# A Model Implementation of Incremental Risk Charge

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## Abstract

In 2009 the Basel Committee on Banking Supervision released the final guidelines for computing capital for the Incremental Risk Charge, which is a complement to the traditional Value at Risk intended to measure the migration risk and the default risk in the trading book. Before Basel III banks will have to develop their own Incremental Risk Charge model following these guidelines. The development of such a model that computes the capital charge for a portfolio of corporate bonds is described in this thesis. Essential input parameters like the credit ratings of the underlying issuers, credit spreads, recovery rates at default, liquidity horizons and correlations among the positions in the portfolio will be discussed. Also required in the model is the transition matrix with probabilities of migrating between different credit states, which is measured by historical data from Moody´s rating institute. Several sensitivity analyses and stress tests are then made by generating different scenarios and running them in the model and the results of these tests are compared to a base case. As it turns out, the default risk contributes for the most part of the Incremental Risk Charge.

# Acknowledgement

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# Contents

# 1. Introduction

Incremental Risk Charge is a one year Value at Risk measure based on credit risk elements of market risk, at a 99.9% confidence level. The purpose of the measure is to capture the risk of losing portfolio value due to migrations between credit ratings and defaults of the underlying issuers in the trading book. This specific risk charge will, together with the standard and a stressed ten day 99% Value at Risk, constitute the total capital requirement in the upcoming Basel III. [Basel Committee on Banking Supervision, 2009b].

## 1.1 Background

Basel II is a set of recommendations on banking regulations issued by the Basel Committee on Banking Supervision with the purpose of creating an international standard for banks to use when calculating the amount of capital necessary to cover up for financial and operational risks. The Basel II standard has, ever since it's first round of proposals in 1999, been evolving. After the publication of the Basel II framework in June 2004 and especially after the global financial crises that originated during 2007, many improvements have been made which are presented in additional documents.

According to the Basel II framework banks have to make a separation between assets held in the trading book and assets held in the banking book. The assets held in the banking book are securities that are not actively traded on the market, while assets in the trading book are invested at short horizons to make profits or to hedge other investments in the book. These assets are associated with market risk. During the financial crises the Basel Committee noticed that most of the losses occurred in the trading book, where the minimum capital requirement is calculated using Value at Risk measures. Also, the losses were much larger than expected. Hence, one could conclude that the models used for calculating the Value at Risk clearly underestimated the risk. A general disadvantage with Value at Risk measures is that, because they are based on historical data to some extent, the worst case generated by the Value at Risk model cannot be worse than what has happened before. Since the period before the financial crises was benign the Value at Risk became too low. Aside from the fact that Value at Risk models in general could underestimate the risk, traditional Value at Risk models don´t capture risks like default risk and migration risk for portfolios including credit positions sufficiently. Losses raised from credit migrations combined with widening of credit spreads and the loss of liquidity were far more common than losses raised from actual defaults during the credit market turmoil.

In response to the increasing amount of exposure to credit risk related products in banks trading books, the Basel Committee has introduced a new measure called the Incremental Risk Charge. Together with a stressed Value at Risk, which is a Value at Risk conditioned to a year of economic turmoil, the Incremental Risk Charge will make a complement to the traditional Value at Risk when calculating the capital requirement. The purpose of the measure is to capture risks that Value at Risk does not cover. In July 2009 the Basel Committee released the final version of guidelines regarding this subject (hereafter denoted the guidelines), where the principles for calculating the Incremental Risk Charge are stated [Basel Committee on Banking Supervision, 2009a]. Banks are expected to develop their own models following these guidelines.

In the guidelines, the Incremental Risk Charge is specified to represent an estimate of the default and migration risks of insecuritised credit products over a one year capital horizon at a 99.9% confidence level, taking into account the liquidity horizons applicable to individual trading positions or sets of positions. The liquidity horizon represents the time required to sell the position or to hedge the risks covered by the Incremental Risk Charge model in a stressed market. The minimum length of the liquidity horizon is three months. It has to be sufficiently long so that the act of selling or hedging does not materially affect market prices.

The model also assumes a constant level of risk of the portfolio. This means that at the end of each liquidity horizon, the bank has to rebalance its trading positions in order to maintain the initial risk level. A position whose credit characteristics have changed over the liquidity horizon, for example its rating has decreased, needs to be replaced by a position that matches the characteristics of those that the original position had at the beginning. When an issuer defaults the position is always rebalanced, whether it has reached its liquidity horizon or not.

Correlations among obligors affect the Incremental Risk Charge and should therefore be considered. There are two types of dependencies:

- Between default and migration risks.
- Between default or migration risks and other risks in the trading book.

The first type of dependence causes clustering of default and migration events and must be included in the Incremental Risk Charge model. A portfolio that is concentrated to a specific issuer or market should require a higher capital charge than a more diversified portfolio. The second one is, according to the Basel Committee, not currently well understood and should therefore not be reflected when computing the capital charge.

## 1.2 Aim & Scope

The task of this thesis is to develop an Incremental Risk Charge model for products that are sensitive to credit risk in accordance with the guidelines of the Basel

Committee. The model should measure risks due to migration of credit ratings and defaults. Starting with a portfolio of simple corporate bonds, a Value at Risk conditioned on these risks may be compiled at a 99.9% confidence level. The Value at Risk is a risk measure that answers the question of what the minimum expected loss is given a bad period, where that specific period is defined to occur once in every 1000 periods at the 99.9% confidence level [Saunders & Allen, 2010]. From a simulated distribution of 100000 values, the 99.9% Value at Risk is simply the 100th worst outcome.

The aim is to develop an elementary model for Incremental Risk Charge that yields a reasonable result and to analyze the effect on calculated risk using various model specifications, in particular the effects of liquidity horizons, credit spreads, correlations and transition probabilities. This is of interest since there will be new credit risk capital requirements in Basel III based on the Incremental Risk Charge measure. The model could be used in the future for development of a more advanced model at Handelsbanken.

The structure of the thesis will be as following. The second chapter will briefly review some of the literature on the methods for computing the migration and default risk and present the methodology used to create an Incremental Risk Charge model. In chapter three, sensitivity analyses and stress tests will be performed on the input data of the model to see how the outcome of the model responds. The results are then discussed and conclusions are presented in chapter four.

# 2. Theory

Since the final guidelines from the Basel Committee regarding the Incremental Risk Charge were released only a few years ago, there are not many books covering this specific risk measure. However, methods for computing the migration and default risk have been discussed and some articles regarding the modelling framework exist on the internet. Together with the guidelines of the Basel Committee these will be used as inspiration for development of the model.

## 2.1 Incremental Risk Charge

CreditMetrics is a method developed to measure the credit risk caused by changes in credit ratings of the counterparties exposed to in a credit portfolio, which includes the default risk [Gupton et al, 2007]. It is a good starting point for an Incremental Risk Charge model. The method uses a transition matrix with probabilities of migrating to the different credit ratings given the initial credit rating, see Table I in section 2.2.2 for an example. Up- and downgrades of credit ratings are considered as functions of the underlying asset value of a firm, which is an extension of a framework proposed by Robert Merton. Asset values are random and simulated by some distribution with underlying issuer correlations. If the simulated asset value falls below a certain threshold, calculated from the transition matrix, the issuer changes its credit rating.

The transition matrix is combined with the simulated asset values and credit ratings of the underlying issuers in the portfolio are created. Depending on the changes in credit ratings, the total portfolio value is measured. The simulation is repeated many times to get a distribution of portfolio values after which a Value at Risk at the confidence level 99.9% can be determined. This is the Incremental Risk Charge.

According to CreditMetrics there are mainly two problems in modelling portfolio risk in credit portfolios compared to modelling equity price risk. Equity returns are typically symmetric and well approximated by a normal distribution, whilst the distribution of credit returns are more skewed with a heavy downside tail, see Figure I. This is due to the default risk. In fact, the distribution seems to match a so called Skew-normal distribution. Aside from the mean and standard deviation, the skewness and the kurtosis are necessary parameters to recreate this distribution [Koski, 2011]. The skewness is a measure of lack of symmetry and the kurtosis is a measure of whether the distribution is peaked or flat compared to a normal distribution. The second problem is that credit correlations are not easily measured compared to market correlations, due to lack of historical data.

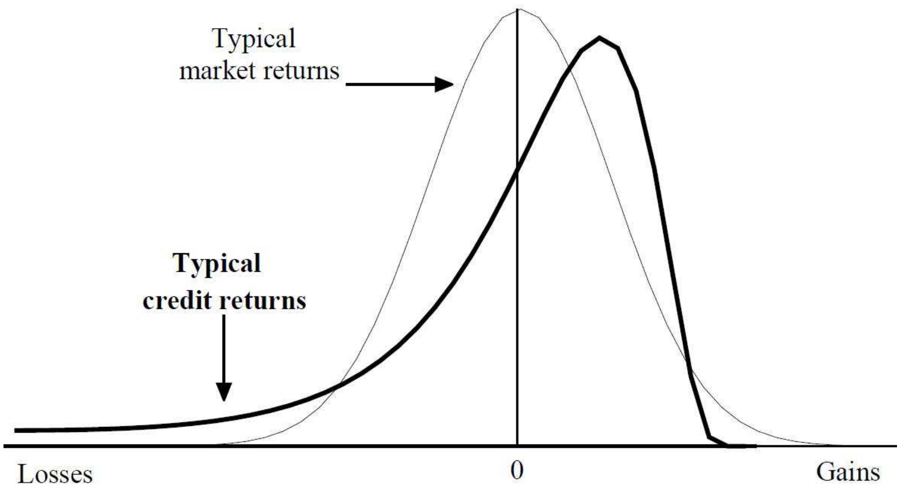


Figure I. *Comparison between the distribution of equity returns and credit returns.*

A drawback of the CreditMetrics model is that only a single horizon of one year is assumed for all positions and rebalancing of the portfolio will therefore not be possible. This means that two criteria from the guidelines of the Basel Committee are not allowed for; retaining a constant level of risk and adjustment of the liquidity horizon for the positions to match their credit quality. To be able to set different liquidity horizons for different positions and allowing for rebalancing, this single-period model has to be extended to a multi-period model. A four-period simulation model is incorporated in The Incremental Risk Model [Stel, Yannick V.D., 2009]. Each period is three months so all four periods add up to the total capital horizon of one year. The four-period model could be seen as four individual simulations according to the methods used in CreditMetrics. Note that the transition probabilities have to be recalculated from one year to three months. At the end of each single period, a position may be rebalanced if the liquidity horizon of that position is reached or if a default occurs. When rebalancing occurs, the position is replaced by a new hypothetical position with characteristics equal to the initial position. The initial risk level of a position is defined by its class, size, credit rating and correlations with other issuers. The total value of each position at the end of the capital horizon (one year from simulation date) will be the value of the bond at the end of the year added with rebalancing results from earlier periods.

## 2.2 Methodology

The general structure of an Incremental Risk Charge model is described in Figure II. Starting from the outcome of the model, a 99.9% Value at Risk has to be calculated from a distribution of portfolio value changes. To get a distribution of portfolio value changes, a single change in portfolio value is calculated at each simulation by taking the sum of all position value changes at each rebalancing time point between simulation date and one year from simulation date.

The value of each position at each time point depends on the state in which the underlying issuer of the position is, where the state is defined as any of the credit ratings or a default. If the state is not default, the future cash flows of the position are discounted using discount factors. The discount factors include the forward rates of which depend directly of the credit rating. If the issuer has defaulted a recovery value is calculated.

The credit rating is the outcome of the simulated asset return for each underlying issuer in the portfolio combined with asset return thresholds. The asset returns are simulated by a multivariate distribution with an issuer correlation matrix as primary input. This matrix has to be measured using historical data. The asset return thresholds are outcome of a transition matrix with probabilities of migrating between all the possible states for an issuer to be in (the credit ratings and the default state). For a complete list of the data required for computing the Incremental Risk Charge, see Appendix (5.1).

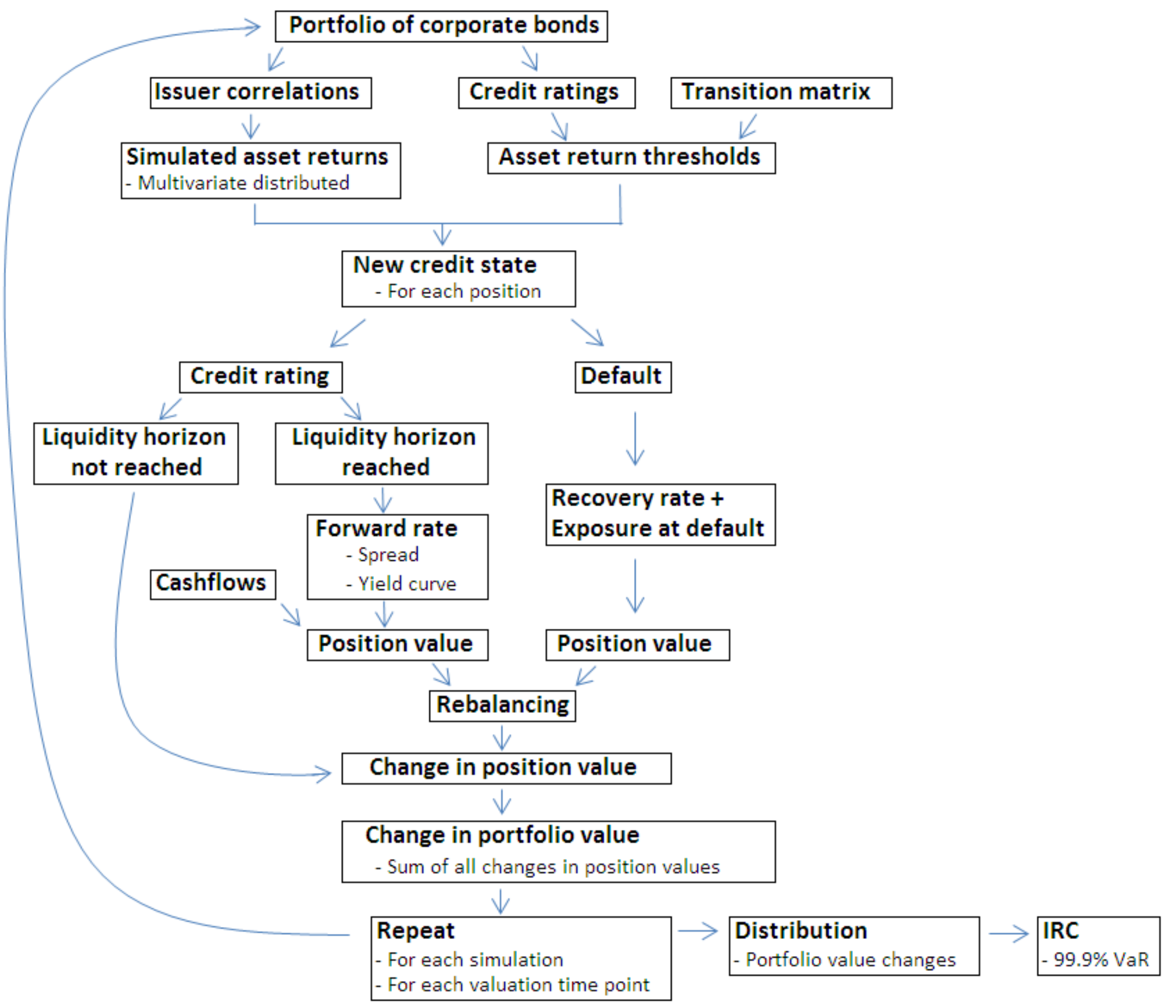


Figure II. *General structure of an Incremental Risk Charge model.*

### 2.2.1 Base Setup

The portfolio used in the model is defined in a table where data for each position in the portfolio is available. For simplicity, all positions are assumed to be corporate bonds where the underlying issuer of each bond belongs to a specific category. There are 36 categories in total that vary across different industries and regions. A category could be either a combination of a region and an industry or just a region. Among these categories, correlations are measured by historical data. A portfolio specific correlation matrix is then created using these correlations.

It is assumed that all relevant information of an issuer is captured by its probability of default. Hence, each issuer of a bond in the portfolio is rated by a credit rating that is directly related to its probability of default. There are eight possible states for an issuer to be in; the seven ratings (Aaa, Aa, A, Baa, Ba, B, Caa) and the default state. The liquidity horizon should be at least three months and it is reasonable to assume a short liquidity horizon for the highest rated issuers and, conversely, a long liquidity horizon for the lowest rated issuers.

### 2.2.2 Transition Probabilities

The credit rating $CR^i$ for underlying issuer $i$ is the most crucial parameter in the Incremental Risk Charge model and a scenario generator will be used to simulate this for every position in the portfolio. The simulation will generate credit rating migrations with certain probabilities. See Table I for a transition probability matrix conditioned on a one year basis, called $P_{one\ year}$. This matrix is taken from CreditMetrics [Gupton et al, 2007]. It is originally measured by Moody's using historical data of rating transitions during a period of 26 years, which does not include the recent financial crises. The following few adjustments were made manually to make it more realistic:

- All transition probabilities of 0% were changed to 0.001%, except those specifying the transition probabilities from the default state.
- The probabilities of default for the initial ratings Aa and A were changed from 0.02% and 0% to 0.01% and 0.0164% respectively.
- All diagonal elements were recalculated in order for each row to sum up to one.

In the table, look up the initial credit rating to the far left and on that row, find the probability to migrate to any of the other rating states or to keep initial rating. As seen in the table, the probabilities on the main diagonal, i.e. those specifying the probability of keeping the initial rating, are the highest. One might find some probabilities deviating from what they intuitively would be. For example, most of the probabilities of downgrading to Caa are lower than the corresponding probabilities of default. Apparently, historical data shows this pattern and therefore these deviations are left untouched in the base case.

| Initial rating | Rating at year-end (%) Aaa | Aa | A | Baa | Ba | B | Caa | Default |
|---|---|---|---|---|---|---|---|---|
| Aaa | 93.396% | 5.940% | 0.640% | 0.001% | 0.020% | 0.001% | 0.001% | 0.001% |
| Aa | 1.610% | 90.559% | 7.460% | 0.260% | 0.090% | 0.010% | 0.001% | 0.010% |
| A | 0.070% | 2.280% | 92.424% | 4.630% | 0.450% | 0.120% | 0.010% | 0.016% |
| Baa | 0.050% | 0.260% | 5.510% | 88.480% | 4.760% | 0.710% | 0.080% | 0.150% |
| Ba | 0.020% | 0.050% | 0.420% | 5.160% | 86.910% | 5.910% | 0.240% | 1.290% |
| B | 0.001% | 0.040% | 0.130% | 0.540% | 6.350% | 84.219% | 1.910% | 6.810% |
| Caa | 0.001% | 0.001% | 0.001% | 0.620% | 2.050% | 4.080% | 69.187% | 24.060% |
| Default | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 100.000% |

Table I. *Credit rating migration probabilities in one year.*

Assume a simulation time horizon equal to the shortest liquidity horizon, i.e. three months. This means that the migration probabilities are measured on a three months basis and the one year based transition probability matrix needs to be transformed into a three months based transition probability matrix. The transitions between the different credit states could be considered as time homogeneous discrete markov chains if the transition probabilities only depend on the current states of the positions and not the previous states, which is the case in this model. Let $p_{ij,n}$ denote the transition probability of going from $i$ to $j$ in $n$ steps and assume that $p_{ij,1}$ is known. Then, according to the Chapman-Kolmogorov theorem [Enger & Grandell, 2010], a transition from $i$ to $j$ in $n$ steps could be done by first transition from $i$ to $r$ in $n-k$ steps $(0 < k < n)$ and then transition from $r$ to $j$ in $k$ steps, where

$$p_{ij,n} = \sum_r p_{ir,n-k} \cdot p_{rj,k},$$

or in matrix form,

$$P_n = P_{n-k} \cdot P_k.$$

Let $k = n - 1$. Then

$$P_n = P_{n-(n-1)} \cdot P_{n-1} = P_1 \cdot P_{n-1} = P_1^2 \cdot P_{n-2} = \ldots = P_1^n.$$

Hence, the transition matrix with probabilities of moving between states in $n$ years is equal to the $n$:th power of the corresponding transition matrix with probabilities of moving between states in one year. Since three months is equal to one quarter of a year, the following will hold;

$$P_{three\ months} = P_{one\ year}^{1/4}.$$

Note that the matrix has to be a square matrix in order for this to work. That is why an extra row with default probabilities is added.

One problem with the transformation is that when raising a matrix to a power less than one (here ¼) it could result in negative elements [Yavin et al, 2011]. Probabilities cannot be negative, so the three months transition matrix needs to be slightly adjusted if any elements are less than zero. There exist a number of techniques on how to solve this problem. [Kreinin & Sidelnikova, 2001] for example introduces a framework that allows one for solving the problem of finding the root of a transition matrix by regularization. It becomes an optimization problem where the object is to find a transition matrix that, when raised to a power $t$, most closely matches the annual transition matrix. This optimization problem leads to a distance minimization problem which is beyond the scope of this thesis. A simple but not very accurate method to get rid of the negative matrix elements is to take the magnitude of the matrix and recalculate the probabilities of not migrating for each credit state in order for each row in the matrix to sum up to one, i.e.

$$p_{ii,1/4} = 1 - \sum_{i \neq j} p_{ij,1/4}$$

for all credit states $i$. Taking the fourth root of the matrix defined in Table I yields three negative elements; the transition probabilities for Aaa->Baa, Caa->A and Caa->Aa. Another drawback of the transformation is that it ignores the issue of autocorrelation in the credit quality changes [Gupton et al, 2007]. Despite these issues, the three months transition matrix provides fine input to the model. It is shown in Table II. As expected, probabilities larger than 50% increased by the transform from one year to three months, whilst probabilities smaller than 50% decreased.

| | **Rating in three months (%)** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Initial rating** | Aaa | Aa | A | Baa | Ba | B | Caa | Default |
| Aaa | 98.289% | 1.581% | 0.121% | 0.003% | 0.005% | 0.000% | 0.000% | 0.000% |
| Aa | 0.428% | 97.522% | 1.994% | 0.032% | 0.021% | 0.001% | 0.000% | 0.002% |
| A | 0.014% | 0.608% | 98.003% | 1.247% | 0.096% | 0.027% | 0.002% | 0.002% |
| Baa | 0.013% | 0.056% | 1.484% | 96.929% | 1.308% | 0.164% | 0.021% | 0.027% |
| Ba | 0.005% | 0.011% | 0.081% | 1.421% | 96.478% | 1.659% | 0.056% | 0.289% |
| B | 0.000% | 0.010% | 0.030% | 0.109% | 1.780% | 95.739% | 0.583% | 1.748% |
| Caa | 0.000% | 0.000% | 0.005% | 0.170% | 0.580% | 1.232% | 91.179% | 6.834% |
| Default | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 100.000% |

Table II. *Credit rating migration probabilities in three months.*

### 2.2.3 Asset Returns

In the Merton model, the asset value of an issuer is considered to determine its credit wealth. A default is defined as the case when the asset value of the counterparty falls below its level of debt. Let $AV_t^i$ be the asset value of issuer $i$ at time $t$ and $\Delta AV_t^i$ be the change in asset value of issuer $i$ between time points $t$ and $t-0.25$ years (a three months interval). This yields

$$AV_t^i = AV_{t-0.25}^i + \Delta AV_t^i$$

for $t = 0.25, 0.5, 0.75, 1$ years. The approximation that the log return of assets follows a normal distribution is well known and for relatively small values of asset returns, $AR_t^i$ the following holds:

$$AR_t^i = \frac{\Delta AV_t^i}{AV_{t-0.25}^i} = \frac{AV_t^i - AV_{t-0.25}^i}{AV_{t-0.25}^i} \approx \ln\left(\frac{AV_t^i}{AV_{t-0.25}^i}\right).$$

Hence, the asset return between $t$ and $t-0.25$ years of issuer $i$ is assumed to follow a standard normally distributed variable scaled by $1/AV_{t-0.25}^i$. Let $\mathrm{N}(\mu,\sigma)$ denote the standard normal distribution with mean $\mu$ and standard deviation $\sigma$. Then

$$AR_t^i \sim \mathrm{N}\left(0, \frac{1}{AV_{t-0.25}^i}\right).$$

By scaling the asset return with this factor, the change in value of issuer $i$ from $t-0.25$ to $t$ becomes standard normally distributed.

$$\Delta AV_t^i = AV_t^i - AV_{t-0.25}^i = AR_t^i \cdot AV_{t-0.25}^i \sim \mathrm{N}(0,1).$$

Here, the asset value is normalized with respect to its standard deviation, which means that a change in asset value is measured as a quantity of standard deviations. The changes in asset value for the positions in the portfolio could now be simulated using a multivariate standard normal distribution with the issuer correlation matrix as input, i.e.

$$\overline{\Delta AV_t} \sim \mathrm{N}(\bar{0}, \Sigma),$$

where the bar indicates a vector and $\Sigma$ is the covariance matrix.

### 2.2.4 Asset Return Thresholds

Asset return thresholds define the boundaries of the asset returns for keeping certain credit ratings. An asset value threshold defines a limit measured in standard deviations which, if exceeded or fallen below by the simulated change in asset value, results in a credit rating upgrade or downgrade. Using the transition probabilities from $P_{three\ months}$, thresholds for the simulated change in asset value corresponding to changes in credit rating could be determined. Let $Z_{CR_t}(CR_{t-0.25})$ be the threshold value for an issuer with credit rating $CR_{t-0.25}$ to migrate to $CR_t$, where

$$CR_{t-0.25} = \{Aaa, Aa, A, Baa, Ba, B, Caa\},$$

$$CR_t = \{Aa, A, Baa, Ba, B, Caa, Default\}.$$

Consequently, there are 49 thresholds; seven for each initial rating. An alternative using only seven thresholds in total were investigated and the conclusion is that it does not yield an appropriate result, see part 'Alternative thresholds' further down. If the simulated change in asset value of an issuer with rating $CR_{t-0.25}$ has fallen just below threshold $Z_{CR_t}(CR_{t-0.25})$ at time $t$ the credit rating change to $CR_t$. Note that whatever happens between these time points is of no interest. Technically a default could have occurred if the change in asset value was compounded continuously and temporarily fell below the threshold of default between these time points. Here, this is not the case and the value is only considered at time $t$. Thresholds for the highest rating $CR_t = \text{Aaa}$ are not required since a change in value exceeding $Z_{Aa}(CR_{t-0.25})$ implies the rating Aaa.

Let $e = \Delta AV^i_{t_1,t_2}$ and $\Phi(x)$ denote the cumulative standard normal distribution function evaluated at $x$. The notations of thresholds are simplified so that $Z_{CR_t} = Z_{CR_t}(CR_{t-0.25})$. Then the thresholds are calculated recursively by the following formulas;

$$P\big(\mathrm{e} < Z_{Default}\big) = \Phi\big(Z_{Default}\big),$$

$$P\big(Z_{Default} < e < Z_{Caa}\big) = \Phi(Z_{Caa}) - \Phi\big(Z_{Default}\big),$$

$$P(Z_{Caa} < e < Z_B) = \Phi(Z_B) - \Phi(Z_{Caa}),$$

$$P(Z_B < e < Z_{Ba}) = \Phi(Z_{Ba}) - \Phi(Z_B),$$

$$P(Z_{Ba} < e < Z_{Baa}) = \Phi(Z_{Baa}) - \Phi(Z_{Ba}),$$

$$P(Z_{Baa} < e < Z_A) = \Phi(Z_A) - \Phi(Z_{Baa}),$$

$$P(Z_A < e < Z_{Aa}) = \Phi(Z_{Aa}) - \Phi(Z_A),$$

$$P(e > Z_{Aa}) = 1 - \Phi(Z_{Aa}),$$

for each credit rating. The probabilities to the left hand side of the equations are found in the transition probability matrix $P_{three\ months}$ for some initial credit rating. Using these, the thresholds for each specific rating can be determined recursively by first computing the default threshold $Z_{Default}$ by the first formula, then $Z_{Caa}$ by the second formula and so on through all formulas until all thresholds are known.

For example, consider an issuer rated A. Transition probabilities over a three months period for an A-rated issuer are found in Table II in section 2.2.2. The probability of default is approximately 0.00223% (shown as 0.002% in the table) which yields

$$Z_{Default}(A) = \Phi^{-1}\big[P\big(\mathrm{e} < Z_{Default}(A)\big)\big] \approx \Phi^{-1}(0.0000223) \approx -4.08.$$

This means that an A-rated issuer defaults if the simulated change in asset value is less than $-4.08$ standard deviations. The threshold for the next credit state; Caa, becomes

$$\begin{aligned} Z_{Caa}(A) &= \Phi^{-1}\big[P\big(Z_{Default}(A) < e < Z_{Caa}(A)\big) + \Phi\big(Z_{Default}(A)\big)\big] \\ &= \Phi^{-1}\big[P\big(Z_{Default}(A) < e < Z_{Caa}(A)\big) + P\big(\mathrm{e} < Z_{Default}(A)\big)\big] \\ &= \Phi^{-1}(0.0000215 + 0.0000223) \approx -3.92. \end{aligned}$$

Continuing like this yields all the thresholds for the A-rating. These are shown in Figure III. Repeating this for each credit state of each initial rating yields the threshold matrix shown in Table III. To read this table, find current rating to the far left and from that row, find threshold $Z_{CR_t}$ where $CR_t$ is any credit rating from the top row. If the simulated asset value of an A-rated issuer stays between $-2.20$ and 2.50, the issuer keeps the rating A.

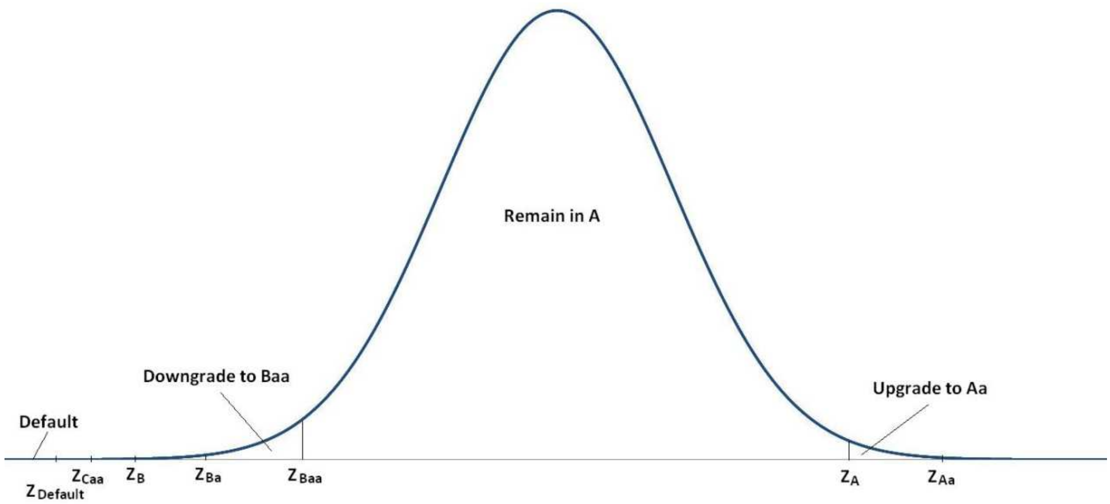


Figure III. *Thresholds for an A-rated issuer.*

| | **Thresholds** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Initial rating** | Aa | A | Baa | Ba | B | Caa | Default |
| Aaa | -2.12 | -3.01 | -3.76 | -3.88 | -4.43 | -4.45 | -4.68 |
| Aa | 2.63 | -2.04 | -3.26 | -3.49 | -3.96 | -4.05 | -4.07 |
| A | 3.63 | 2.50 | -2.20 | -3.02 | -3.42 | -3.92 | -4.08 |
| Baa | 3.66 | 3.20 | 2.16 | -2.17 | -2.86 | -3.30 | -3.46 |
| Ba | 3.89 | 3.60 | 3.10 | 2.17 | -2.05 | -2.70 | -2.76 |
| B | 4.99 | 3.71 | 3.35 | 2.97 | 2.07 | -1.99 | -2.11 |
| Caa | 4.60 | 4.49 | 3.86 | 2.92 | 2.43 | 2.06 | -1.49 |

Table III. *Thresholds measured in standard deviations.*

### 2.2.5 Alternative Thresholds

One could intuitively think that it would be sufficient to use only seven thresholds in total, where each threshold is independent of the initial credit rating. The following analysis will show the results of this and it is based on a one year horizon (all probabilities are based on one year). Assume that the probability of default, $PD^i$, is all that is known for each issuer $i$ and that this is the only parameter controlling the initial asset value $AV_0^i$ of the issuer in the Merton model. Also assume a level of debt equal to zero; $LoD = 0$, so that an issuer defaults if the asset value falls below zero. This yield

$$PD^i = P\left(AV_1^i < LoD\right) = P\left(AV_0^i + \Delta AV_{0,1}^i < LoD\right) = P\left(\Delta AV_{0,1}^i < LoD - AV_0^i\right)$$
$$= \left\{\Delta AV_{0,1}^i \sim N(0,1)\right\} = \Phi\left(LoD - AV_0^i\right) = \{LoD = 0\} = \Phi\left(-AV_0^i\right).$$

Hence

$$AV_0^i = -\Phi^{-1}\left(PD^i\right).$$

An expected initial asset value for each credit rating, measured as a quantity of standard deviations above the level of debt, may now be calculated using the expected probability of default for each rating respectively. These could be collected from the transition probability matrix $P_{one\ year}$. Table IV shows these $PD$:s among with the expected asset values ($AV$:s) and thresholds for all ratings. Here, all thresholds except the default threshold are defined so that the distance to each threshold is equal for the two nearest expected asset values. The last column of the table shows the probabilities of keeping initial credit rating assuming an initial asset value equal to the expected asset value and thresholds defined as in the table. Clearly these probabilities are too low comparing to the corresponding probabilities in Table I in section 2.2.2 (the diagonal elements). It is possible to adjust the thresholds slightly but not enough to get a pleasant result.

To put it simple, in order to have realistic probabilities (at least 80%) of not migrating to any other state for each credit state, the distances between every adjacent pair of thresholds have to be large. This yields a very large total distance between the thresholds of the lowest and the highest rating which in turn yields too low probabilities of default. Hence, the assumption of using only seven thresholds does not hold.

| Rating | Expected PD | Expected AV | Threshold | P(no migration) |
|---|---|---|---|---|
| Aaa | 0.001% | 4.265 | - | 60.755% |
| Aa | 0.010% | 3.719 | 3.992 | 13.279% |
| A | 0.016% | 3.592 | 3.656 | 14.784% |
| Baa | 0.150% | 2.968 | 3.280 | 26.664% |
| Ba | 1.290% | 2.229 | 2.598 | 28.818% |
| B | 6.810% | 1.490 | 1.860 | 29.693% |
| Caa | 24.060% | 0.704 | 1.097 | 41.219% |
| Default | - | - | 0.000 | 100.000% |

Table IV. *Thresholds independent of initial credit rating.*

### 2.2.6 Position Valuation

For valuation of the portfolio, bond information like face values, coupon amounts and maturity dates are necessary. The maturity date of each bond should be at least one year from simulation date since the Value at Risk is measured on a one year horizon and the model does not make any assumptions of repurchasing matured bonds. Coupon payments are assumed to occur annually. Future cash flows of the bonds are discounted using forward rates which are forward zero risk-free rates added by a credit spread depending on the credit rating. Since the credit rating of each underlying issuer is the only position parameter that varies through the model simulation, the credit spreads alone cause the change in portfolio value at each simulation, except if an issuer defaults. In that case the position value depends on the recovery rate and exposure at default, which will be explained in more detail later.

The forward zero risk-free rates are calculated from the current zero yield curve. Which yield curve to use depends on the currency of which the position is exposed to. The model can handle different yield curves for different positions in the portfolio. Let $f_{t_1,t_2}^{risk-free}$ denote the forward zero risk-free rate between the future time points $t_1$ and $t_2$, measured in years. In this model, no market risk is considered, so the forward rate between two time points is a deterministic function of the interest rates at these time points.

To calculate the forward zero risk-free rate, consider two alternate ways to invest money from $t = 0$ to $t = t_2$ [Asgharian & Nordén, 2007]. Either buy a zero coupon bond at time 0 maturing at time $t_2$ with interest rate $r_{t_2}$, or buy a zero coupon bond at time 0 maturing at time $t_1$ with interest rate $r_{t_1}$ and reinvest a new zero coupon bond at time $t_1$ with maturity at time $t_2$ to the forward rate $f_{t_1,t_2}^{risk-free}$. Since the forward rate is known at time 0, both alternatives are risk-free and the two investments should be equivalent, i.e.

$$\left(1 + r_{t_1}\right)^{t_1}\left(1 + f_{t_1,t_2}^{risk-free}\right)^{t_2-t_1} = \left(1 + r_{t_2}\right)^{t_2}.$$

This yields

$$f_{t_1,t_2}^{risk-free} = \frac{(1 + r_{t_2})^{t_2/(t_2-t_1)}}{(1 + r_{t_1})^{t_1/(t_2-t_1)}} - 1.$$

The zero yield curve is obtained from data of Handelsbanken [Handelsbanken, 2011]. A credit spread $s^{CR^i}$ for credit rating of issuer $i$ is then added to get the forward zero risky debt rate of issuer $i$ between time point $t_1$ and $t_2$,

$$f_{t_1,t_2}^{i} = f_{t_1,t_2}^{risk-free} + s^{CR^i}.$$

The credit spreads for each credit rating are defined in Table V in the beginning of section 3. They are hypothetical and grow exponentially as the credit rating decreases.

Knowing which forward rate to use, each position value could be calculated using the formula

$$V_t^i = \sum_k^M \frac{CF_k^i}{(1 + f_{t,k}^i)^{(k-t)}}$$

for issuer $i$ at time $t$, measured in years. $CF_k^i$ denotes the cash flow of the bond paid at time $k$ for issuer $i$ and $t < k = \ldots, (M-2), (M-1), M$, where $M$ is the maturity of the bond. The cash flow could be a coupon only or a coupon plus the face value of the bond.

If an issuer defaults, the value of that position becomes the product of its recovery rate ($RR$) and exposure at default ($EAD$). For simplicity, the exposure at default is assumed to be the face value of the bond and the recovery rate is the proportion of the face value recovered at default. The recovery rate is currently set to 0.37%, which is the mean of all recovery rates for bonds measured by [Moody's Investors Service, 2007]. This could have been stochastic and simulated by a distribution, e.g. a beta distribution, but for now it is assumed to be constant.

### 2.2.7 Simulation

The change in asset value $\Delta AV_t^i$ is the only parameter that needs to be simulated in the model. A simulation should be done for each position in the portfolio and each time interval. The changes in asset value are assumed to follow a multivariate normal distribution with means equal to zero and standard deviations normalized to one given a time interval of three months. Consequently the covariance matrix is equal to the correlation matrix. Let $t = 0$ be the simulation date, e.g. the time point at which the simulation starts. Since the time horizon of the Incremental Risk Charge measure is one year and the shortest liquidity horizon is three months, the simulation time intervals become $[t - 0.25\colon t]$ for $t = 0.25, 0.5, 0.75, 1$ years. For each time interval, each underlying issuer to the positions in the portfolio receives a credit rating depending on its previous credit rating and the change in asset value. The new credit rating will determine which specific spread to add to the forward rate used for discounting the future cash flows of the position.

At the start of the simulation ($t = 0$) an initial position value will be determined for each position in the portfolio depending on the initial credit rating of the underlying issuer. At all the other time points, $t = 0.25, 0.5, 0.75, 1$ years, two position values will be determined for each position in the portfolio; $V_t^i(CR_t^i)$ and $V_t^i(CR_0^i)$. The first of these position values depends on the simulated credit rating while the second value depends on the initial credit rating of the position. By taking the difference of these;

$$\Delta V_t^i = V_t^i(CR_t^i) - V_t^i(CR_0^i),$$

the change in position value due to rebalancing of the position is measured. Rebalancing should occur if an issuer defaults or if the position reached its liquidity horizon. If neither of these conditions is true for a position, the position value should be unchanged ($\Delta V_t^i = 0$).

In order to calculate the effect of these rebalancing results at the end of the capital horizon (one year from the simulation date), these changes in position values need to be pushed into the future using forward rates. That is, for each position $i$, the total change in position value carried to the end of the capital horizon becomes

$$\Delta V^i = (1 + f^i_{0.25,1})^{0.75} \cdot \Delta V^i_{0.25} + (1 + f^i_{0.5,1})^{0.5} \cdot \Delta V^i_{0.5} + (1 + f^i_{0.75,1})^{0.25} \cdot \Delta V^i_{0.75} + \Delta V^i_1.$$

The forward rates in this formula include the spread corresponding to the initial credit rating of the issuer. By subtracting the initial portfolio value with the sum of all these total position value changes, the difference in value of the total portfolio is captured. This is done for each simulated data points and the simulation will be repeated 100 000 times for each time interval in order to create a distribution of changes in the total portfolio value. From this distribution a 99.9% Value at Risk is determined, simply by sorting the distribution by size and taking out the 100th worst outcome.

A two-sided confidence interval around the 99.9% Value at Risk percentile ($\rho$) could be constructed by calculating the standard deviation ($s$) of the percentile. The confidence bounds are defined as the standard deviation scaled by a factor $\alpha$ depending on the confidence level. For a 95% two-sided confidence interval, $\alpha = 1.96$ and hence the confidence bounds become $\rho \pm \alpha s = \rho \pm 1.96s$ where

$$s = \sqrt{\rho \cdot (0.999)}$$

for the 99.9% Value at Risk. With $\rho = 100$ the 95% confidence interval is defined by the percentiles [80,120].

# 3. Analysis

The Incremental Risk Charge model results in a Value at Risk indicating how much one should expect to lose during a really bad year due to the migration risk and the default risk, which is highly dependent on a number of assumptions made about the underlying risk factors. These assumptions may constitute a base case meant to reflect normal economic conditions, or a more stressed period of economic turmoil. To be able to analyze the outcome of different scenarios, model parameters have to be varied and the impact of each individual risk factor has to be measured. In order to do this, an initial portfolio setup needs to be defined.

A portfolio of 100 fabricated corporate bonds is used as input to the model. Discount factors based on the SEK yield curve only will be used. Face values, coupon rates and maturity dates are fixed and equal to 100000 SEK, 5% and 8 years respectively for all positions. The impact of varying any of the so far mentioned parameters is not very interesting when analyzing the outcome of the model. More important parameters are those describing the credit quality of the portfolio, i.e. the credit ratings, the liquidity horizons, the credit spreads and also the issuer concentration in the portfolio. Varying the credit ratings and the issuer concentrations is simply a matter of position allocation in the portfolio. An example of a position allocation based on credit ratings is shown in Table V.

| Rating | Liquidity horizon | Credit Spread | Proportion | Total face value |
|---|---|---|---|---|
| Aaa | 3 months | 0.60% | 15% | 1 500 000 SEK |
| Aa | 3 months | 0.80% | 20% | 2 000 000 SEK |
| A | 6 months | 1.00% | 20% | 2 000 000 SEK |
| Baa | 6 months | 1.60% | 15% | 1 500 000 SEK |
| Ba | 6 months | 3.00% | 15% | 1 500 000 SEK |
| B | 9 months | 5.00% | 10% | 1 000 000 SEK |
| Caa | 12 months | 10.00% | 5% | 500 000 SEK |
| Total | - | - | 100% | 10 000 000 SEK |

Table V. *An example of a position allocation.*

The liquidity horizons are chosen to be directly dependent of the credit ratings; a simple yet reasonable assumption. Therefore, lower rated bonds are assumed to have longer horizons, reflecting the longer time required to sell those positions compared to higher rated positions. The credit spreads for each rating are also shown in Table V.

This should reflect an actual portfolio, where the exposures towards lower rated issuers are smaller than the other exposures. The underlying issuers are not shown here but the portfolio is well diversified, so there are no issuer concentrations. The impact of higher issuer concentrations will be discussed later.

This setup will be used when analyzing the effects of varying the model parameters that are not portfolio specific, such as the correlation matrix and the transition matrix. Values at Risk are consistently presented in absolute terms, although they represent negative changes. Given the transition matrix defined in Table II in section 2.2.2 and the portfolio specific correlation matrix, the model yields a 99.9% Value at Risk of 17.8% with 95% confidence interval [18.7%, 17.1%]. From now on, this Value at Risk will be referred to as the standard Value at Risk, determined from the base case. The portfolio distribution created by this simulation is shown in Figure IV. The Value at Risk is illustrated with a blue circle (close to $-2$ million SEK) and the circles next to it (red and green) are the confidence bounds. The typical skewness of a credit portfolio is emphasized in the figure. The three peaks on the negative side close to zero are caused by default losses.

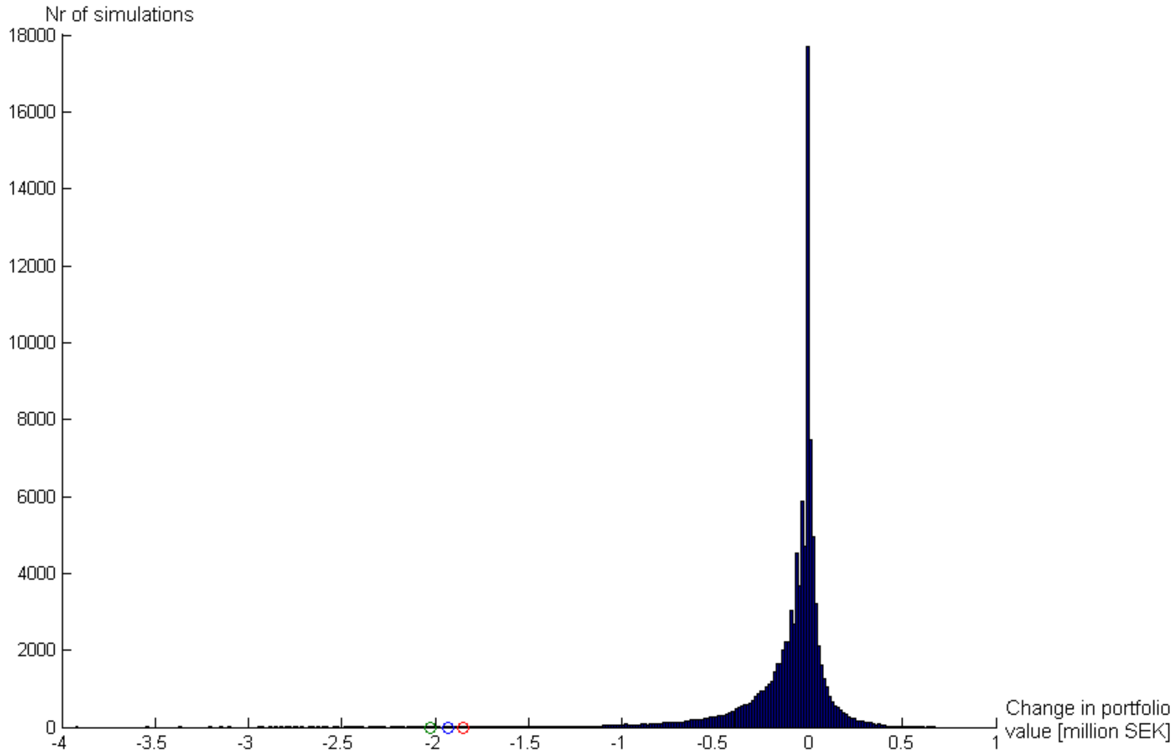


Figure IV. *Simulated portfolio distribution. Values in million SEK.*

## 3.1 Portfolio Sensitivity Analysis

The main purpose in this section is to determine if the model acts as expected. There are mainly two ways to adjust the risk in a credit portfolio from the perspective of an

Incremental Risk Charge model. Perhaps the most obvious way is to change the allocation of credit ratings by taking positions where the underlying issuers have the desirable ratings. The lower the overall credit quality is in a portfolio, the higher the risk, due to the larger differences in credit spreads and the higher probabilities of default. If hypothetically the probability of default was zero, the risk in a portfolio containing the lowest rating only would also be zero, because it could only gain credit quality and increase in value. The other way to adjust risk in the portfolio is to take advantage of the well known method to reduce risk in any portfolio; diversification. By spreading the positions among issuers in different industries and countries rather than concentrating the portfolio to a certain issuer class, risk reduces. This is due to the fact that asset values of issuers do not tend to move up and down in perfect synchrony if the issuers are not alike. As a consequence, the total risk in a well diversified portfolio will be less than the weighted average risk of the assets in the portfolio.

### 3.1.1 Adjusting the Credit Ratings

The credit ratings of the underlying issuers are the most crucial portfolio property and changing these will affect the Value at Risk significantly. To get an idea of how much impact the ratings have on Value at Risk, seven portfolios are created where every position in each portfolio has the same rating. Then, simulation is made for each of these portfolios, generating seven different Values at Risk seen in Figure V. By comparing these, it is obvious that the potential loss increases rapidly when the rating falls below A.

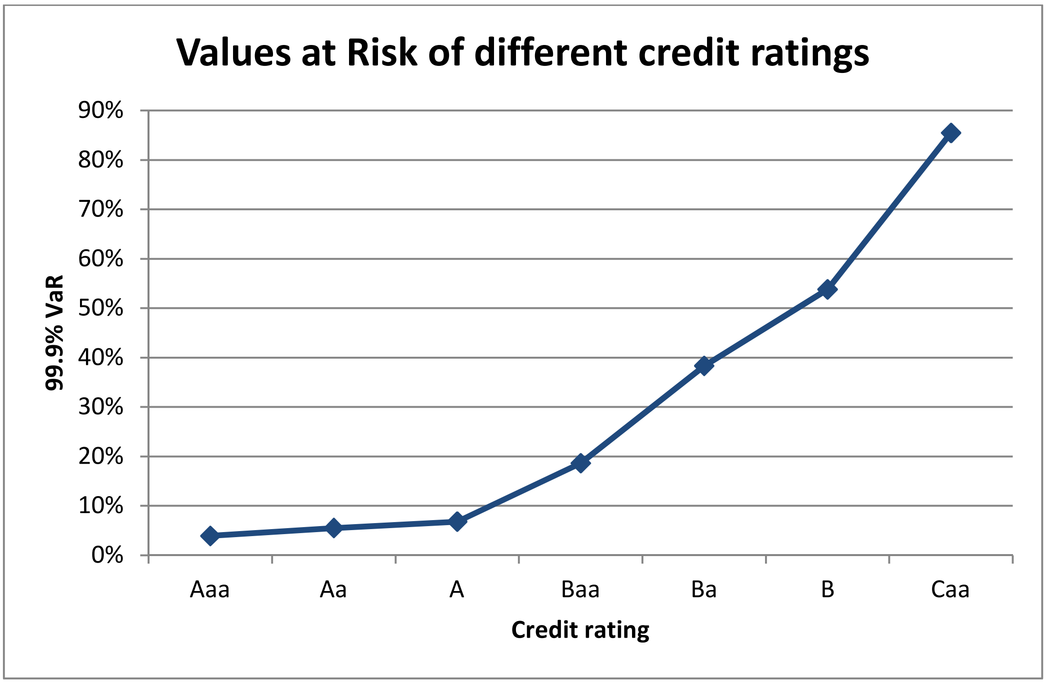


Figure V. *Values at Risk for portfolios with equal rating among all positions.*

A portfolio where all ratings are the same is not very realistic. An interesting analysis would be to take the initial portfolio (defined in Table V) and push 50% of each rating except Aaa (Caa) one step higher (lower). See Case 1 and Case 2 in Table VI for these allocations and the resulting Values at Risk.

| | Rating allocation | | | | | | | | VaR & CI | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Case** | Aaa | Aa | A | Baa | Ba | B | Caa | Total | VaR | Lower bound | Upper bound |
| Case 1 | 25% | 20% | 18% | 15% | 13% | 7% | 2% | 100% | 14.1% | 14.9% | 13.4% |
| Original | 15% | 20% | 20% | 15% | 15% | 10% | 5% | 100% | 17.8% | 18.7% | 17.1% |
| Case 2 | 7% | 17% | 20% | 18% | 15% | 13% | 10% | 100% | 21.6% | 22.4% | 21.1% |

Table VI. *Three rating allocation alternatives and corresponding Values at Risk.*

The changes in Value at Risk are close to $\pm$ four percentage points when shifting 50% of the ratings up and down respectively.

### 3.1.2 Issuer Concentrations

The portfolio has so far been well diversified among the 36 different issuer categories. However, a good diversification is not always possible, so an investigation of how much a more concentrated portfolio affects Value at Risk could be interesting to do. Just to see how bad things could go, a portfolio of 100 positions with the same underlying issuer was tested in the model. This generated a 99.9% Value at Risk of 32,4%, which should be compared to the Value at Risk generated by the well diversified portfolio; 17,8%. The difference is clear.

A more realistic assumption would be a portfolio with a wide distribution of different issuers but where a significant number of issuers belonged to a certain category. Given 36 categories, the best diversification would be to have no more than three issuers of the same category in the portfolio. This was the case when the standard Value at Risk of 17,8% was generated. In order to make the portfolio more concentrated, the categories of two issuers for each credit rating were switched into one specific category; Sweden. Consequently, 17% of the portfolio is perfectly correlated and risk should be increased. Using this portfolio as input to the model generates a Value at Risk of 19.1% with confidence interval [19.7%, 18.3%]; an additional 1.3 percentage points above the standard Value at Risk. This change in risk is rather small compared to the changes in risk caused by adjusted credit ratings (shown in the previous section). This could be explained by the correlation matrix; correlations between different categories are generally high, in average above 50%, so the change in risk created by making 17% of the portfolio perfectly correlated is not that large. For modification of the correlation matrix, see section 3.2.5.

## 3.2 Stress Testing

Parameters of the model that are not directly dependent of the portfolio but affect the risk and are likely to change with the business cycle will be discussed here. Fluctuations on the market could lead to variations in liquidity horizons, correlations between issuers, credit spreads and transition probabilities between credit states. A financial crisis is an example of a period when these fluctuations are large and stress testing is meant to reflect such a scenario.

### 3.2.1 Change in Liquidity Horizons

The liquidity horizon of each position is an important parameter defining the frequency of rebalancing in the portfolio. During stressed market events one cannot assume that the markets remain liquid. The model is limited to four different liquidity horizons; three months, six months, nine months and one year. The choice of liquidity horizon for each position should reflect the time required to sell the position or to hedge all material risks covered by the model in a stressed market [Basel Committee on Banking Supervision, 2009a]. Lower rated positions may be more difficult to sell than higher rated positions. Hence it is reasonable to assume a strong relation between the credit ratings and the length of the liquidity horizon. A distribution of liquidity horizons among the different credit ratings is shown in Table V in the beginning of section 3.

To test whether the liquidity horizons have big impact on the result or not, two experiments were made. First, the liquidity horizons were raised by three months for all positions except those that already had the maximum length of one year. Then contrary, the liquidity horizons were lowered by three months for all positions except those that already had the minimum length of three months. The impact on Value at Risk turned out to be small, almost negligible, which may come as a surprise. A deeper investigation of the liquidity horizons has to be done to understand the underlying reason for this.

How should the variety of liquidity horizons affect the Incremental Risk Charge intuitively? First of all, one should keep in mind that whenever a position defaults, that position is rebalanced whether it reached its liquidity horizon or not. In that sense it is possible for an issuer to default up to four times in the model regardless of the liquidity horizon for that position. The risk of losing value due to migrating between credit ratings is clearly affected by the liquidity horizon. At the rebalancing time points, the change in position value due to the new credit state is carried out and that position is then replaced by a new hypothetical position with the same credit worthiness that the original position had at the beginning of the simulation.

Consider two positions, position A with a liquidity horizon of one year and position B with a liquidity horizon of three months, both initially rated Ba. Assume for simplicity that the positions are 100% correlated and that the probabilities of migrating one step (up or down) is equal for all credit rating states. Assume also three possible scenarios of rating shifts during one year, shown as Case 1,2 and 3 respectively in Table VII. The arrows represent either an up- or a downgrade of one credit rating, or no migration at all. X/Y means rebalancing of the position where rating X is replaced by rating Y.

| Case | Time [months] 0 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|
| **Case 1** | → | ↘ | → | → | ↗ |
| Position A | Ba | B | B | B | Ba/Ba |
| Position B | Ba | B/Ba | Ba/Ba | Ba/Ba | Baa/Ba |
| **Case 2** | → | → | ↘ | → | ↘ |
| Position A | Ba | Ba | B | B | Caa/Ba |
| Position B | Ba | Ba/Ba | B/Ba | Ba/Ba | B/Ba |
| **Case 3** | → | → | ↘ | ↘ | ↘ |
| Position A | Ba | Ba | B | Caa | Default/Ba |
| Position B | Ba | Ba/Ba | B/Ba | B/Ba | B/Ba |

Table VII. *Three possible scenarios of rating shifts and two example positions.*

In case 1 there are two rating shifts. The change in position value for position A is zero since its rating at the rebalancing time point is equal to its initial rating. For position B however, there is a loss at three months due to a downgrade and a gain at one year due to an upgrade. To determine the total value change caused by these two, one has to find out the individual sizes of the gain and the loss, which completely depend on the differences between the credit spreads. By inspecting the credit spread curve shown in Figure VI in section 3.2.2, one could see that the differences between spreads increase as the rating decreases. Specifically, the difference between the spreads of rating Ba and B is larger than the difference between the spreads of rating Ba and Baa, i.e. $s^{B} - s^{Ba} > s^{Ba} - s^{Baa}$. Due to this fact, the magnitude of the loss will be greater than the magnitude of the gain, and position B will suffer a total value change in case 1 which is negative. The opposite case, where $s^{B} - s^{Ba} < s^{Ba} - s^{Baa}$, will of course yield the opposite result, though it is not a realistic assumption since differences in credit spreads tend to be larger between lower ratings than between higher ratings. Therefore, position A with the longer liquidity horizon will be better off in this kind of scenario.

In case 2 there are two rating downgrades and the difference between the positions is that position A downgrades two rating steps once while position B downgrades one rating step twice. The value changes of each position given this scenario will also completely depend on the spreads, in this case the credit spreads for Ba, B and Caa. Assume, as in Figure VI in section 3.2.2, that $s^{Caa} - s^{B} > s^{B} - s^{Ba}$, i.e. that the difference between the credit spreads of rating Caa and B is larger than the difference between the credit spreads of rating B and Ba. Position A with the liquidity horizon of one year would then suffer a loss caused by the difference between the credit spreads $s^{Caa}$ and $s^{Ba}$ which would be larger than the sum of the two losses of position B caused by the difference between $s^{B}$ and $s^{Ba}$. However, the two downgrades might as well have been two upgrades. Assume a scenario with two upgrades and the same assumption about spreads, i.e. larger differences between spreads of lower rating than of higher. Position A will not gain as much as position B by upgrading one more rating step, since it is more beneficial to upgrade one rating step twice. Hence position B would be better off and the final conclusion is that position B will gain more value than position A if there were two upgrades and lose less value than position A if there were two downgrades; shorter liquidity horizon pays off in this case.

In the final case there are three downgrades. Position A will suffer from one default and position B will lose value three times due to each downgrade. If neither the difference in credit spreads between the ratings Ba and B nor the recovery rate at default is unreasonably high, position A will lose more than position B. Once again, position B with the shorter liquidity horizon is better off.

After studying these three cases in detail it is clear that changes in liquidity horizons control risk in both directions. On this basis, two final tests on the liquidity horizons were made, one isolating the migration risk (case a) and one isolating the default risk (case b). The model parameters were set as follows:

a) Zero probabilities of default and standard spreads.
b) Standard probabilities of default and all spreads set to be 2.1% (expected value of all standard spreads based on the position allocation of the standard portfolio).

Both these tests were made on a portfolio with Baa-rated positions only, first with liquidity horizons all equal to three months and then with liquidity horizons all equal to one year. The results are shown in Table VIII. When isolating the migration risk (case a), Value at Risk decreases by almost one percentage point with the longer liquidity horizon. On the other hand, when isolating the default risk (case b), Value at Risk increases by almost two percentage points with the longer liquidity horizon. To summarize, a shorter liquidity horizon should be marginally more beneficial than a longer.

| | **Liquidity horizon** | |
|---|---|---|
| **Case** | 3 months | 1 year |
| a | 12.72% | 11.84% |
| b | 11.65% | 13.41% |

Table VIII. *Values at risk for different liquidity horizons.*

### 3.2.2 Varying the Credit Spreads

The credit spreads define the differences in yield due to different credit quality among the positions. The variation of credit spreads should have a significant impact on the risk since greater differences in spreads raise the potential loss. As discussed in the previous section, the credit spreads also matter a lot along with the length of the liquidity horizons. The credit spreads used in the model have an exponential growth, as seen in Figure VI.

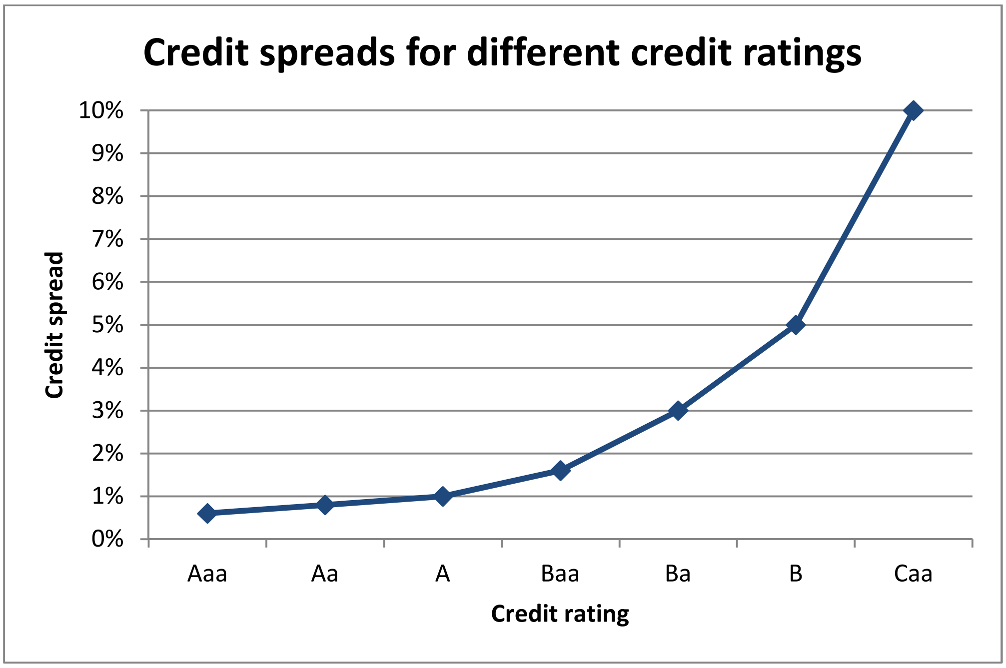


Figure VI. *Credit spreads used in the model.*

Varying the credit spreads is simply a matter of shifting this curve, scaling each spread by different factors or replacing the spreads to completely reshape the curve. The following adjustments of the credit spreads were made:

a) Standard spreads shifted up by five percentage points.
b) All spreads set to be 0%.
c) All spreads set to be 2.1% (expected value of all standard spreads based on the position allocation of the standard portfolio).
d) Standard spreads scaled by a factor of 0.5 (reduction by 50%).
e) Standard spreads scaled by a factor of 1.5 (raise by 50%).
f) Standard spreads scaled by a factor of 2.5 (raise by 150%).

Each of the above mentioned modifications of the credit spreads was tested in the model, once using the standard transition matrix and once using a transition matrix where the probabilities of default were set to zero. The purpose of the latter is to make the impact of spread changes more clear. A third case was also tested where the change in credit spreads appear right in the middle of the simulation, just before the rebalancing time point of six months. The results are shown as 99.9% Values at Risk in Table IX.

| Case | Standard | a) Shift 5pp up | b) All 0% | c) All 2.1% | d) Down 50% | e) Up 50% | f) Up 150% |
|---|---|---|---|---|---|---|---|
| 1. Standard transition matrix, spreads constant through the simulation. | 17.64% | 14.80% | 18.25% | 16.80% | 17.28% | 17.80% | 17.04% |
| 2. Zero default probabilities, spreads constant through the simulation. | 9.32% | 8.60% | 0.00% | 0.00% | 5.10% | 12.55% | 17.32% |
| 3. Standard transition matrix, spreads change just before six months through the simulation. | | 69.20% | 5.20% | 22.44% | 6.75% | 27.07% | 42.66% |

Table IX. *Values at Risk for different modifications of the credit spreads in different cases.*

Starting with modification a), the Value at Risk actually decreases when shifting the spread curve up five percentage points for case 1 and 2. Remember that in this model, losses (or gains) occur due to position value changes at rebalancing time points. These value changes depend on differences in credit spreads during migrations (or the recovery rate when a default occurs). Hence, the sizes of the credit spreads are of less importance, it´s the differences between them that count. When raising all the spreads by the same amount, the differences between each spread stay the same. Hence this should not affect the Value at Risk. However, since higher spreads reduces the position values and the recovery value at default is constant and independent of the spreads, the potential loss at default become relatively lower. If, hypothetically thinking, spreads were very high, the discounted position values would be very small and the recovery value would be close to the discounted position value. Hence, potential losses would be small. This is also clarified by comparing modification b) and c) (still case 1 and 2), where the larger spreads in c) yield a smaller Value at Risk than in b). Since all spreads are equal in these modifications respectively, only defaults affect the Value at Risk and hence the Value at Risk becomes zero when the probability of default is set to zero.

In case 3 there is a change in the credit spreads during the simulation, which obviously has a huge effect on the Value at Risk, especially for modification a). A shift affects every position in the portfolio equally and there will be a loss for each position at six months for certain. Note that a shift by five percentage points is an extreme case which is not likely to happen in reality. Modification b) and c) have no unexpected impact on case 3. If all spreads are lowered to 0% right before six months there will be a gain in portfolio value which reduces the Value at Risk. By setting all spreads equal to 2.1%, most of the positions in the portfolio will increase their spreads slightly and therefore lose value at six months, which rises the Value at Risk.

In the final modifications d), e) and f) the credit spreads are scaled by different factors. In these cases, the differences between the spreads changes (although the percentage differences stay the same). The potential loss increases with the larger differences between the spreads, but it also decreases due to the spreads being larger. As seen in the results of case 1, using the standard transition matrix, Value at Risk is almost unaffected by this kind of scaling of the spread curve. However, when the probabilities of default are set to zero as in case 2, the impact of the spreads being larger in size almost disappears and only the impact of the larger differences between the spreads is reflected. In case 3 the impact of these modifications is clear and expected.

### 3.2.3 The Impact of the Recovery Rate

The recovery rate (indicating the percentage of the face value of the bond returned when a default occurs) is just a constant in this model, but in reality this is highly variable. Therefore it is good to get an idea of how different recovery rates affect the risk. Six different recovery rates were tested and the resulting Values at Risk are shown in Figure VII. As seen there is an approximately linear relationship of the Value at Risk among different recovery rates.

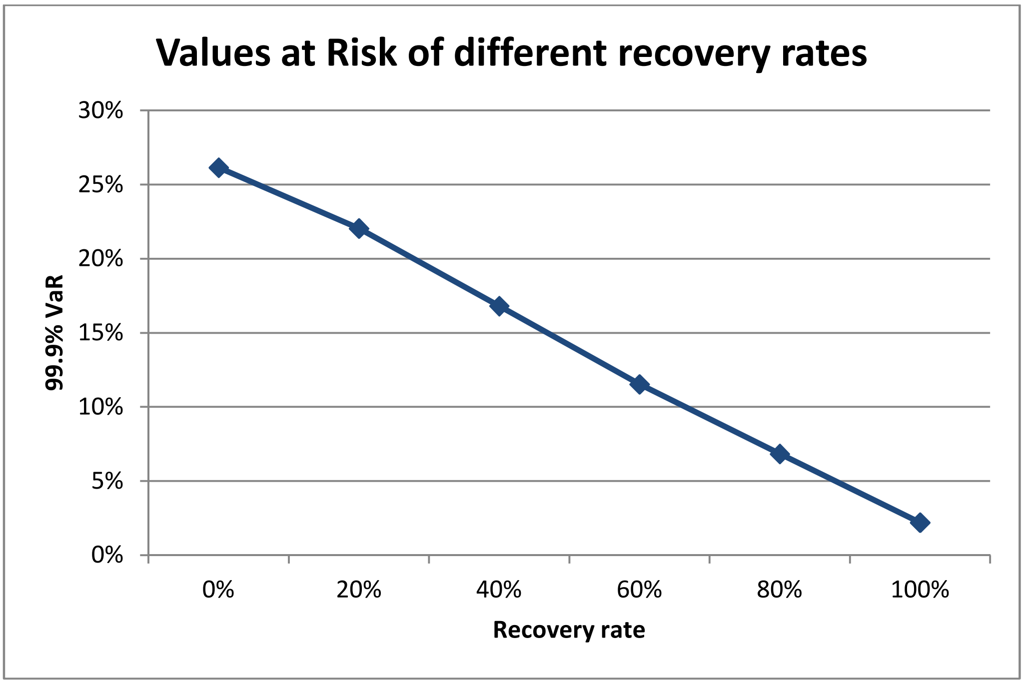


Figure VII. *Values at Risk of six different recovery rates.*

Another assumption that can be made about the recovery rate is that it should vary depending on which credit rating the position had before it defaulted. Consider an allocation of recovery rates as shown in Table X, where the recovery rate increases with higher-rated positions. The average is consistent with the previously used recovery rate of 37%.

| Rating | Aaa | Aa | A | Baa | Ba | B | Caa |
|---|---|---|---|---|---|---|---|
| **Recovery rate** | 52% | 47% | 42% | 37% | 32% | 27% | 22% |

Table X. *Recovery rates by credit rating.*

Using this input, a Value at Risk of 19.7% is generated, with confidence interval [20.7%, 19.15%]. This is about two percentage points higher than the Value at Risk generated with the constant recovery rate of 37%. The reason for the risk growth is simply that more lower rated issuers default in the model than higher rated issuers, so recovery rates below 37% are more anticipated than recovery rates above 37%.

### 3.2.4 Adjusting the Transition Matrix

The transition matrix is based on historical data and hence it depends entirely on the period of which the data were taken from. Intuitively, the probabilities of migrating to other ratings would be higher (and probabilities of keeping the original rating would be lower) if the matrix was generated by data from the past 5 years only rather than the past 30 years because of the recent financial crises. This is because of the fact that migrations among ratings appear more frequently during stressed scenarios. An experiment on real historical data could not be done due to lack of data, but a transition matrix conditioned on a hypothetical financial crisis may be constructed. Take the standard transition matrix based on three months defined in Table II in section 2.2.2 as a starting point. Assume fluctuations on the market leading to a 100% raise of the downgrade probabilities (including the default probabilities) and a 50% reduction of the upgrade probabilities. The probabilities of keeping the initial rating are recalculated in order for each row in the transition matrix to sum up to one and the matrix ends up as in Table XI. Using this as input to the model yields a 99.9% Value at Risk of 22.7% with a 95% confidence interval of [24.2%, 22.3%]. The difference from using the standard transition matrix is almost five percentage points.

| | **Rating in three months (%)** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Initial rating** | Aaa | Aa | A | Baa | Ba | B | Caa | Default |
| Aaa | 96.579% | 3.161% | 0.243% | 0.007% | 0.009% | 0.000% | 0.001% | 0.000% |
| Aa | 0.214% | 95.687% | 3.987% | 0.063% | 0.041% | 0.002% | 0.000% | 0.005% |
| A | 0.007% | 0.304% | 96.940% | 2.494% | 0.192% | 0.054% | 0.004% | 0.004% |
| Baa | 0.006% | 0.028% | 0.742% | 96.185% | 2.616% | 0.327% | 0.042% | 0.053% |
| Ba | 0.003% | 0.006% | 0.040% | 0.710% | 95.232% | 3.317% | 0.113% | 0.579% |
| B | 0.000% | 0.005% | 0.015% | 0.055% | 0.890% | 94.373% | 1.165% | 3.497% |
| Caa | 0.000% | 0.000% | 0.003% | 0.085% | 0.290% | 0.616% | 85.339% | 13.667% |
| Default | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 100.000% |

Table XI. *A hypothetical transition matrix conditional on a financial crisis.*

What´s interesting here is to see how much of the higher risk that depends on the higher probabilities of default. Assume the standard transition matrix defined in Table II in section 2.2.2 where only the probabilities of default are increased by 100%, i.e. the far right column is consistent with the one in Table XI. Again, the probabilities of no migration at all have to be recalculated for each row to sum up to one. The Value at Risk using this hypothetical matrix as input becomes 21.2% with confidence interval [22.1%, 20.4%]. As one might have expected, the higher probabilities of default contribute almost solely to the additional Value at Risk.

### 3.2.5 Adjusting the Correlation Matrix

Making a few adjustments of the default correlation matrix may yield the same effect as reorganizing the portfolio by concentrating the portfolio towards certain issuers, as made previously in the portfolio sensitivity analysis. The difference is that the correlation matrix is directly dependent on economic and industry conditions, and independent of the portfolio setup. The idea of changing the correlations to see how it affects the result is not only based on the fact that correlations may vary depending on the economic situation, but also because the estimation of the correlation matrix may not be perfect due to lack of good data and hence it could be interesting to see the results of using different matrices.

The average correlation between two issuers in the standard correlation matrix used in the model is 58%. What happens to the Value at Risk when adjusting the correlations so that the average correlation changes significantly? To analyze this, six different correlations matrices were constructed where all the correlations in each matrix between different issuers were set to 0%, 20%, 40%, 60%, 80% and 100%, respectively. The result is shown in Figure VIII where the horizontal axis shows the correlations and the vertical axis shows the 99.9% Value at Risk.

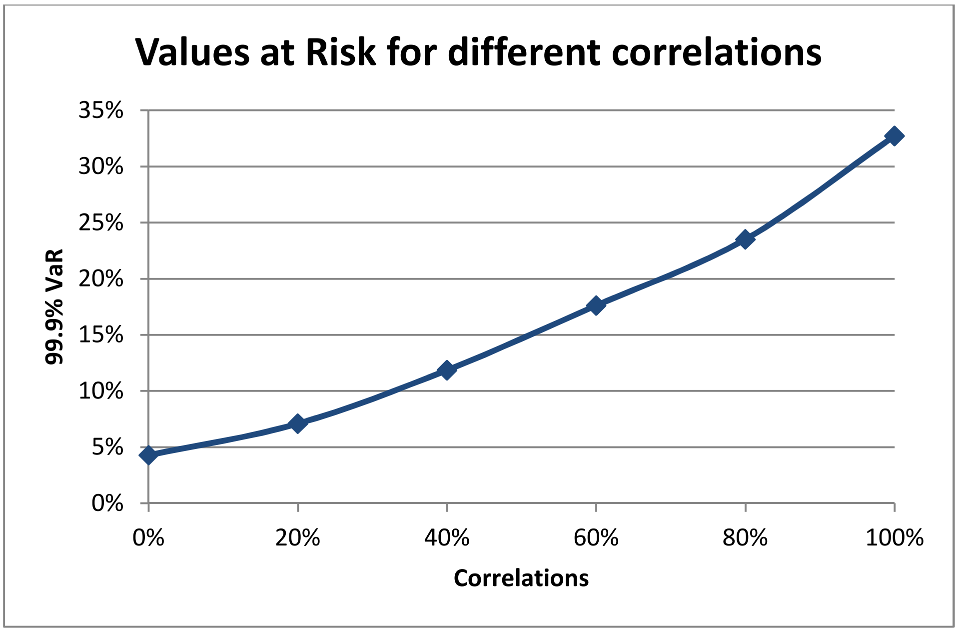


Figure VIII. *Values at Risk for six different correlation matrices.*

As seen in the figure, the correlation matrix containing correlations of 60% yields a Value at Risk close to the one generated by the standard correlation matrix with an average of 58% correlations. Also and quite obviously, the correlation matrix with correlations of 100% yields the same result as a portfolio where all issuers belong to the same category. Worth noticing is how much Value at Risk reduces with correlations. Value at Risk becomes remarkably small using correlations close to zero as input.

# 4. Summary & Conclusions

The aim of this thesis was to build a model that computes the Incremental Risk Charge for a simple portfolio in accordance with the rules of the Basel Committee. It should mainly detect potential losses caused by the migration risk and the default risk of unsecuritised credit products over a one year capital horizon. An important property of the model is the assumption of a constant level of risk; each position in the portfolio is rebalanced either if the liquidity horizon of that position is reached or if a default has occurred. Rebalancing of the position means replacing the position with the initial position and taking out the change in value. A portfolio of corporate bonds only was tested in the model since it results in simple calculations although it covers all the essential risks related to the specific risk charge. Extensions of the model to cover other positions should not be difficult to make as long as the positions can be evaluated by their credit quality.

When constructing a portfolio, the allocation of credit ratings among the positions is the single most important property of the portfolio from a credit risk perspective. The risk increases rapidly with the number of lower rated issuers and issuer concentrations in the portfolio are not nearly as risky. Aside from the portfolio, the model developed consists of a number of parameters that are sensitive to economic fluctuations. By generating hypothetical stress scenarios, individual analyses of the model inputs were made. Stress testing the model shows that the default risk accounts for a greater part of the risk than the migration risk. Two crucial sources of risk are the probabilities of default for each initial credit state, found in the transition matrix, and the recovery value at default. What´s interesting to see is the nearly perfect linear relationship between the recovery rate and the Value at Risk. The Value at Risk also varies a lot depending on the correlations among the different issuers in the portfolio.

The liquidity horizon of each position in the portfolio defines how often the position should be rebalanced in order to maintain a constant risk level through the simulation and hence it is an important parameter. It is not self-explanatory how the specific length of this affects the risk. Small changes in the liquidity horizons did not seem to have a significant impact on the Value at Risk in the model. As it turned out for a Baa-rated issuer, the reason for this was that by increasing the horizon, the default risk was raised but the migration risk was decreased slightly. Therefore, the total change in Value at Risk caused by longer liquidity horizons became almost negligible. One might find it strange that one of the main risks in the model decreases as the liquidity risk increases. The discussion of this is left for future research.

Changes in the credit spreads affect the risk in two ways. If the gaps between the spreads increase, the potential loss rises because of the larger differences between the discounted position values for each credit state. With higher spreads though, the difference between a discounted position value before default and the recovery value at default reduces. In that sense the potential loss reduces. Since these two counteract each other in terms of the risk, the effect on Value at Risk is small. However, if the changes in credit spreads appear during the simulation period, there is a clear impact on the risk. The Value at Risk increases significantly if the whole credit spread curve rises at some point during the simulation.

## 4.1 Further Research

The model built in this thesis fulfills the basic requirements stated in the guidelines of the Basel Committee. However, extensions can be made for a more accurate model. One example is adding issuer specific autocorrelation to the correlation structure, which introduces rating drift in the simulation. Rating drift means a higher probability for an issuer to be downgraded again if the issuer was recently downgraded than if it was not.

Another example is to let the recovery rate be a stochastic variable. Historical data from Moody´s Investors Service, 2007 shows that, for bonds, the probabilities of a recovery rate close to 0% or close to 100% are the largest. There are other documents in the literature supporting this and suggesting a beta distribution for the recovery rate. Aside from a mean and a standard deviation there are two parameters of the beta distribution that may (but does not have to) be calibrated in order for the distribution to fit the data. If this is done, implementation of the distribution function into the model is the only thing remaining, which is straight forward.

Regarding the issuer asset returns and the corresponding correlations, it is assumed that the log returns are outcomes from a normal distribution and the correlations are of Gaussian nature. There might be a better choice for this, for example the Student´ t-distribution. This has similarities to the normal distribution except its larger and longer tails, and it has proven to fit financial data well. To implement this in the model, the asset returns have to be simulated using a Student´s t-copula and the asset return thresholds have to be recalculated using the cumulative Student´s t-distribution function. A big advantage with the distribution is that it can be calibrated by varying the number of degrees of freedom. The higher this number is, the more similar it becomes the normal distribution.

# 5. Appendix

## 5.1 Data Required for Computing the Incremental Risk Charge

Following is a list of all the data necessary as input to the model for computing the specific risk charge.

- A portfolio of positions where the information below is available for each position.
  - Position type (e.g. coupon bond)
  - Issuer category
  - Issuer credit rating and liquidity horizon
  - Position currency
  - Cash flow amounts (e.g. coupon and face value for a bond)
  - Cash flow dates (e.g. payment frequency and maturity date for a bond)
  - Recovery rate (in case of default)
- An interest rate term structure (yield curve) for each currency in the portfolio that extends to the longest maturity date in the portfolio.
- Credit spreads for each credit rating and possibly even for different maturities.
- A default correlation matrix among the different issuer categories.
- A transition matrix with migration probabilities among the different credit states, where the credit state is either a credit rating or a default.

# 6. Bibliography

Asgharian, Hossein & Nordén, Lars. (2007). *Räntebärande instrument. Värdering och riskhantering.*

Basel Committee on Banking Supervision. (2009a). *Guidelines for computing capital for incremental risk in the trading book.* http://www.bis.org/publ/bcbs159.htm

Basel Committee on Banking Supervision. (2009b). *Revisions to the Basel II market risk framework.* http://www.bis.org/publ/bcbs158.htm

Enger, Jan & Grandell, Jan. (2010). Markovkompendium. http://www.math.kth.se/matstat/gru/markovkomp.pdf

Gupton, Greg M., Finger, Christopher C. & Bhatia, Mickey. (2007). *CreditMetrics. Technical Document.* http://www.msci.com/resources/technical_documentation/CMTD1.pdf

Handelsbanken. (2011). Internal documents.

Koski, Timo. (2011). Lecture notes of Computer intensive methods: Skew-normal distributions. http://www.math.kth.se/matstat/gru/sf2955/skewnormalal.pdf

Kreinin, Alexander & Sidelnikova, Marina. (2001). *Regularization Algorithms for Transition Matrices.* http://www.algorithmics.com/EN/media/pdfs/arq4-1_regularization.pdf

Moody´s Investors Service. (2007). *Moody´s Ultimate Recovery Database.* http://www.moodys.com/sites/products/DefaultResearch/2006600000428092.pdf

Saunders, Anthony & Allen, Linda. (2010). *Credit Risk Measurement In and Out of the Financial Crisis. New Approaches to Value at Risk and Other Paradigms.* 3rd Edition.

Stel, Yannick V.D. (2009). *The Incremental Risk Model. An Assessment of Alternate Inputs and Assumptions in a Multi-Period Setting.* http://oaithesis.eur.nl/ir/repub/asset/6469/298369stelma0110.docx

Yavin, Tzahi., Zhang, Hu., Wang, Eugene., Clayton, Michael A. (2011). *Transition Probability Matrix Methodology for Incremental Risk Charge.* http://mpra.ub.uni-muenchen.de/28740/1/MPRA_paper_28740.pdf