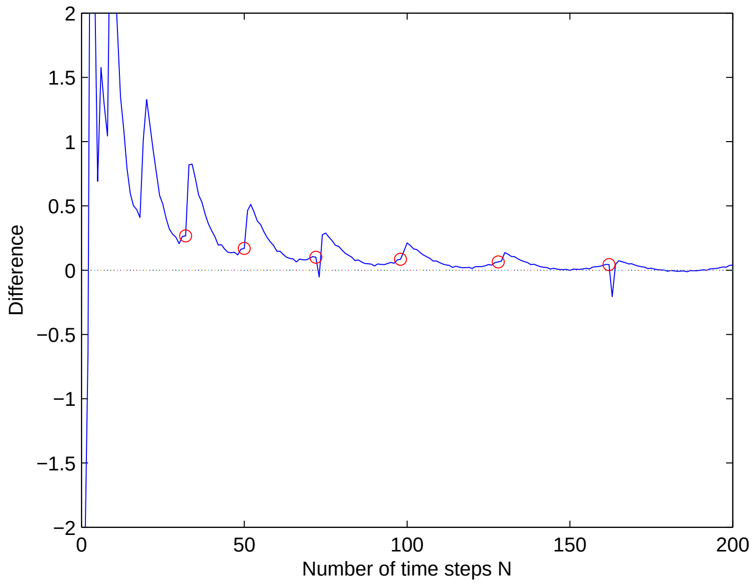

Figure 6.4: Difference between tree and analytical option prices for the CoCo.

The red circles show the Ritchken optimal points. The N depicted on the x-axis in figure 6.4 is the number of steps used for the three options maturing at the CoCo maturity. The rest of the coupon-matching binary barrier options are computed using the method elaborated above. The convergence looks very good, in fact too good. As can be seen in figure 6.2, the errors of the down and in call and put have opposite signs. These two options generate the largest errors and the error of the call is also bigger than that of the put. In the CoCo structure, these two are superpositioned, leading to a much smaller error. The binary barrier options further reduce the size of the resulting error. The spike at $N = 163$ is more how the convergence actually "should" look. There is a Ritchken optimal point at $N = 162$. After Ritchken optimal points, there is always a jump in the errors. The reason for the jump after the $N = 162$ optimal point is the fact that the binary options' points corresponding to $N = 163$ are mapped onto the points corresponding to $N = 162$, which is an effect of the floor rounding required to make N an integer. Hence, they do not offset the effect of the call error being larger than that of the put (due to the fact that their spike only show up on $N = 164$).

Below is a table with data on the CoCo price for different Ritchken optimal Ns:

N	Analytical Price	Computed Price	Difference	in %
32	94.1848	94.4513	0.2666	0.28
50	94.1848	94.3546	0.1698	0.18
72	94.1848	94.2858	0.1010	0.11
98	94.1848	94.2699	0.0851	0.09
128	94.1848	94.2491	0.0644	0.07
162	94.1848	94.2278	0.0430	0.05
200	94.1848	94.2247	0.0399	0.04
242	94.1848	94.2171	0.0324	0.03
288	94.1848	94.2074	0.0227	0.02

Table 6.1: CoCo price for different Ritchken optimal N.

6.1.3 The Adaptive Mesh Model

When the share price is located too close to the barrier, even with $B = 1$ in equation 5.8, the number of required time steps will be very large. This is when the adaptive mesh model is useful. Using the same CoCo parameters as in section 4.3.1 but setting the initial stock price to $S = 3.1$, the number of time steps needed for the Hillard Schwartz approach (with $B = 1$ in equation 5.8) is $N = 1339$, which takes very long time for all 8 options to compute. In a Matlab setup[1], it takes 10585 seconds (just under 3 hours). The AMM needs $N = 334$ steps to compute the same value in only 71 seconds. Below is a table showing the discrepancies versus the analytical price for the Hillard Schwartz approach for a few different initial stock prices S close to the barrier (all with $B = 1$):

S	Analytical Price	HS Price	Difference	in %	N	Running time (s)
3.40	74.4285	74.4559	0.0274	0.04	91	3.8
3.30	73.4917	73.5084	0.0167	0.02	158	18
3.25	73.0187	73.0270	0.0083	0.01	224	50
3.20	72.5434	72.5477	0.0043	0.006	345	184
3.15	72.0663	72.0681	0.0018	0.003	604	976
3.10	71.5883	71.5890	0.0007	0.001	1339	10585
3.08	71.3970	71.3974	0.0004	0.0006	2079	40100

Table 6.2: CoCo price for different S with Hillard and Schwartz trinomial.

Furthermore, the following table shows the result of the AMM approach with the same setup:

[1] Intel Core i5-2430M 2.4GHz machine with 8GB RAM and no specific multiple core usage.

S	Analytical Price	AMM Price	Difference	in %	N	Running time (s)
3.40	74.4285	76.4862	2.0577	2.76	22	0.7
3.30	73.4917	74.7766	1.2849	1.75	39	0.9
3.25	73.0187	73.9670	0.9483	1.30	56	1.4
3.20	72.5434	73.1839	0.6405	0.88	86	2.7
3.15	72.0663	72.4495	0.3832	0.53	151	9.4
3.10	71.5883	71.7698	0.1816	0.25	334	71
3.08	71.3970	71.5164	0.1194	0.17	519	252

Table 6.3: CoCo price for different S with Hillard and Schwartz trinomial and the AMM.

Comparing the results in the two tables the AMM approach reduces the computational time significantly as the share price approaches the barrier. However, there is a certain trade-off between speed and accuracy. The AMM produces rougher figures when the share price is far away from the barrier, due to lower number of time steps. In a practical setup, typically a maximum number of time steps should be defined for choosing between HS and AMM.

6.2 Model discussion

Using the example from section 4.3.1, a comparison between the GBM and the JD approaches can be performed. With the same total variance, both processes produce comparable results. The formula for the total variance of the log-normal jump-diffusion process presented in [2] is the following:

$$v^2 = \sigma_{\mathrm{JD}}^2 + \lambda(\gamma^2 + \delta^2) \tag{6.1}$$

With $\sigma_{\mathrm{GBM}}^2 = v^2$ and by setting λ, γ and δ, and having σ_{JD} implied from equation 6.1, the following table showing CoCo prices for different start prices S is the result:

S	σ_{GBM}	GBM Price	λ	δ	σ_{JD} (implied)	JD Price
3.1	0.3	72.08	5	0.05	0.28	72.12
				0.1	0.2	72.00
			10	0.05	0.25	72.05
				0.1	Im	-
	0.4	71.59	5	0.05	0.38	71.63
				0.1	0.33	71.52
			10	0.05	0.37	71.54
				0.1	0.24	71.16
7	0.3	99.02	5	0.05	0.28	99.25
				0.1	0.2	99.36

			10	0.05	0.25	99.27
				0.1	Im	-
	0.4	94.18	5	0.05	0.38	94.20
				0.1	0.33	94.20
			10	0.05	0.37	94.20
				0.1	0.24	94.25

Table 6.4: CoCo prices for different GBM and JD parameters.

The table shows that the quantitative differences between the two approaches are small when the total variance is kept at the same level. However, the setup is overly theoretical and does not do the the jump-diffusion justice. The advantages of the it are rather shown by some practical remarks, some of which brought up in [18].

First of all, price jumps (for example when new information about the company is available or on a stock market crash) are observed frequently in the empirical studies of many assets' returns. A basic criterion for an asset process model is that it captures as good as possible the real world behavior of the asset. To capture the empirically observed jumps, the volatility in a pure diffusion model has to be set at very high levels, which is not consistent with the behavior of the asset in jump-infrequent time regions. Furthermore, when Merton introduced the jump-diffusion in [13], he stated that Black and Scholes model critically assumes a "local Markov property". This property only allows for small changes in the underlying price in small periods of time. Even though the volatility in the Black-Scholes model can be set to high levels as to capture the variability in itself, it does still not describe the movements of the price process since the large movements tend to occur as jumps rather than as many small changes in the same direction. In the case of CoCos this jump effect is perceived to be even more manifestable since the trigger would most likely introduce an extra jump-inducing property over other types of assets.

Second of all, the jump-diffusion has the possibility to explain the most well-known inconsistency in option pricing (which is most likely present in the CoCo case) - the volatility smile. For options with different strikes, the implied volatilities in the market are different, which is not permitted in the Black-Scholes model. This effect is most pronounced when the time to maturity is short. By making it possible for the underlying asset price to jump in this short time period, the smile inconsistency can be explained.

Third of all, the jump-diffusion model allows for incomplete markets. In the Black-Scholes world, every asset is in itself redundant and does not introduce any new feature in the market. The payoff of a vanilla option can be replicated exactly by a combination of positions in the underlying and the risk free asset. If this was the case in the real world, then all types of options would have been redundant, which is an overly simplistic assumption. Perfect hedging in a jump-diffusion world is not possible by only taking positions in the underlying and the risk-free asset. Additional options are needed in a hedging structure, which is more in line with real world observations.

Fourth of all, the jump-diffusion approach allows for more realistic risk management. Different type of dynamic portfolio strategies can, under a pure diffusion, be constructed so as to have no downside risk and still provide a decent upside return possibility. Examples include the CPPI structured products[6] for which the gap risk is eliminated in the absence of jumps. However, the largest risk for the issuer is exactly this gap risk. High premiums are being charged on the products covering for this risk, which suggests that there are jumps involved in the underlying process.

Last but not least, the jump-diffusion allows for more customization of the asset process towards the needs of the user. The jump-diffusion model allows the implementer to price the asset according to her own opinion in a more thorough way, due to the additional parameters. Specifically, it allows for more detailed specification of the skewness and kurtosis of the underlying distribution. The third and fourth moments are generally statistically significantly different from zero for asset returns[10].

6.3 Application on Live CoCos

6.3.1 Market Data

In general, the CoCo market is too immature to draw any far-reaching conclusions on modeling and calibration from the current market data. Each CoCo issued so far is unique. In addition, only about half of the issues are public. Moreover, there is most often less than one year of historical data available. Furthermore, the most prominent challenge is the fact that no CoCo has been triggered to date, so there is not a single observation of precise stock behavior around the trigger. Even though the underlying stock may have been around for quite some time, its historical data is not applicable as the CoCo may change the game plan and introduce new behavior.

The usual hardships connected to the gathering of market data for option pricing is also present for CoCos. What is the risk free rate of return? How to model and determine the continuous dividend yield? In addition, the jump-diffusion parameters (σ_{JD}, λ, γ and δ) have to be chosen in some structured way. Last but not least, if the CoCo does not have a market trigger, one has to guesstimate its level somehow.

6.3.2 Credit Suisse Buffer Capital Notes

The Credit Suisse CoCos are publicly traded on Börse Berlin. At the time of writing (December 2nd 2011) the price is 88.55%. The coupon is 7.875%, paid semi-annually with 10 coupons remaining. The next payment is due in 0.2 years (2.5 months). Its time to maturity (to the first call date) is 4.7 years. It will convert fully and the conversion price is the higher of \$20 or the share price at conversion (the trigger is of accounting or regulatory type). The stock is trading at \$23.75 and pays a continuous dividend of 6%. Furthermore, $\sigma = 0.25$, $\lambda = 6$, $\gamma = 0$ and $\delta = 0.15$ are assumed. The US 10 year treasury bond (which works as the risk free rate) pays a rate of 2.11%. The trigger share price is guesstimated at \$12. The estimated CoCo price for the Ritchken optimal point at $N = 153$ is 84.16%.

Chapter 7

Conclusion

The new pricing approach to CoCos introduced in this paper provides for a more flexible way of specifying the behavior of the underlying asset, and in the end do a more sound valuation of the CoCo, than the already existing pricing approaches. Since the CoCo perceptively introduces behavior which may affect the underlying price process in ways not compatible with the existing price methods, this new development is necessary to capture the full scope of the valuation. However, the new approach introduces computational challenges. These challenges are taken care of with reasonable methods and they make the pricing approach produce sensible results within manageable time frames.

In general, there is too little data available to perform a thorough investigation of the underlying price processes and to evaluate the new approach. Instead, the tractability of the new approach is reinforced by qualitative arguments.

Further research in this field is definitely required. Firstly, when the standards of the CoCo layout have been set, a more unified approach on valuation can be outlined. Secondly, to take the modeling to the next level, one or a few observations of triggering or close to triggering CoCos are required in order to further pinpoint the behavior. Last but not least, more empirical studies can be performed when the available time series are longer and more numerous than is the case today.

Chapter 8

Appendix

8.1 Cumulants

$$
\begin{aligned}
\kappa_1 &= \lambda \Delta t \gamma' \\
\kappa_2 &= \lambda \Delta t \Big((\gamma')^2 + \delta^2 \Big) \\
\kappa_3 &= \lambda \Delta t \Big((\gamma')^3 + 3\delta^2 \gamma' \Big) \\
\kappa_4 &= \lambda \Delta t \Big((\gamma')^4 + 6\delta^2 (\gamma')^2 + 3\delta^4 \Big) \\
\kappa_5 &= \lambda \Delta t \Big((\gamma')^5 + 10\delta^2 (\gamma')^3 + 15\delta^4 \gamma' \Big) \\
\kappa_6 &= \lambda \Delta t \Big((\gamma')^6 + 15\delta^2 (\gamma')^4 + 45\delta^4 (\gamma')^2 + 15\delta^6 \Big)
\end{aligned}
$$

Bibliography

[1] Michael Albert, Jason Fink, Kristin Fink, *Adaptive Mesh Modeling and Barrier Option Pricing Under a Jump-Diffusion Process*. James Madison University, 2006.

[2] David S. Bates, *The Crash of '87: Was it Expected? The Evidence from Option Markets*. The Journal of Finance Vol. 46 No. 3, pp. 1009-1044, American Finance Association, 1991.

[3] Emanuel Derman, Iraj Kani, Deniz Ergener, Indrajit Bardhan, *Enhanced Numerical Methods for Options with Barriers*. Quantitative Strategic Research Notes, Goldman Sachs, 1995.

[4] Stephen Figlewski, Bin Gao, *The adaptive mesh model: a new approach to efficient option pricing*. Journal of Financial Economics 53, pp. 313-351, 1999.

[5] Espen Gaarder Haug, *The Complete Guide to Option Pricing Formulas*, Second Edition. McGraw-Hill, New York, 2006.

[6] Arnaud Gallais, *CPPI Structures on Funds Derivatives*. KTH Department of Mathematical Statistics, Stockholm, 2011.

[7] Jimmy E. Hillard, Adam Schwartz, *Pricing European and American Derivatives under a Jump-Diffusion Process: A Bivariate Tree Approach*. The Journal of Financial and Quantitative Analysis, Vol. 40, No. 3, pp.671-691, 2005.

[8] Jens Hilscher, Alon Raviv, *Bank stability and market discipline: The effect of contingent capital on risk taking and default probability*. Brandeis University, Massachusetts 2011.

[9] John C. Hull, *Options, Futures, and other Derivatives*, Seventh Edition. Pearson Prentice Hall, New Jersey, 2009.

[10] Benoit B. Mandelbrot, Richard L. Hudson, *The (mis)behaviour of markets: a fractal view of risk, ruin and reward*. Basic Books, New York 2006.

[11] Robert C. Merton, *Theory of rational option pricing*. The Bell Journal of Economics and Management Science, Vol. 4, No. 1, Spring 1973, pp 141-183, 1973.

[12] Robert C. Merton, *On the pricing of corporate debt: the risk structure of interest rates*. Journal of Finance 29, pp.449-470, 1974.

[13] Robert C. Merton, *Option Pricing when Underlying Stock Returns are Discontinuous*. Journal of Financial Economics 3, pp.125-144, 1976.

[14] Peter Ritchken, *On Pricing Barrier Options*. The Journal of Derivatives 3, pp. 19-28, 1995.

[15] Nouriel Roubini et al., *The Financial Development Report 2008*. World Economic Forum, 2008.

[16] Jan de Spiegeleer, Wim Schoutens, *Contingent Convertible (CoCo) Notes: Structuring and Pricing*. Euromoney Books, London, 2011.

[17] Jan de Spiegeleer, Wim Schoutens, *Pricing Contingent Convertibles: A Derivatives Approach*. Department of Mathematics, Katholieke Universiteit Leuven, 2011.

[18] Peter Tankov, Ekaterina Voltchkova, *Jump-diffusion models: a practitioner's guide*. Banque et Marchés, No. 99, Université Paris 7, 2009.