Master Thesis

Pricing and parameters influencing the Basis:

is it a profitable arbitrage opportunity?

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Abstract

The purpose of this report is to present the basis and analyze if it is an arbitrage opportunity. This master thesis explores products needed to compute the basis. Two main products are treated: the credit default swap and the asset-swap. We will investigate how to price these products, in order to get a method to obtain the value of the basis.

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Contents

Introduction

After the subprime mortgage crisis in the autumn 2008 credit derivatives desks in banks have to face difficulties to price their products: CDS, RMBS, CMBS... Moreover, most of the major banks had a huge portfolio of toxic assets they wanted to sell quickly in order to avoid contagion. The key point to price products traded on a credit derivatives desk is the spread, which represents the credit risk of the company involved in these deals. During the crisis values of spread had no more logic and most of the products could not be priced anymore.

When the situation was getting better and spread values got back to a regular level in 2010, a new crisis has started in Europe. Fears of a sovereign debt crisis developed among investors concerning rising government debt levels across the globe together with a wave of downgrading of government debt of certain European states like Greece, Ireland or Spain and more recently France. The consequences of this downgrading have been a change in the value of the credit risk of european countries and european companies. Consequently the spread of sovereign bonds and corporate bonds had changed a lot the past months.

The combination of these crisis has created some discrepancies in the value of the credit risk used to compute the spread of the credit default swaps, the bonds or the asset-swaps creating some arbitrage opportunities. One of them takes advantages of the difference of spread between the asset-swap and the CDS on the same underlying.

The difference between the spread of the CDS and the asset-swap on the same underlying is called the basis. In a market without any arbitrage opportunities the basis is supposed to be equal to 0. Nevertheless with the recent crisis described just above the basis moved away from 0 in some cases creating some arbitrage opportunities. Depending on the sign of the basis, taking a long or a short position on the CDS and the asset-swap will allow the investor to earn money with no risk.

In this master thesis we will study the pricing of the basis and parameters influencing its value. As explained previously the basis is the difference between the spread of the CDS and the one of the asset-swap on the same underlying. A perfect understanding of the credit default swap and the asset-swap is needed to price and describe the parameters influencing the basis.

The first part will be a description of my missions in Societe Generale Corporate and Investment Banking during these 11 months of trainee position. In a second time I will describe the credit default swap and be focused on the method to price it. Then, the same description will be done with the asset-swap. The last part will define the basis. We will see which parameters change its value, how can it be priced and if it is a good and easy arbitrage opportunity as it seems to be.

1 Presentation

In this chapter I will describe briefly the bank and my missions during these 11 months of trainee in the Front Office Commando team for an exotic credit derivatives desk in Societe Generale in London. The first part is dedicated to a global presentation of the group Societe Generale and the department I worked for in the part of the group named Societe Generale Corporate and Investment Banking. The second part will present the missions I performed as a Commando and my work environment.

1.1 Societe Generale Group presentation

Founded the 4th of May 1864, Societe Generale is today one of the three main companies of the French bank Industry with BNP Paribas and CALYON.

The group is located in more than 82 countries and employs 120 000 people across the world with $75,000$ based in Europe. The CEO is Frederic Oudea since May 2009

With a market capitalizations of €15 billion at the beginning of September Societe Generale belong to the top 8 ranking of World Bank groups. In 2008 the market capitalization of the bank was three times bigger. The Jerome Kerviel's massive loss of $5 billion in the beginning of 2008, the subprimes' crisis and the recent European financial crisis with a partial default from Greece explain the fall of the stock price and so the capitalization. For example, since March 2011 the stock price has dropped from €50 to €20. Nevertheless the bank has showed good results in the third trimester of 2011 with a profit of €622 million and the group remains the 6^{th} largest French company by market capitalization

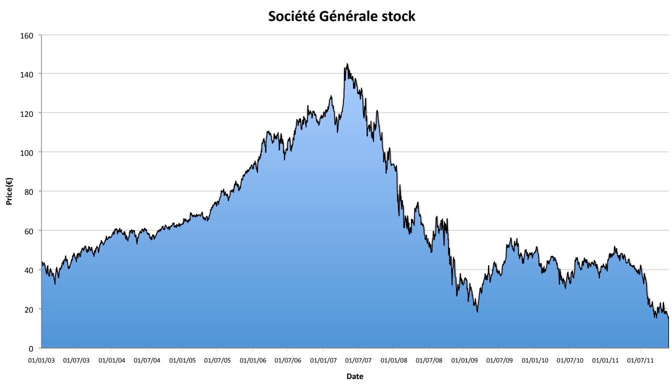


Figure 1.1: Price of the Societe Generale share

In november 2011 the group announced it has halved its sovereign debt exposure to the troubled eurozone countries of Greece, Italy, Ireland, Portugal and Spain and between September and October and decided to scrap its dividend and is cutting bonuses by a significant amount to preserve capital.

The group Societe Generale is focused on 3 main activities:

- Retail Banking in France and abroad with more than 17 million of clients.
- Societe Generale Asset Management (SGAM)
- Societe Generale Corporate and Investment Banking (SGCIB)

1.2 SGCIB

Societe Generale Coporate and Investment Banking is the third largest corporate and investment bank in the euro zone by net banking income. It serves corporates, financial institutions and investors in over 45 countries across Europe, the Americas and Asia.

The corporate part of SGCIB is the part of the bank covering all the missions of merges and acquisitions, structured financing, cash management, syndication. All these services are offered to companies in order to provided them advisory on investment operations.

The investment banking activity advise private or institutional investors on financial markets and provide them an access to the market thanks to the emission of actions, bonds or complex financial products.

SGCIB is structured in 5 different business lines:

- BIEN (Mid-Cap Investment Banking) is the service of strategic and capital evolution for companies
- CAFI (Capital raising and financing) deals with financing on the capital markets
- CORI (Corporate, Institutions and Advisory) is the commercial direction of SGCIB
- FICC (Fixed Income Currencies and Commodities) deals with structured financing
- GEDS (Global Equities and Derivatives Solutions) is the business line which handles the derivatives products

These business lines are rely on different resources:

- AUD (Audit)
- COM (Communication)
- HUMN (Human Ressources)
- ITEC (Information Technology)

- MARK (Market)
- OPER (Financial Operations)
- PMO (Project Management Office)

My trainee position took place in the exotic Credit Derivatives Commando team. Our team is in the ITEC department for the FICC business line. We were dedicated to a trading desk dealing with exotic Credit Derivatives.

In the London branch of SGCIB there are two different trading desk relative to credit derivatives. One of them is dedicated to the correlation trading and the secondary market on credit derivatives and is supported by half of our team. The second trading desk, named "bad bank" in most of the bank or IEC i SGCIB, deals with toxic assets which were at the origin of the subprimes' crisis in 2008 like RMBS or CMBS US. During my 11-months in SGCIB I worked for this trading desk.

1.2.1 IEC: the "Bad Bank"

In the middle of 2007 the sub primes crisis starts in United States of America, characterized by a rise in subprime mortgage delinquencies and foreclosures, and the resulting decline of securities backed by said mortgages. Some financial products like ABS (asset-backed security), RMBS (Residential Mortgage-Backed Security), CMBS (Commercial Mortgage-Backed Security) and CDO (Collateralized Debt Obligation) become totally illiquid and toxic.

With a position of €50 billion on these products SGICB decided at the end of 2007 to create a trading desk, named SSG (Special Situation Group), dedicated to this portfolio of assets in London and New-York. The aim of this exotic credit derivatives desk was to decrease the value of the exposure by trading these toxic assets.

In January 2010, the value of the exposure was of €35 billion and the group decided to transfer all the assets in a new structure named IEC (Inter Europe Conseil). With 8 traders in London and 10 in New-York the exposure has decreased by €10 billion last year. Traders have to hedge each asset of the portfolio and sold them when there are some opportunities.

1.2.2 Commando role

My trainee position took place in the Exotic Credit derivatives Commando team, a fifteen persons team split between New-York City and London offices and working for two exotic credit derivatives trading desk: IEC and correlation desk. The "Commando" or Rapid Application Developer (RAD) are seated on the trading floor with quants, structurors, traders, analysts and sales. The main role is to serve the trading desk for short term (1-2 day(s)) projects and middle term projects (1-4 week(s)). All long term projects are transferred to industrial team. Commando main duty is to provide tactical and flexible solutions to cope with the Front Office needs. All daily tasks cover a wide range of applications such as risk analysis, reporting (P&L), automating pricers, service desk, building up a referential database and communication with others teams (R&D, MO, SupportÉ)

The key point to success as commando in to understand the need of the clients (traders) and find the best way to answer this need by elaborating a solution quickly and user-friendly without any bugs.

1.2.3 My missions

The daily tasks are just a small part of the work performed by the commando team. Our main role is to provide tools to make traders' life easier. Programming languages and frameworks used to create tools were C#, SQL, VBA, Microsoft.NET Framework 3.5 and Excel DNA which allows to ingrate .NET into Excel in order to create Excel Add-Ins.

The main tools, used directly in Excel, I have developed this 11 months of internship are:

- **Liquidity Ratio:** It is asked by regulator to be sure the desk is standard of the market and has enough short term liquidity, meaning Societe Generale has enough assets to deleverage. This tool computes automatically the liquidity ratio in Excel using Fx rates from the European Central Bank or Societe Generale's referential with details on bonds and funding products used to compute the liquidity ratio.
- **Negative Basis Trade Pricing Excel spreadsheet:** Just before I arrived in this team a new model to price negative basis trade had been developed with quantitative research team. I had to calibrate and make some linear regressions on this new model to be sure it was consistent with the previous and make an excel spreadsheet used by traders to price these trades using this new model. My results had to be validated by the research quantitative team.
- **Board Pack:** This tool shows all the positions and the assets managed by the trading desk. Around 50 pivot tables are automatically generated to summarize the impact of the currency, the monoline and the maturity date of the asset. This report is showed to the top management twice a month.
- **Funding strategies:** It is an excel report showing all the funding strategies. Traders can see what are the funding links between loans and bonds.

2 Credit Default Swap

Nowadays the CDS market is one of the biggest financial derivatives market. In july 2011, just in the US, the value of the assets protected by credit default swaps was around $15,227$ billions of dollars, equaling the gross domestic product of the US. Moreover the credit default swap is a key point to understand and compute the basis.
Consequently, in this chapter we will describe and explain the credit default swap. It will start with the definition and the characteristics of this product. We will then model the risk of default in order to price the value of a CDS.

2.1 Definition

A Credit Default Swap is an OTC (Over-The-Counter) contract between the buyer and the seller of the protection to swap the credit risk of an obligation issued by a reference entity. The CDS contract specifies exactly the legal entity for which it provides the default protection but also the seniority of the covered capital structure. The CDS protects the buyer against losses that may occur on the obligation in case of credit events in exchange of a periodic premium over the lifetime of the contract. It can be seen as an insurance against the risk of default.
A credit event is an event that occurs on the reference entity and will trigger settlement in a credit derivative contract :

- Downgrade in S&P, Fitch and/or Moody's credit rating below a specific minimum level
- Financial or debt restructuring, for example under administration or as required under U.S. bankruptcy protection
- Bankruptcy or insolvency of the reference asset obligor
- Default on payment obligation such as bond coupon and continued nonpayment after a specific time period
- Technical default, for example the nonpayment of interest or coupon when it falls value
- A change in credit spread payable by the obligor above a specific maximum level

In case of a credit event before the maturity date of the contract a payment by the protection seller to the protection buyer will happen. This payment is equal to the difference between par and the price of the assets of the reference entity on the face value of the protection, and compensates the protection buyer of the loss.

This protection leg can be settled by two different ways decided at the initiation of the contact:

Physical settlement: This is the most widely used settlement procedure. It requires the protection buyer to deliver the notional amount of deliverable obligations of the reference entity to the protection in return for the the notional amount paid in cash.
If the deliverable obligations trade with different prices following a credit event, which they are most likely to do if the credit event is a restructuring, the protection buyer can take advantage of this situation by buying and delivering the cheapest asset. The protection buyer is therefore long a cheapest to deliver option.

Cash settlement: This is the alternative to physical settlement, and is used less frequently in standard CDS but is more common in tranched CDOs. In cash settlement, a cash payment is made by the protection seller to the protection buyer equal to par minus the recovery rate of the reference asset. The recovery rate is calculated by referencing dealer quotes or observable market prices over some period after default has occurred.

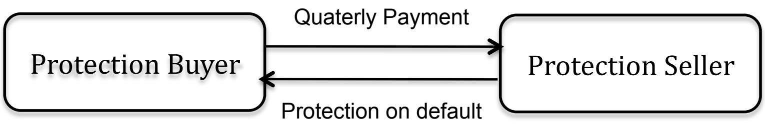


Figure 2.1: CDS CashFlows with No Default

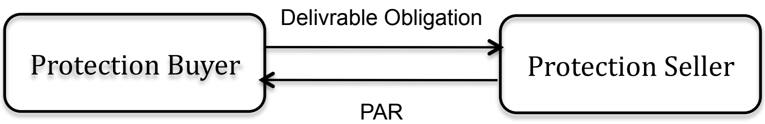


Figure 2.2: CDS CashFlows with Physical Settlement

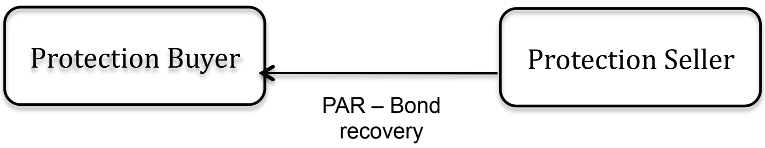


Figure 2.3: CDS CashFlows with Cash Settlement

Since 2002, most of the CDS contracts have standardized quarterly payments and maturity dates : March 20^{th}, June 20^{th}, September 20^{th} and December 20^{th}. The value of the quarterly payments is computed using the spread and the value of the protection.

The spread of a CDS is the annual amount the protection buyer must pay the protection seller over the length of the contract, expressed as a percentage of the notional amount. For example, if the CDS spread of Risky Corp is 50 basis points, or 0.5%, then an investor buying \$10 million worth of protection from AAA-Bank must pay the bank \$50,000 per year. These payments continue until either the CDS contract expires or a credit event occurs.

The value of the spread depends on the risk of default of a company or a country. As you can imagine, for few months the spread of European countries or companies is very high due to the Europe financial crisis and thus the large probability of default. It can be illustrated by watching the value of the spread to get a 5 year protection on Greece [1], France[2] and Societe Generale [3]. The historical data of spreads came from the Bloomberg website.

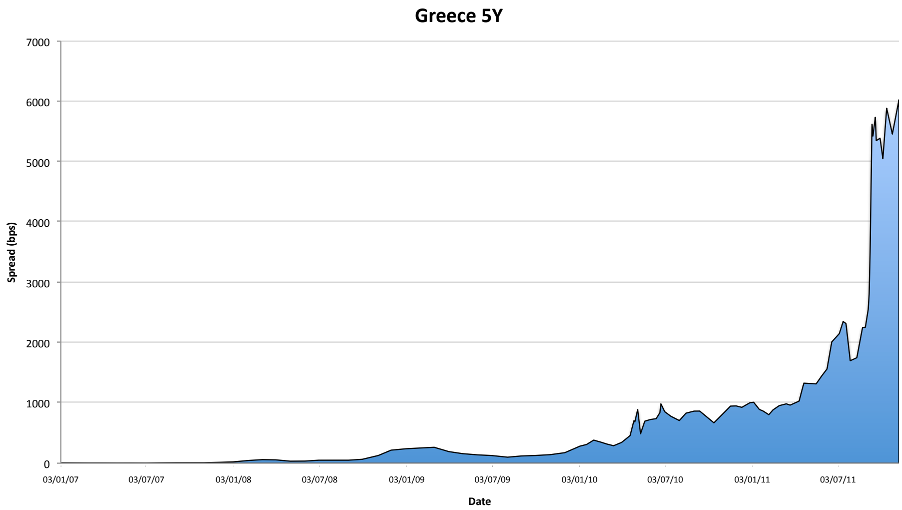


Figure 2.4: Spread for a 5 years protection on Greece

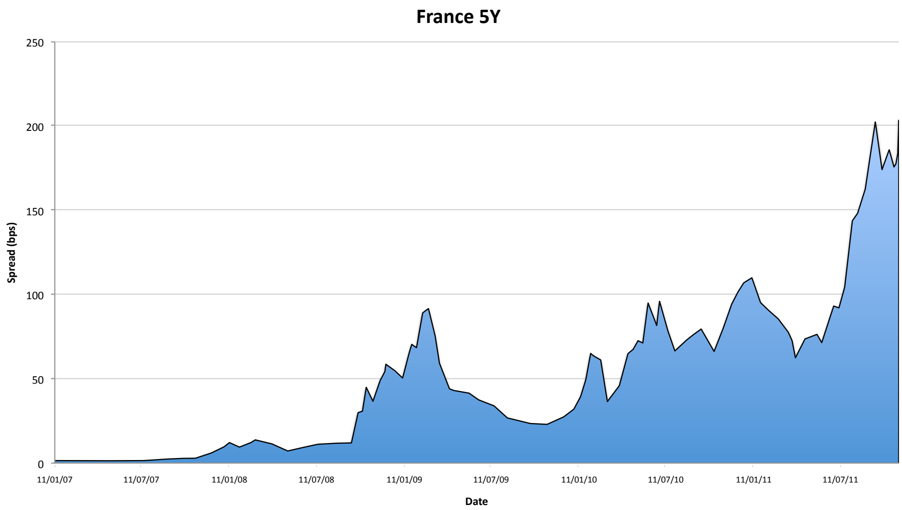


Figure 2.5: Spread for a 5 years protection on France

[1]http://www.bloomberg.com/quote/CGGB1U5:IND
[2]http://www.bloomberg.com/quote/CFRTR1U5:IND
[3]http://www.bloomberg.com/quote/CSOC1E5:IND

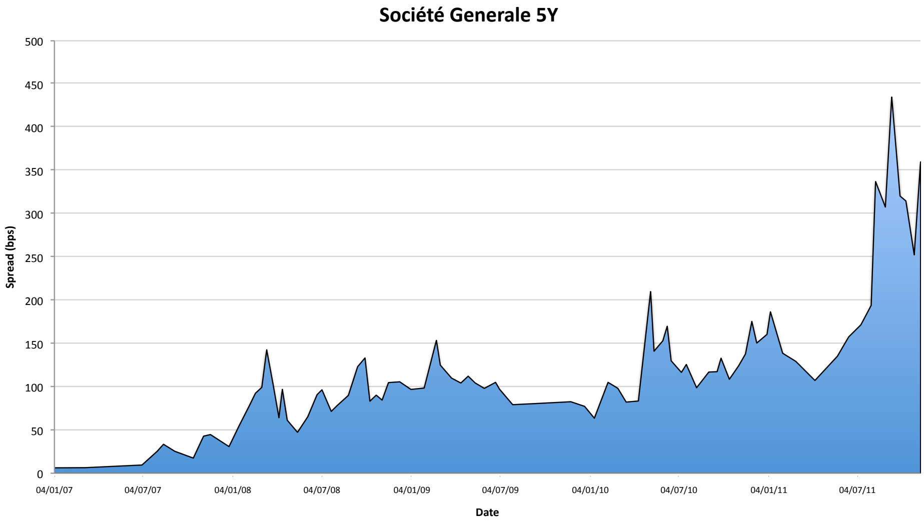


Figure 2.6: Spread for a 5 years protection on Societe Generale

We will take a concrete example to illustrate these graphics:
Let us suppose you buy \$100,000,000 of Societe Generale bonds maturing in 5 years. Nevertheless you are afraid by the fact that Societe Generale might make default within the five next years. So you decide to take a long position (buy) on a CDS. The credit default swap concerned is a 5 years Societe Generale contract.
Let us suppose now we were the 4^{th} of July 2009. The price to get a long position on Societe Generale bonds during 5 years is 100 bips per year. Thus you would have to pay $100,000$ per year to be protected by a Societe Generale default. If it is a semi-annual contract you would have to pay $50,000$ every six months during the next 5 years.

2.2 Reduced Form Model

In the reduced-form approach, the credit process is modeled directly via the probability of the credit event itself. Reduced-form models also generally have the flexibility to refit the prices of a variety of credit instrument of different maturities.They can also be extended to price more exotic credit derivatives. It is for these reasons that reduced-form models are used for credit derivatives pricing.

The other main approach is the structural form. It is generally used to say what spread corporate bonds should trade based on the internal structure of the company. Thence it requires some informations about the balance sheet of the company concerned in order to determine a link between debt markets and pricing in the equity. Nevertheless it is impossible to price complex credit derivatives explaining the reason why the reduced form is preferred to price them.

The most widely used reduced-form approach is based on the work of Jarrow and Turnbull in 1995 characterizing a credit event as the first event of a Poisson counting process that occurs at some time t with a probability defined as:

$$\mathbb{P}(\tau \in [t, t + dt[) = \lambda(t)dt$$

It means that the probability of defaulting in the next dt instants conditional on surviving to time t is proportional to some time-dependent function $\lambda(t)$ called the intensity or hazard rate and the length of the time interval dt.

Moreover the intensity does not take into account the credit spread volatility since λ is deterministic. It means that λ has the following structure:

$$d\lambda(t) = (\dots)dt + \boxed{0}dW_t$$

2.3 Poisson Process

The main parameters of a Poisson process are the size of the jumps and the intensity which can be::

- constant: standard Poisson process, time homogeneous Poisson process
- time varying: time inhomogeneous Poisson process

2.3.1 Standard Poisson process, time homogeneous Poisson process

We call standard Poisson process or time homogeneous Poisson process a Poisson process where intensity λ is constant and jumps equal to 1. It is a Markov process with stationary independent increments.

Definition: The stochastic process in continuous time $M = (M_t)_{t\in\mathbb{R}_+}$ is a Poisson process with intensity λ if:

1. $M_0 = 0$
2. **the process has stationary independent increments**
3. **the increments satisfy :**

$$\mathbb{P}(M_{t+h} - M_t = n) = \mathbb{P}(M_h = n) = \begin{cases} 1 - \lambda h + o(h) & \textbf{if } n = 0 \\ \lambda h + o(h) & \textbf{if } n = 1 \\ o(h) & \textbf{if } n \geq 2 \end{cases}$$

Another way to define the Poison process is to use the probability of the time waited between two jumps :

Definition: Assume T_n, $n = 1, 2, \dots$ be random variables independent and identically distributed following an exponentially distribution with a parameter λ and $\tau_n = \sum_{i=1}^{n} T_i$ with $n \geq 1$. Then

$$M_t = \sum_{n\geq 1} \mathbb{I}_{\tau_n \leq t} = \begin{cases} 0 & \text{if } 0 \leq t < \tau_1 \\ 1 & \text{if } \tau_1 \leq t < \tau_2 \\ \vdots & \\ n & \text{if } \tau_n \leq t < \tau_{n+1} \end{cases}$$

Using these definitions some properties of the Poisson process can be demonstrated:

1. *It is an increasing process*
2. *It is a Markov process*
3. *The number of jump is finite in [0, T]*
4. *The process is continuous in probability:*

$$\forall t,\, \forall \epsilon > 0 \lim_{h \to 0} \mathbb{P}(M_{t+h} - M_t \geq \epsilon) = 0$$

We will demonstrate $\mathbb{P}(\tau \in [t, t+dt[) = \lambda(t)dt$ using the fact that :

$$M_t \text{ is a time homogeneous Poisson process} \iff \mathbb{P}(M_{t+h} - M_t = n) = \frac{\lambda(h)^n}{n!} e^{-\lambda h}$$

Assume τ_i be the i^{th} time jump. Then $\tau_1, \tau_2 - \tau_1, \ldots, \tau_n - \tau_{n-1}$ are independent and identically distributed following an exponential distribution with a parameter λ.
If we define the default time $\tau = \tau_1$ the first jump time of $M(t)$ and thus λt is exponential distributed with a parameter equal to 1.

$$\begin{aligned}\mathbb{P}(\tau > t) &= \mathbb{P}(\lambda\tau > \lambda t) \\ &= e^{-\lambda t}\end{aligned}$$

It implies:

$$\begin{aligned}\mathbb{P}(\tau \in [t, t+dt[/\tau > t) &= \frac{\mathbb{P}(\tau \in [t, t+dt[\cap \tau > t)}{\mathbb{P}(\tau > t)} \\ &= \frac{\mathbb{P}(\tau \in]t, t+dt[)}{\mathbb{P}(\tau > t)} \\ &= \frac{\mathbb{P}(\tau > t) - \mathbb{P}(\tau > t+dt)}{\mathbb{P}(\tau > t)} \\ &= \frac{e^{-\lambda t} - e^{-\lambda(t+dt)}}{e^{-\lambda dt}} \\ &= 1 - e^{-\lambda dt} \\ &= \lambda dt\end{aligned}$$

So as conclusion the Poisson process jump property allows to consider survival probabilities as discount factors and thus default intensities as credit spreads.

2.3.2 Time inhomogeneous Poisson process

Consider now a deterministic time varying intensity $\lambda(t)$ which is a positive piecewise continuous function. The cumulated intensity or cumulated hazard rate is defined as follows

$$\Lambda(t) = \int_0^t \lambda(u)du$$

The definition of a time inhomogeneous Poisson process is :

Definition: The stochastic process in continuous time $M = (M_t)_{t \in \mathbb{R}_+}$ is a time inhomogeneous Poisson process with intensity $\lambda(t)$ if:

1. $M_0 = 0$
2. **the process has stationary independent increments**
3. **the increments satisfy :**

$$\mathbb{P}(M_{t+h} - M_t = n) = \mathbb{P}(M_h = n) = \begin{cases} 1 - \lambda h + o(h) & \textbf{if } n = 0 \\ \lambda h + o(h) & \textbf{if } n = 1 \\ o(h) & \textbf{if } n \geq 2 \end{cases}$$

This definition implies the following expression:

$$\mathbb{P}(M_{t+s} - M_s) = \frac{1}{n!} \left(\int_s^{s+t} \lambda_u du \right)^n exp \left(- \int_s^{s+t} \lambda_u du \right), \quad n \geq 0$$

Denoting as previously the default time τ the first jump time of $M(t)$ we get:

$$\begin{aligned} \mathbb{P}(\tau \geq t) &= exp \left(- \int_s^{s+t} \lambda_u du \right) \\ &= e^{-\Lambda_t} \end{aligned}$$

It implies:

$$\begin{aligned} \mathbb{P}(s < \tau < t) &= \mathbb{P}(\Lambda(s) < \Lambda(\tau) < \Lambda(t)) \\ &= \mathbb{P}(\Lambda(s) < \Lambda(\tau) < \Lambda(t)) \\ &= \mathbb{P}(\Lambda(\tau) > \Lambda(s)\} - \mathbb{P}(\Lambda(\tau) > \Lambda(t)) \\ &= e^{-\Lambda(s)} - e^{-\Lambda(t)} \\ &= e^{-\int_0^s \lambda(u) du} - e^{-\int_0^t \lambda(u) du} \\ &\approx 1 - \int_0^s \lambda(u) du - (1 - \int_0^t \lambda(u) du) \\ &\approx \int_0^t \lambda(u) du - \int_0^s \lambda(u) du \\ &\approx \int_s^t \lambda(u) du \end{aligned}$$

It follows $\mathbb{P}(\tau \in [t, t + dt[/ \tau > t) = \lambda dt$

2.4 CDS Pricing

In the following part we will consider that the protection buyer pays rate S at times $T_{a+1}, \ldots, T_b$, ending payments in case of default. We indicate the default time by τ and the year fraction between T_{i-1} and T_i with α_i.
Moreover the protection seller agrees to make a single protection payment if the pre-specified default event happens between T_a and T_b. We will denote $T_{\beta(\tau)}$ the first of the T_i's following τ and the price of a riskless zero-coupon bond, which is the price at time 0 of one unit of currency made maturing at T will be written as $P(0, T)$.

2.4.1 Hypothesis

To compute the price of a Credit Default Swap we will consider the following hypothesis:

1. There is no arbitrage opportunity in the market
2. There is no market frictions
3. The buyer and the seller have no counter-party risk

An arbitrage opportunity is equivalent to the possibility of earning money out of nothing without taking any risk. It might by defined by:

Definition: An arbitrage possibility on a financial market is a self-financed portfolio h such that:

1. $V^h(0) = 0$

2. $\mathbb{P}(V^h(T) \geq 0) = 1$

3. $\mathbb{P}(V^h(T) > 0) > 0$

with $V(t)$ the value of the portfolio h at time t. Moreover we consider a self-financed portfolio as a portfolio with no exogenous infusion or withdrawal of money.

An arbitrage possibility is a serious case of mispricing in the market. To avoid this phenomenon, we will suppose in the following that the there is no arbitrage opportunity.
A market friction is defined as anything that interferes with trade. It includes transaction costs, taxes and the liquidity of the market.
Under the assumptions of no arbitrage opportunities and no market frictions there exists a risk neutral measure denoted $\mathbb{Q}$.

No counter-party risk means that the buyer and the seller have no risk of no-payment until the maturity of the contract.

Moreover we will perform our computations on a CDS single-name. A single-name is a contract on which the reference-entity is just one obligation. If the reference-entity is a panel or a pool of different obligations the contract is said to be a CDS Index. There are two mains families of CDS Index: iTraxx and CDX. A CDS index is literally a portfolio of CDS single-name having all the following properties:

1. Each obligation has the same notional
2. The maturity is the same, generally $5y$, $7y$ or $10y$
3. The same spread for each reference entity, named contractual-spread, is defined at the beginning of the contract.

2.4.2 Bank account

Assume the following complete filtered probability space $(\Omega, F, \mathbb{P}, F_t)$ where F_t satisfies $F_0 = (\Omega, \emptyset)$ and $F = F_T$ for $T \in [0, \infty[$ and the bank account numeraire B_t associated to the risk neutral measure $\mathbb{Q} \sim \mathbb{P}$

$$\begin{aligned} dB(t) &= r(t)B(t)dt \quad \text{where} \quad B(0) = 1 \\ B(t) &= e^{\int_0^t r(s)ds} \end{aligned}$$

The stochastic discount factor $D(t, T)$ between two instants t and T is the equivalent value of one unit of currency at time t and payable at T.

$$D(t, T) = \frac{B(t)}{B(T)} = e^{-\int_t^T r(s)ds}$$

Let X be a F measurable random variable such that $\frac{X}{B_T}$ is $\mathbb{Q}$-integrable. The claim is given by $X_t = \mathbb{E}^{\mathbb{Q}}[D(t, T)X \mid F_t]$ where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the risk neutral measure.

2.4.3 Running Credit Default Swap

The value of a credit default is the difference between the discounted value of the money received by the protection seller to offer the protection, named the premium leg and the value of the part paid by the seller to the buyer of the protection if a default occurs named the protection leg.

In a running Credit Default Swap the buyer receives the protection leg by the protection seller at the default time τ.
It can be written as:

$$\text{value of the CDS} = \substack{\text{discounted value of} \\ \text{the premium leg}} - \substack{\text{discounted value of} \\ \text{the protection leg}}$$

Using the notation described in the beginning of this section, S as the CDS spread and R as the recovery rate, we get the discount value of the running Credit Default Swap given by:

$$\text{RCDS}_t^{T_a,T_b} = \underbrace{D(t,\tau)(\tau - T_{\beta(\tau)-1})S\mathbf{1}_{\{T_a<\tau<T_b\}} + \sum_{i=a+1}^{b} D(t,T_i)\alpha_i S\mathbf{1}_{\{\tau>T_i\}}}_{\textit{premium leg}}$$

$$- \underbrace{(1-R)\mathbf{1}_{\{T_a<\tau\leqslant T_b\}}D(t,\tau)}_{\textit{protection leg}}$$

A detailed computation of these expression will be performed in the following of this chapter.

2.4.4 Postponed Credit Default Swap

In some case a slightly different payoff is considered. Instead of considering the exact default time τ, the protection payment is postponed to the first T_i following default.

Under this formulation where the protection payment is moved from τ to $T_{\beta(\tau)}$ the CDS discounted payoff can be written as:

$$PCDS_t^{T_a,T_b} = \sum_{i=a+1}^{b} D(t,T_i)\alpha_i S\mathbf{1}_{\{\tau>T_{i-1}\}} - (1-R)\mathbf{1}_{\{T_{i-1}<\tau\leqslant T_i\}}D(t,T_i)$$

As we can see, in a Postponed Credit Default Swap the buyer of the protection has to pay the premium leg for the full period containing the default time. Consequently this kind of contract is more expensive for the buyer of protection.

2.4.5 CDS Legs

Hazard rate:

As we have seen previously we have the following properties:

$$\begin{aligned}
\mathbb{P}(\tau > T_i) &= e^{-\int_0^{T_i}\lambda(u)du} \\
\mathbb{P}(\tau \leqslant T_i) &= 1 - e^{-\int_0^{T_i}\lambda(u)du} \\
d\mathbb{P}(\tau \leqslant T_i) &= \lambda(T_i)e^{-\int_0^{T_i}\lambda(u)du} \\
d\mathbb{P}(\tau \leqslant T_i) &= -d\mathbb{P}(\tau > T_i)
\end{aligned}$$

Premium Leg:

The premium leg $\Phi_t^{T_a,T_b}(S)$ of a credit default swap is defined as follows:

$$\Phi_t^{T_a,T_b}(S) = \text{Accrued rate at default} + \text{CDS payments if no default}$$

$$\Phi_t^{T_a,T_b}(S) = D(t,\tau)(\tau - T_{\beta(\tau)-1})S\mathbf{1}_{\{T_a<\tau<T_b\}} + \sum_{i=a+1}^{b} D(t,T_i)\alpha_i S\mathbf{1}_{\{\tau>T_i\}}$$

At time 0, the expected premium leg is computed under risk neutral measure $\mathbb{Q}$ as follows:

$$\Phi_0(S) = \mathbb{E}^{\mathbb{Q}}[D(0,\tau)(\tau - T_{\beta(\tau)-1}))S\mathbf{1}_{\{T_a<\tau<T_b\}} + \sum_{i=a+1}^{b} D(0,T_i)\alpha_i S\mathbf{1}_{\{\tau>T_i\}}]$$

$$= \mathbb{E}^{\mathbb{Q}}[\mathbb{E}^{\mathbb{Q}}[e^{-\int_0^\tau r(u)du}(\tau - T_{\beta(\tau)-1}))S\mathbf{1}_{\{T_a<\tau<T_b\}}/\tau > t]] + \mathbb{E}^{\mathbb{Q}}[\sum_{i=a+1}^{b} e^{-\int_0^\tau r(u)du}\alpha_i S\mathbf{1}_{\{\tau>T_i\}}]$$

$$= \int_0^\infty \mathbb{E}^{\mathbb{Q}}[e^{-\int_0^t r(u)du}(t - T_{\beta(s)-1})S\mathbf{1}_{\{T_a<\tau<T_b\}}\mathbf{1}_{\{u\in[t,t+ds]\}}]dt + S\sum_{i=a+1}^{b}\alpha_i\mathbb{E}^{\mathbb{Q}}[e^{-\int_0^{T_i} r(u)du}\mathbf{1}_{\{\tau>T_i\}}]$$

$$= \int_{T_a}^{T_b} \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r(u)du}(t - T_{\beta(t)-1})S\mathbf{1}_{\{u\in[t,t+dt]\}}\right] + S\sum_{i=a+1}^{b}\alpha_i\mathbb{E}^{\mathbb{Q}}[e^{-\int_0^{T_i} r(u)du}\mathbf{1}_{\{\tau>T_i\}}]$$

$$= S\sum_{i=a+1}^{b}\alpha_i\left(\int_{T_{i-1}}^{T_i}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r(u)du}\frac{t-T_{i-1}}{T_i-T_{i-1}}\mathbf{1}_{\{u\in[t,t+dt]\}}\right]dt + \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^{T_i} r(u)du}\right]\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau>T_i\}}\right]\right)$$

$$= S\sum_{i=a+1}^{b}\alpha_i\left(\int_{T_{i-1}}^{T_i}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r(u)du}\right]\left(\frac{t-T_{i-1}}{T_i-T_{i-1}}\right)\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{u\in[t,t+dt]\}}\right]dt + P(0,T_i)Q\left(\tau\geq T_i\right)\right)$$

$$= S\sum_{i=a+1}^{b}\alpha_i\left(\int_{T_{i-1}}^{T_i}\left[P(0,t)\left(\frac{t-T_{i-1}}{T_i-T_{i-1}}\right)Q(\tau\in[t,t+dt])\right]dt + P(0,T_i)Q(\tau\geq T_i)\right)$$

$$= S\sum_{i=a+1}^{b}\alpha_i\left(\int_{T_{i-1}}^{T_i}\left[P(0,t)\alpha_i\left(\frac{t-T_{i-1}}{T_i-T_{i-1}}\right)\right]dQ(\tau\leq t) + P(0,T_i)Q(\tau\geq T_i)\right)$$

$$= S\sum_{i=a+1}^{b}\alpha_i\left(P(0,T_i)Q(\tau\geq T_i) - \int_{T_{i-1}}^{T_i}\left[P(0,t)\alpha_i\left(\frac{t-T_{i-1}}{T_i-T_{i-1}}\right)\right]dQ(\tau>t)\right)$$

The risky basis point value, most of the time named $PV01$ or $DV01$, is used by traders to determine the risk on a Credit Default Swap. It defines the value of the premium leg when the spread is equal to 1.
Its value is given by:

$$RBP_t^{T_a,T_b} = -\sum_{i=a+1}^{b}\int_{T_{i-1}}^{T_i}[P(t,u)\alpha_i[\frac{u-T_{i-1}}{T_i-T_{i-1}}]dQ(\tau\geq u)] + \sum_{i=a+1}^{b}[\alpha_i P(t,T_i)Q(\tau\geq T_i)]$$

Protection Leg:

The discounted value of the default leg or protection leg:

$$\Psi_T^{T_a,T_b}(R) = \text{Protection pays at default}$$

$$= (1-R)\mathbf{1}_{\{T_a<\tau\leq T_b\}}D(t,\tau)$$

The protection leg is calculated under risk neutral measure as follows:

$$
\begin{aligned}
\Psi_0^{T_a,T_b}(R) &= \mathbb{E}^{\mathbb{Q}}\left[(1-R)\mathbf{1}_{\{T_a<\tau\leqslant T_b\}}e^{\int_0^\tau r(u)du}\right] \\
&= (1-R)\mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{T_a<\tau\leqslant T_b\}}e^{\int_0^\tau r(u)du}/\tau > t]\right] \\
&= (1-R)\mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty e^{\int_0^\tau r(u)du}\mathbf{1}_{\{T_a<\tau\leqslant T_b\}}\mathbf{1}_{\{\tau\in[u,u+du]\}}\right] \\
&= (1-R)\int_{T_a}^{T_b}\mathbb{E}^{\mathbb{Q}}\left[e^{\int_0^\tau r(u)du}\mathbf{1}_{\{\tau\in[u,u+du]\}}\right] \\
&= (1-R)\int_{T_a}^{T_b}\mathbb{E}^{\mathbb{Q}}\left[e^{\int_0^\tau r(u)du}\right]\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau\in[u,u+du]\}}\right] \\
&= (1-R)\int_{T_a}^{T_b}P(0,u)Q\left(\tau\in[u,u+du]\right)du \\
&= (1-R)\int_{T_a}^{T_b}P(0,u)dQ(\tau\leq u) \\
&= (1-R)\int_{T_a}^{T_b}P(0,t)dQ(\tau\leq t) \\
&= -(1-R)\int_{T_a}^{T_b}P(0,t)dQ(\tau\geq t)
\end{aligned}
$$

2.4.6 CDS Mark to Market

The fair value price of a running credit default swap and a postponed credit default swap under the risk neutral measure $\mathbb{Q}$ are defined as:

$$
\begin{aligned}
\Pi(RCDS^{T_a,T_b}(0,S,R)) &= \mathbb{E}^{\mathbb{Q}}[RCDS_0^{T_a,T_b}] \\
\Pi(PCDS^{T_a,T_b}(0,S,R)) &= \mathbb{E}^{\mathbb{Q}}[PCDS_0^{T_a,T_b}]
\end{aligned}
$$

At CDS issuance, the contract spread S_c is calculated to give the value of the contract a fair value:

$$
\begin{aligned}
MtM_{CDS}(t=0) &= \Phi_0^{T_a,T_b}(S) - \Psi_0^{T_a,T_b}(R) = 0 \\
&= S_c \times RBP_0^{T_a,T_b} - \Psi_0^{T_a,T_b}(R)
\end{aligned}
$$

$$
\iff S_c = \frac{\Psi_0^{T_a,T_b}(R)}{RBP_0^{T_a,T_b}}
$$

The contractual spread determines the premium paid by the CDS protection buyer.

At time t, the value of the CDS contract is $MtM_{CDS}(t) = (S - S_c)RBP^{T_{k(t)-1},T_b}(t)$ where $k(t) = \min(i \;/\; T_i > t)$

Stripping intensity from CDS quotes

We will assume that the hazard rate λ is deterministic and piecewise constant:

$$\forall i \in [1, \beta(t)-1] \ \lambda(t) = \lambda_i \text{ for } t \in [T_{i-1}, T_i[$$

$$\begin{aligned}\Lambda(t) &= \int_0^t \lambda(s)ds \\ &= \sum_{i=1}^{\beta(t)-1} [(T_i - T_{i-1})\lambda_i] + \left(t - T_{\beta(t)}\right)\end{aligned}$$

We set $\Lambda_j = \int_0^{T_j} \lambda(s)ds = \sum_{i=1}^{j}(T_i - T_{i-1})\lambda_i$ and so we get:

$$\begin{aligned}\Phi_0^{T_a,T_b}(S) &= S \sum_{i=a+1}^{b} \alpha_i \left(\int_{T_{i-1}}^{T_i} P(0,t) \left[\frac{t - T_{i-1}}{T_i - T_{i-1}}\right] dQ(\tau \leq t) + P(0,T_i)Q(\tau \leq T_i) \right) \\ &= S \sum_{i=a+1}^{b} \alpha_i \left(\int_{T_{i-1}}^{T_i} P(0,u) \left[\frac{u - T_{i-1}}{T_i - T_{i-1}}\right] \lambda(u) e^{-\int_0^u \lambda(s)ds} du + P(0,T_i) e^{-\int_0^u \lambda(s)ds} \right) \\ &= S \sum_{i=a+1}^{b} \alpha_i \left(\int_{T_{i-1}}^{T_i} P(0,u) \left[\frac{u - T_{i-1}}{T_i - T_{i-1}}\right] \lambda_i e^{-\Lambda_{i-1} - \lambda_i(u - T_{i-1})} du + P(0,T_i) e^{-\Lambda(T_i)} \right)\end{aligned}$$

$$\begin{aligned}\Psi_0^{T_a,T_b}(R) &= (1-R) \int_{T_a}^{T_b} P(0,u) dQ(\tau \leq u) \\ &= (1-R) \sum_{i=a+1}^{b} \int_{T_{i-1}}^{T_i} P(0,u)\lambda(u) e^{-\int_0^u \lambda(s)ds} du \\ &= (1-R) \sum_{i=a+1}^{b} \lambda_i \int_{T_{i-1}}^{T_i} e^{-\Lambda_{i-1} - \lambda_i(u - T_{i-1})} P(0,u) du\end{aligned}$$

The price of the CDS at time 0 can be deducted from:

$$\begin{aligned}CDS^{T_a,T_b}(0,S,R,\Lambda) &= \Phi_0^{T_a,T_b}(S) - \Psi_0^{T_a,T_b}(R) \\ &= S \sum_{i=a+1}^{b} \int_{T_{i-1}}^{T_i} P(0,u)\alpha_i \left[\frac{u - T_{i-1}}{T_i - T_{i-1}}\right] \lambda_i e^{-\Lambda_{i-1} - \lambda_i(u - T_{i-1})} du \\ &+ S \sum_{i=a+1}^{b} [\alpha_i P(0,T_i) e^{-\Lambda(T_i)}] \\ &- (1-R) \sum_{i=a+1}^{b} \lambda_i \int_{T_{i-1}}^{T_i} e^{-\Lambda_{i-1} - \lambda_i(u - T_{i-1})} P(0,u) du\end{aligned}$$

3 Asset-Swap

3.1 Recall on the bonds

Bonds are a key-point in the study of the asset-swaps. In this part we will recall some definitions and characteristics of bonds.

3.1.1 Amortization schedule

There exist different amortization schedules depending on the nature of the bond.

General case

Date	Nominal	Interest	Principal	Payment
1	N_0	I_1	P_1	CF_1
2	N_1	I_2	P_2	CF_2
⋮	⋮	⋮	⋮	⋮
t	N_t	I_t	P_t	CF_t
⋮	⋮	⋮	⋮	⋮
T	N_{T-1}	I_T	P_T	CF_T

with $N_0 = \sum_{k=1}^{T} P_k$ is the original balance
$N_t = \sum_{k=t+1}^{T} P_k$ is the outstanding balance at the beginning of the period
$I_t = rN_{t-1}$ is the interest due on the remaining balance
P_t is the scheduled principal reimbursement
$CF_t = P_t + I_t$ is the payment to the lender

No amortization schedule

In this case the principal is redeemed at the maturity of the loan.

Date	Nominal	Interest	Amortization	Payment
1	N_0	rN_0	0	rN_0
⋮	⋮	⋮	⋮	⋮
T	N_0	rN_0	N_0	$(1+r)N_0$

Constant amortization schedule

The principal is redeemed periodically with the same amount: $P_1 = \ldots = P_T = \frac{N_0}{T}$

Date	Nominal	Interest	Principal	Payment
1	N_0	rN_0	$\frac{N_0}{T}$	$rN_0 + \frac{N_0}{T}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$N_0 - \frac{tN_0}{T}$	$r(N_0 - \frac{tN_0}{T})$	$\frac{N_0}{T}$	$rN_0(\frac{T-t}{T}) + \frac{N_0}{T}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
T	$N_0 - \frac{(T-1)N_0}{T}$	$r(N_0 - \frac{(T-1)N_0}{T})$	$\frac{N_0}{T}$	$rN_0\frac{1}{T} + \frac{N_0}{T}$

Constant payment schedule

The payment is constant for each period:

Date	Nominal	Interest	Principal	Payment
1	N_0	rN_0	$rN_0\frac{(1+r)^{-T}}{1-(1+r)^{-T}}$	$rN_0\frac{1}{1-(1+r)^{-T}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$N_0\frac{1-(1+r)^{T-t+1}}{1-(1+r)^{-T}}$	$rN_0\frac{1-(1+r)^{T-t+1}}{1-(1+r)^{-T}}$	$rN_0\frac{1-(1+r)^{T-t+1}}{1-(1+r)^{-T}}$	$rN_0\frac{1}{1-(1+r)^{-T}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
T	$N_0\frac{1-(1+r)^{-1}}{1-(1+r)^{-T}}$	$rN_0\frac{1-(1+r)^{-1}}{1-(1+r)^{-T}}$	$rN_0\frac{1-(1+r)^{-1}}{1-(1+r)^{-T}}$	$rN_0\frac{1}{1-(1+r)^{-T}}$

3.1.2 Bond pricing

General Case

$$\begin{aligned}\Pi_B &= \sum_{t=1}^{T} \frac{cN_0}{(1+r)^t} + \frac{N_0}{(1+r)^T} \\ &= cN_0\frac{1-(1+r)^{-T}}{r} + \frac{N_0}{(1+r)^T}\end{aligned}$$

with Π_B the bond price,
T the maturity date,
N_0 the original balance,
c the coupon rate,
r interest rate

Remarks: If $r = c$: $\Pi_B = N_0$ and the bond is said to be a par-bond
If $r > c$: $\Pi_B < N_0$ and the bond is said to be a discount-bond
If $r < c$: $\Pi_B > N_0$ and the bond is said to be a premium-bond

Zero Coupon Bond

$$\Pi_B = \frac{N_0}{(1+r)^T}$$

3.1.3 Bond Pricing Measures

Yield to maturity

The yield to maturity (YTM) is the internal rate of return when holding a bond r until maturity and when reinvesting all the coupons.

$$\Pi_B = \frac{CF_k}{(1+r)^k}$$

with CF_k is the stream of cash-flows.

Clean Price, Dirty Price and Accrued Interest

The **dirty price** is referred as the effective price. Meanwhile the **clean price** is the quoted price. The relationship is

$$\text{Dirty Price} = \text{Clean Price} + \text{Accrued Interest}$$

The **accrued Interest** is the coupon fraction corresponding to the lag between the last coupon date and the settlement date.

$$\begin{aligned}
\text{Accrued Interest} &= \frac{\text{Settlement Date - Last Coupon Date}}{\text{Next Coupon Date - Last Coupon Date}} \\
\text{Clean Price} &= \sum_{k=1}^{T} \frac{CF_k}{(1+r)^k} \\
\text{Dirty Price} &= \sum_{k=1}^{T} \frac{CF_k}{(1+r)^{x+k-1}}
\end{aligned}$$

Remarks: Usually the settlement date is lagged by x days from the transaction date
There exist different day conventions: $ACT/360$, $ACT/365$

Duration

The duration is an indicator of the bond price sensitivity to a move of the interest rate:

$$\begin{aligned}
& \Pi_B = \sum_{k=1}^{T} \frac{CF_k}{(1+r)^k} \\
\Longleftrightarrow \quad & \frac{\partial \Pi_B}{\partial r} = \sum_{k=1}^{T} -\frac{kCF_k}{(1+r)^{t+1}} = -\frac{1}{1+r} \sum_{k=1}^{T} -\frac{kCF_k}{(1+r)^k} \\
\Longleftrightarrow \quad & \frac{1}{\Pi_B} \frac{\partial \Pi_B}{\partial r} = -\frac{1}{1+r} \sum_{k=1}^{T} -\frac{kCF_k}{(1+r)^k} \frac{1}{\Pi_B}
\end{aligned}$$

The **MacCauley Duration** is referred as the barycenter of the present value of the cash-flows and the **modified duration** is known as the relative price difference to a move of interest rate:

$$
\begin{aligned}
\frac{1}{\Pi_B}\frac{\partial \Pi_B}{\partial r} &= -\frac{\text{MacCauley Duration}}{1+r} \\
\frac{1}{\Pi_B}\frac{\partial \Pi_B}{\partial r} &= -\text{ Modified Duration } (MoD)
\end{aligned}
$$

A good approximation when the yield variation is small, the maturity is not too long and the coupons not too large is :

$$
\frac{\delta \Pi_B}{\Pi_B} = -MoD \times \delta r
$$

However, this approximation underestimate the true value of the duration because of the convexity of the price as a function of the interest rate.

Convexity

To improve the approximation, the quadratic part of the bond price yield relationship have to be considered i.e the convexity of the price to yield function:

$$
\begin{aligned}
\Pi_B(r+\delta r) &= \Pi_B(r) + \frac{\partial \Pi_B}{\partial r}\delta r + \frac{\partial^2 \Pi_B}{\partial r^2}\frac{\delta r^2}{2} + O(\delta r^3) \\
\frac{\delta \Pi_B}{\Pi_B} &= \frac{1}{\Pi_B}\frac{\partial \Pi_B}{\partial r}\delta r + \frac{1}{\Pi_B}\frac{\partial^2 \Pi_B}{\partial r^2}\frac{\delta r^2}{2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Convexity} &= \frac{1}{\Pi_B}\frac{\partial^2 \Pi_B}{\partial r^2} \\
&= -\frac{1}{(1+r)^2}\sum_{k=1}^{T} -\frac{k(k-1)CF_k}{(1+r)^k}\frac{1}{\Pi_B}
\end{aligned}
$$

The convexity describes how a bond's duration changes in response to moves in market interest rates. A better price variation approximation is given by:

$$
\frac{\delta \Pi_B}{\Pi_B} = -MoD \times \delta r + \frac{1}{2}\delta r^2 \text{ Convexity}
$$

3.2 Asset-Swap Principle

An asset swap is a financial product composed by a cash bond with an interest-rate swap. The aim of this product is to transform the interest-rate basis of the bond. Most of the time a fixed rate bond is linked with an interest-rate swap. The investor of the bond received floating rates, in which the coupon is spread over Libor, and pays fixed coupon. The spread, named the asset swap spread, is depending on the credit risk of the bond involved in the asset swap. Usually asset swaps are transacted at par but they can be also at the bond's market price meaning that the asset swap value is composed by the difference between the bond's market price and par, and the difference as the bond coupon and the swap fixed rate.

The curve used to valuate an asset swap is the zero coupon curve. The spread is equal to the difference between between the present value of the bond's cash flows using the swap zero rates and the market price of the bond. It means that it measures a difference between the market price of the bond and the bond's value using zero coupon rates.

3.3 Par Asset-Swap

In this section we will study the par asset-swap. We will start with the description of the par asset-swap and then will price it at a fair value. In the last part we will try to validate the pricing formula on a concrete example.

3.3.1 Description

The par asset-swap might be summarized by the following figure:

Figure 3.1: Asset Swap Structure

The par asset-swap involved two operations for the investor in order to transform the fixed coupons into a floating coupons:

1. The investor buys a risky bond Π_B with a periodic coupon rate c and a face value FV.

2. The investor will enter with a face dealer in a swap contract with the same face value FV, the same maturity T and the same coupon payment dates t_i as the risk bond. This swap contract will create 3 payment legs:

- **Fixed leg:** The investor pays to the swap dealer the fixed coupon c received from the risky bond. In case of default of the bond the investor has to pay the rest of the coupons to the swap dealer.

- **Initial adjustment:** if the bond price Π_B is over the par the investor will receive money from the asset dealer at the beginning of the contract equals to the difference between the price of the bond Π_B and the pair price.

- **Variable leg:** The investor receives *Libor* + *Spread*. The spread of the swap s_a is computed such that the present value of the three legs is equal to 0.

3.3.2 Pricing

The asset-swap seller will sell the bond for par plus accrued interest. If we denote P the full price of the bond in the market, the value of the net up-front payment is $100 - P$. Moreover if we assume that both parties involved in the asset-swap have a credit quality rating equal to AA or similar, these cash flows are priced off the LIBOR curve. We net off the principal payments of par at maturity. To make it easier we will assume that payments are annual and are made on the same dates.

The asset-swap spread s_a is computed by setting the present value of all cash flows equal to 0.

$$\underbrace{(1-P)}_{\substack{\textit{Upfront payment to}\\ \textit{purchase asset in}\\ \textit{return for Par}}} + \underbrace{\underbrace{C\sum_{i=1}^{N_{fixed}} D(0,T_i)}_{\textit{Fixed payments}} - \underbrace{\sum_{i=1}^{N_{float}} \delta_i (L(T_{i-1},T_i)+s_a) D(0,T_i)}_{\textit{Floating payments}}}_{\textit{Interest Rate Swap}} = 0$$

with P the full price of the bond in the market
C the coupon paid annually
$L(T_{i-1},T_i)$ the LIBOR rate at the time $i-1$ and paid at i
Δ_i the accrual factor
$D(0,T_i)$ the discount factor from today to $t = T_i$.

We will suppose $N_{fixed} = N_{float}$.

We have the following relation:

$$\sum_{i=1}^{N} \Delta_i L(T_{i-1},T_i) D(0,T_i) + D(0,T_N) = 1$$

So we get:

$$
\begin{aligned}
& (1-P) + C\sum_{i=1}^{N} D(0,T_i) - \sum_{i=1}^{N} \delta_i (L(T_{i-1},T_i) + s_a) D(0,T_i) = 0 \\
\Longleftrightarrow\quad & 1 - \sum_{i=1}^{N} \delta_i L(T_{i-1},T_i) D(0,T_i) - \sum_{i=1}^{N} \delta_i s_a D(0,T_i) + C\sum_{i=1}^{N} D(0,T_i) - P = 0 \\
\Longleftrightarrow\quad & D(0,T_N) - \sum_{i=1}^{N} \delta_i s_a D(0,T_i) + C\sum_{i=1}^{N} D(0,T_i) - P = 0 \\
\Longleftrightarrow\quad & s_a = \frac{C\sum_{i=1}^{N} D(0,T_i) + D(0,T_N) - P}{\sum_{i=1}^{N} \delta_i D(0,T_i)}
\end{aligned}
$$

If we denote P_{LIBOR} the value of the bond's cash flows discounted at LIBOR, P_{full} the market price of the one and $PV01$ the LIBOR discounted present value of a $1bp$ coupon stream, paid according to the frequency, basis and stub conventions of the floating leg of the interest rate swap we have:

$$
\begin{aligned}
P_{LIBOR} &= C\sum_{i=1}^{N_{fixed}} D(0,T_i) + D(0,T_N) \\
P_{full} &= P \\
PV01 &= \sum_{i=1}^{N} \delta_i D(0,T_i)
\end{aligned}
$$

Finally we obtain the relation:

$$
s_a = \frac{P_{LIBOR} - P_{full}}{PV01}
$$

Validation of the formula:

We will use real data provided by Bloomberg to validate the formula obtained just above. Let us consider the bond IBM 7.5%, 15 June 2013 which priced at \$113.1590 on the 25^{th} of April 2011 and settles the 28^{th} of April 2011. The full price, including accrued interest, is 113.9445. We assume that the floating leg of the swap pays quarterly and is computed using the ACT360 basis. The asset swap spread given by the Bloomberg "Asset Swap Calculator" is 36.6bp.

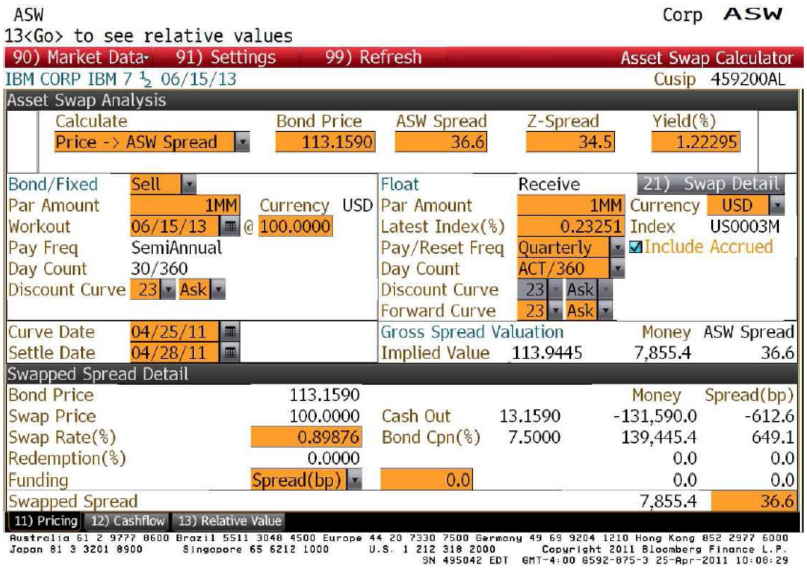


Figure 3.2: Bloomberg Data

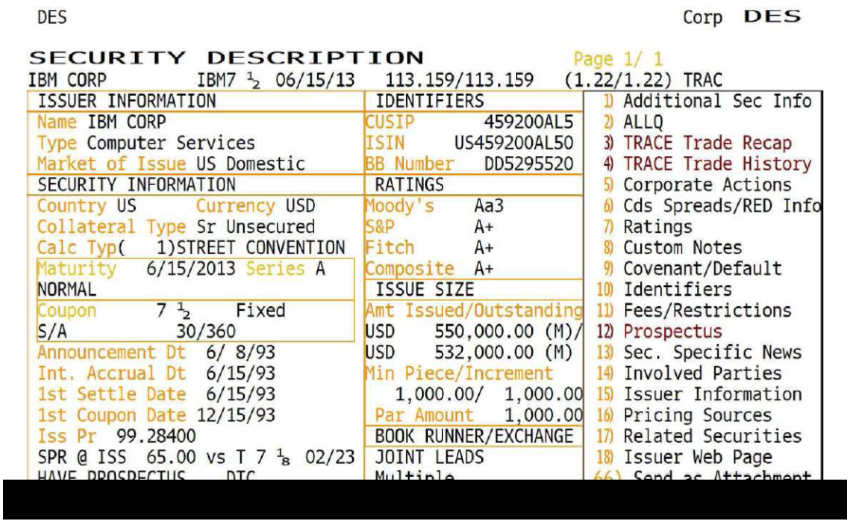


Figure 3.3: Bloomberg Data

Cash flows from this par asset swap spread are shown in the table below. The first two columns of the table show the cash-flow schedule of the bond. LIBOR discount factors are shown in the fourth column. Using these we have:

$$P_{LIBOR} = 114.7207$$

$$P_{full} = 113.9445$$

The floating leg of the swap has 3M frequency and accrues interest on ACT360 basis. These accrual factors for each period are shown in column 5. The PV01 of the floating leg is to:

$$PV01 = 2.1399$$

Using the figures, the asset-swap spread works out to 36.2 bp and validating the formula obtained previously.

Date	**Cash flow on $100 Notional**	**LIBOR Rate**	**LIBOR Discount Factor**	**Accrual Factor (ACT360)**	**Forward LIBOR**	**ASW Fixed Leg**	**ASW Floating Leg**	**Discounted Price**
29-4-11		0%	100%		0%		0%	
29-7-11	3,75	0,581%	99,85%	0,253	0,580%	-3,75	0,239%	3,745
29-10-11		0,808%	99,59%	0,256	1,029%		0,357%	
29-1-12	3,75	0,966%	99,27%	0,256	1,275%	-3,75	0,419%	3,723
29-4-12		1,128%	98,87%	0,253	1,61%		0,4995%	
29-7-12	3,75	1,18%	98,52%	0,253	1,382%	-3,75	0,442%	3,695
29-10-12		1,28%	98,08%	0,256	1,767%		0,545%	
29-1-13	3,75	1,36%	97,63%	0,256	1,826%	-3,75	0,560%	3,661
29-4-13		1,429%	97,16%	0,25	1,908%		0,569%	
16-6-13	103,125	1,48%	96,88%	0,133	2,238%	-103,125	0,347%	99,898

Computations have been performed using an Excel Spreadsheet. The main goal was to obtain the value of the PV01 from the swap spread. The value of the spread has been provided by the Bloomberg Asset-Swap Calculator. The Libor rate Curve has been found on the Internet [1].

[1] http://www.wsjprimerate.us/libor/libor_rates_history.htm

As Of Date	28/04/11
Shift Additif	0,00%
ASW Spread	0,37%
Coupon	7,50

P0	3,98%

t	Date	CF	Zc Rate	Df	Accrual	Fwd Lib	ASW Fixed Leg	ASW Float Leg	Discounted CF
0	28/04/11		0%	100,00%					
0,25	28/07/11	3,75	0,58%	99,85%	0,25	0,58%	-3,75	0,239%	3,7445
0,51	28/10/11		0,81%	99,59%	0,26	1,03%		0,357%	
0,76	28/01/12	3,75	0,97%	99,27%	0,26	1,27%	-3,75	0,419%	3,7226
1,02	28/04/12		1,13%	98,87%	0,25	1,61%		0,499%	
1,27	28/07/12	3,75	1,18%	98,52%	0,25	1,38%	-3,75	0,442%	3,6946
1,53	28/10/12		1,28%	98,08%	0,26	1,77%		0,545%	
1,78	28/01/13	3,75	1,36%	97,62%	0,26	1,83%	-3,75	0,560%	3,6609
2,03	28/04/13		1,43%	97,16%	0,25	1,91%		0,569%	
2,16	15/06/13	103,125	1,48%	96,87%	0,13	2,24%	-103,125	0,347%	99,8981

As Of Date	28/04/11
Shift Additif	0,01%
ASW Spread	0,37%
Coupon	7,50%

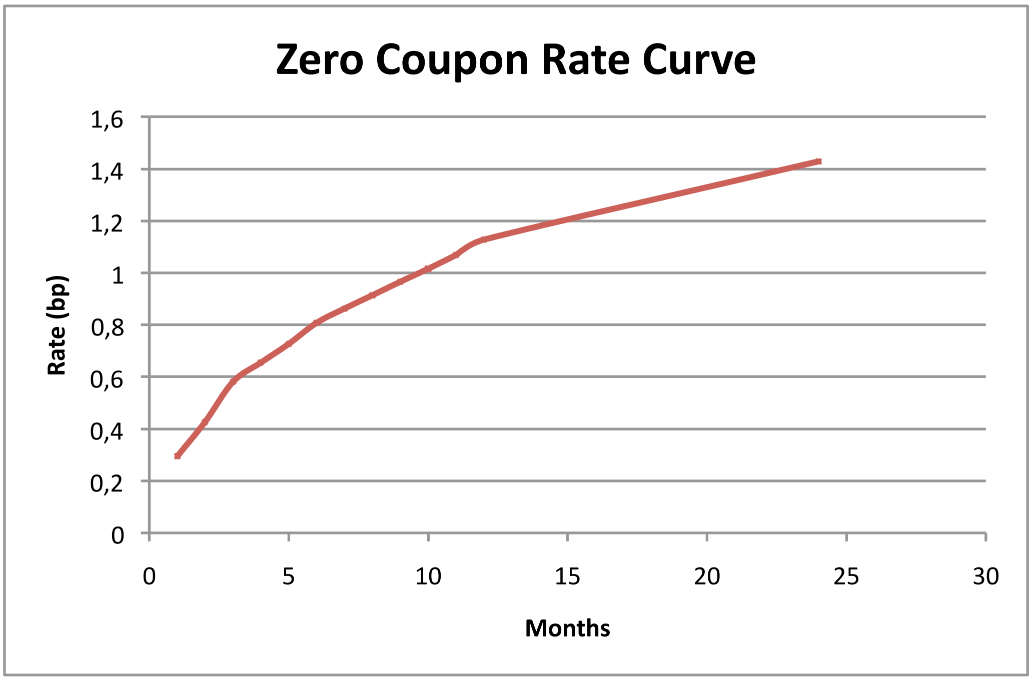
P0	3,98%
P1	4,00%
PV01	2,1399

t	Date	CF	Zc Rate	Df	Accrual	Fwd Lib	ASW Fixed Leg	ASW Floating Leg
0	28/04/11		0%	100,00%				
0,25	28/07/11	3,75	0,58%	99,85%	0,25	0,59%	-3,75	0,242%
0,51	28/10/11		0,81%	99,59%	0,26	1,04%		0,359%
0,76	28/01/12	3,75	0,97%	99,26%	0,26	1,28%	-3,75	0,422%
1,02	28/04/12		1,13%	98,86%	0,25	1,62%		0,502%
1,27	28/07/12	3,75	1,18%	98,51%	0,25	1,39%	-3,75	0,444%
1,53	28/10/12		1,28%	98,06%	0,26	1,78%		0,548%
1,78	28/01/13	3,75	1,36%	97,61%	0,26	1,84%	-3,75	0,563%
2,03	28/04/13		1,43%	97,14%	0,25	1,92%		0,571%
2,16	15/06/13	103,125	1,48%	96,85%	0,13	2,25%	-103,1	0,349%

P_libor	114,7207
P_full	113,945
PV01	2,1399

ASW Spread	36,2713
Discounted Price	114,7207

Zc Rate	
Months	Rate
1	0,295
2	0,427
3	0,581
4	0,655
5	0,727
6	0,808
7	0,863
8	0,914
9	0,966
10	1,016
11	1,069
12	1,128
24	1,429



It is important to note that if the asset in the asset-swap, paid initially 100, defaults immediately after initiation, the investor is left with an asset which can be sold for a recovery price R, and an interest rate swap worth $100-P$, with P the full price of the bond initially.

The loss to the investor is equal to the difference between the full price of the bond and the recovery price of the defaulted bond. It means that the loss is $-100-R-(100-P) = P-R$. If the price of the asset is par, i.e. $P = 100$, then the loss on immediate default is $100-R$. The key point here is that if we hold the credit quality of the asset constant and increase its price, by, for example, considering another bond of the same issuer with the same maturity but with a higher coupon, the loss on default is greater and the asset swap spread should increase.

Pricing example:

In this example we will describe another way to price the asset-swap spread. Let us consider the following Bond flows:

	Year 0	Year 1	Year 2	Year 3
Swap rates		0.50%	1.00%	2.00%
Bond flows	-105	10	10	110

This asset-swap spread is equal to the difference between the fair value bond price and the actual bond price divided by the duration. The fair value bond price assumes bond had no credit risk, i.e., discounted at swap rate.

$$\text{Fair Value:} \quad \frac{10}{(1+0.005)} + \frac{10}{(1+0.010)^2} + \frac{110}{(1+0.020)^3} = 123.41$$

$$\text{Duration:} \quad \frac{1}{(1+0.005)} + \frac{1}{(1+0.010)^2} + \frac{1}{(1+0.020)^3} = 2.92$$

$$\textbf{Asset-Swap Spread:} \quad \frac{123.41 - 105}{2.92} = 6.31\%$$

3.4 Z-Spread

The Z-spread is a purely theoretical concept designed to allow a bond yield to be compared to a swap rate as fairly as possible. It is defined as the size of the shift in the zero coupon swap curve such that the present value of a bonds cash flows is equal to the bonds dirty price.
The Z-spread use the zero-coupon yield curve to calculate spread. It represents the spread that would need to be added to the implied spot yield curve such that the discounted cash-flows of a bond are equal to its present value.

Normally the Z-Spread does not differ greatly from the asset-swap spread. This fact is especially true when the bond is shorter dated and for better credit quality bonds. It is supposed to be slightly higher than the asset-swap spread. If the difference between the Z-spread and the asset-swap spread is too big the bond can be considered as mispriced.

The Z-spread is closely related to the bond price, as shown by:

$$P = \sum_{i=1}^{n} \left[\frac{C_i}{(1 + ((Z + S_i + T_i)/m))^i} \right]$$

with n is the number of interest periods until maturity
P is the bond price
C is the coupon
S_i the swap-spread
T_i is the yield on the Treasury security
Z is the Z-spread
m is the frequency of the coupon payments

It is the standard bond price equation but the discount factor is adjusted with the Z-spread. So when using the correct Z-spread the sum of the bond's discounted cash-flows is equal to the current price of the bond.
Moreover for each i we use the appropriate maturity swap rate and it is actually the main difference between the Z-spread and the I-spread. The I-spread will be study in the next section.

3.5 I-Spread

The Interpolated Spread or I-Spread is the difference between the yield to maturity of the bond and the linearly interpolated yield to the same maturity on an appropriate reference curve.

The simplest way to interpolate the yield off the treasury curve is to find two treasury bonds which straddle the maturity of the defaultable bond. If the maturities of the two government bonds are T_1 and T_2 and the yields to maturity are y_1 and y_2 then the interpolated spread is given by

$$I_{\text{Spread}} = y_D - [y_1 + (\frac{y_2 - y_1}{T_2 - T_1})(T_D - T_1)]$$

with y_D the yield to maturity of the bond
T_D the maturity date of the bond

It is important to specify the reference curve used to quote the I_{Spread} because there are different choices of reference curves: Constant Maturity Treasury rates curve, the LIBOR swap rate curve...

The I-Spread is sometimes used to compare a cash bond with its equivalent CDS price. If the reference curve is the swap rate curve, the interpolated spread can be seen as the difference between the corporate bond spread and the swap spread.

Let us consider the bond Ford Motor Company 6.50%, 1^{st} August 2018 which priced at \$108.4 on the 20^{th} of January 2012 and settles the 15^{th} of February 2012. The yield of the bond is equal to 4.98%. We will use the US treasury curve as reference curve [2]:

Maturity	1M	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
Daily Treasury Yield Curve Rate	0.03	0.05	0.07	0.11	0.26	0.38	0.91	1.47	2.05	2.78	3.10

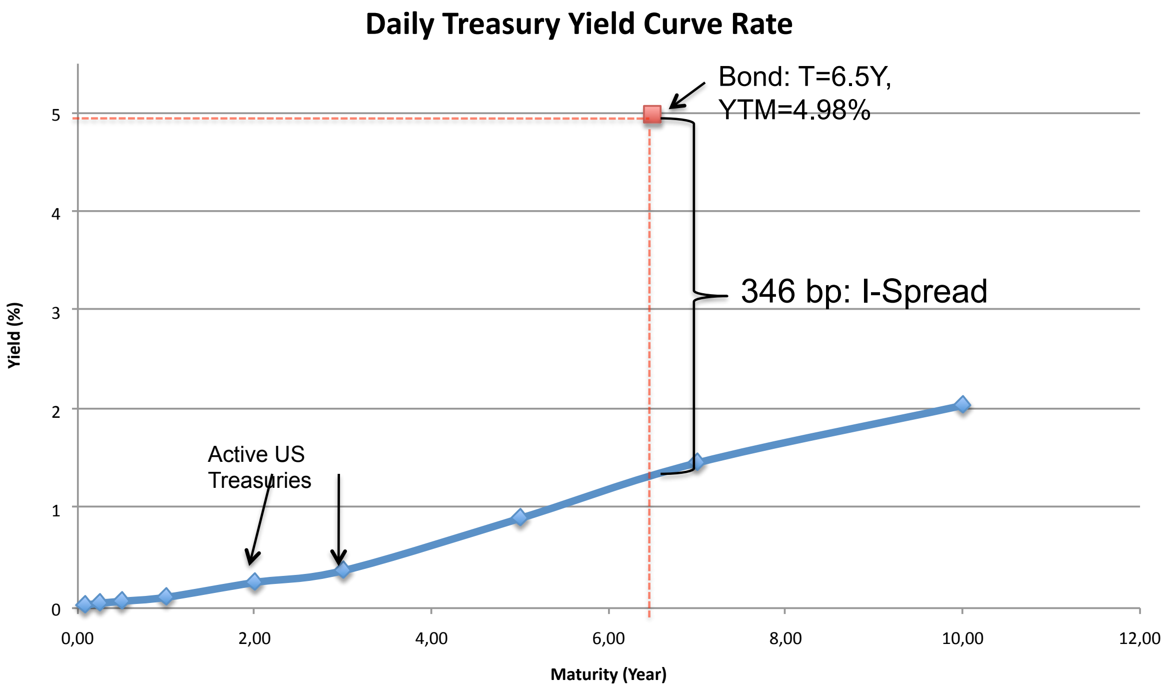


Figure 3.4: Computation of the I-Spread

[2]http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

4 Basis

4.1 Definition

The basis is the difference between the CDS and the asset-swap price. Denoting CDS_t the CDS premium for a given CDS at time t and AWS_t the corresponding maturity matched par asset swap spread the basis at time t is equal to:

$$B_t = CDS_t - ASW_t$$

Most of the time B_t is equal to 0 to avoid any arbitrage opportunity. In that case the relationship between the CDS and the asset-swap is the following:

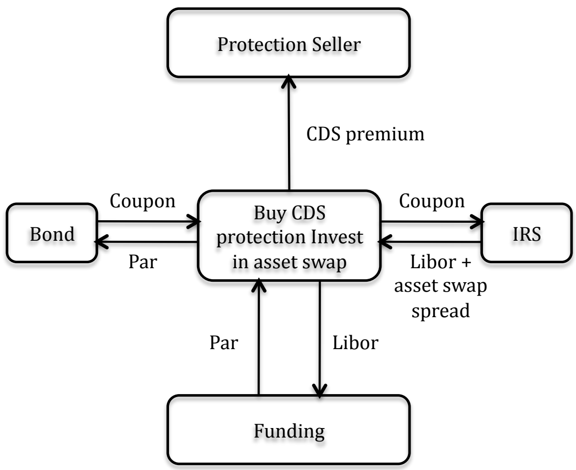


Figure 4.1: with no default

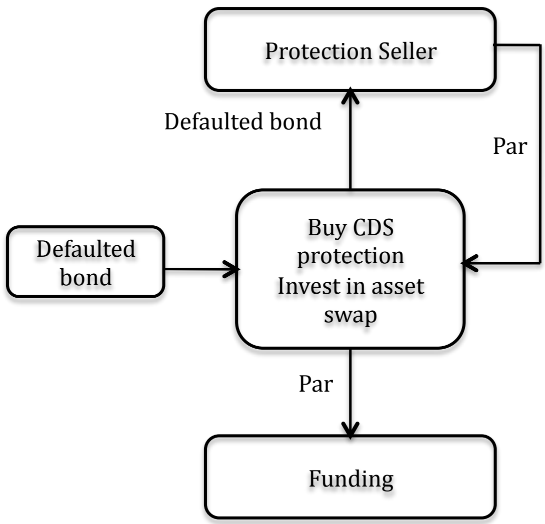


Figure 4.2: with default

B_t diverging from 0 implies an arbitrage opportunity. There exists two difference arbitrage opportunities depending on the sign of the basis:

- $B_t > 0 \Rightarrow$ It means that the CDS spread is higher than the asset-swap spread. A positive basis trade can be set up: sell the CDS protection and sell the bond in an asset swap.
  You will pay the bond coupons and receive the CDS spread. you will receive money (you receive the basis which is positive) and in case of credit event you can paid back the buyer of protection with the cash from the bond.

- $B_t < 0 \Rightarrow$ It means that the asset-swap spread is higher than the CDS spread. A negative basis trade can be set up: buy the CDS protection and buy the bond in an asset swap.
  You will receive the bond coupons and pay the CDS spread: you will receive money (you pay the basis which is negative) and be protected against any credit event on the bond.

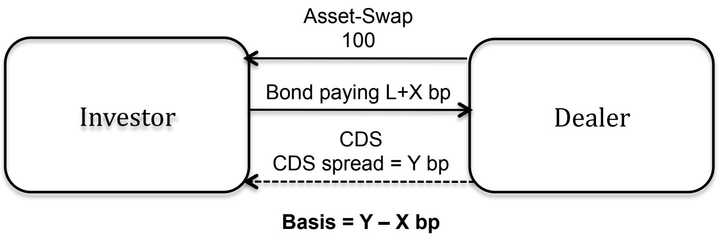


Figure 4.3: Basis trades involving buying a bond and buying protection or vice versa

It is possible to evaluate the basis using the rating of the bond involved in the CDS asset-swap deal. The following graph shows us the basis we can expected relative to the rating of the bond:

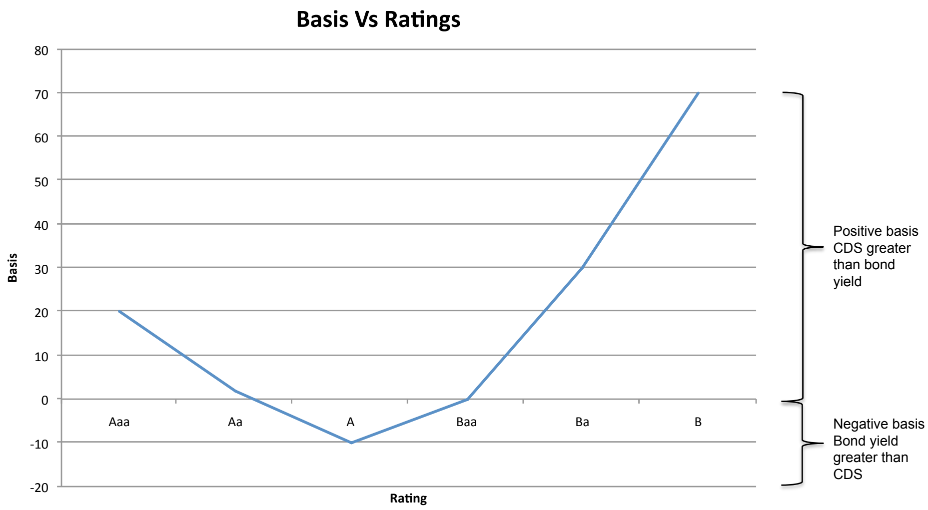


Figure 4.4: Value of the basis in function of the ratings

With the previous definition, obtaining an arbitrage opportunity seems to be quite easy. You check the market and each time you observe a slightly difference between the CDS spread and the asset-swap spread on the same underlying you set up a negative or positive basis spread depending on he value of the basis. However, an investor should be aware that not every apparently attractive basis trade will necessarily be an interesting arbitrage opportunity due to the fact that transaction costs (bid-ask spreads) are significant in credit markets and the basis is determined by a complex set of factors, some of which cannot be controlled in a quantitative manner.
In the following part we will look into these various economic basis drivers.

4.2 Basis Drivers

There exist two different axes to order basis drivers. The first one groups factors according to their expected impact on the basis as they tend to make the basis positive or negative, or even have a mixed effect depending on the situation. Second, factors can be grouped according to whether they are more fundamental or technical in nature:

- **Fundamental factors**: they are also referred to in the market variously as fundamental or contractual factors. These factors are reasons that is due to to the definition and specifications of the CDS contract.
- **Technical factors**: they come from the state of the market where contracts and assets are traded.

Most of the time, if a factor add risk to the CDS relative to the asset-swap the basis will increase. At the opposite a factor adding risk to the asset-swap relative to the CDS will produce a decreasing of the basis. Also, factors that tend to increase the return of an asset-swap relative to a CDS drive the basis upwards, while factors that tend to increase the return of a CDS relative to an asset swap have the effect of depressing the basis.

It is obvious that all the drivers have not the same impact on the basis because they are all not equally powerful and it depends on the specific underlying and the maturity date. Some factors can outweigh others in importance, while some determinants might even be totally irrelevant under certain conditions.

Quantifying the impact and the influence of some individual factors is quite difficult so the study of combined effect of different factors is a very challenging task for a trader or a researcher.

In the next parts of this section we will describe few parameters modifying the value of the basis.

4.2.1 Drivers making the basis positive

Fundamental Factors

- **CDS premiums are above 0:** the quality of a bond issuer is defined mainly by the probability and the risk of default of the bond. Credit rating agencies' function is to rate this companies depending on this default probability. The 3 main agencies are Moody's , Standard & Poor's and Fitch. Certain bonds rated *AAA* or *Aaa* may well trade below Libor in the asset swap market. Nevertheless a bank selling protection on such a bond will except a premium higher than the Libor involving a positive basis.
- **Greater protection level of the CDS contract:** CDS protections are used to be sold with specifics credit events such as technical default (partial default, restructuring of the bond) and not the full default. It involves an additional risk for the seller of the protection. Consequently the premium of CDS will be higher than the bond's premium.

- **CDS cheapest to deliver option:** when a physical delivery is concerned in case of a credit event, the protection buyer is free to choose the cheapest from a basket of deliverable bonds. Since it is likely that protection sellers will end up owning the least favorable option if different deliverable bonds are trading at different spreads, they should receive a higher premium to compensate for this risk. Consequently, the cheapest to deliver option tends to increase the basis.

Technical Factors

- **Demand for protection:** In order to be hedged, banks buys protection in CDS markets. Indeed, shorting the cash market tends to be difficult, as the bond needs to be sourced in a fairly illiquid and short-dated repo market in which bonds additionally might trade on special, making it expensive to borrow the bond. This drives out the CDS premium relative to cash bond spreads, hence widening the basis.
  Moreover, a long bond investor can always fund his positions in the repo market at a rate close to LIBOR. Such a market does not exist for CDS involving a higher spread for CDS than for bonds.

4.2.2 Drivers making the basis negative

Fundamental Factors

- **Counterparty Risk:** The protection seller can default so protection buyers will, as form of compensation tend to pay a lower premium reducing the premium of the CDS and so the basis.

- **Accrued coupon:** In case of default the bond does not paid any accrued coupons. At the opposite, according to the standard CDS documentation, the protection buyer has to pay the accrued premium up to the credit event. It tends to drive the CDS-bonds basis more negative.

- **Bond trading above par:** if a bond trades at a price above 100, the seller of a CDS contract who guarantees the par amount will settle for a correspondingly lower spread.

Technical Factors

- **Demand for protection**: Strong demand from protection sellers drives the basis tighter.

- **Synthetic CDS issuance:** issuance in structured credit markets, and synthetic CDO has been rising over the past years. At the same time, this is a key factor that has been driving CDS spreads tighter and, as a result, depressing the CDS-bond basis.

4.2.3 Drivers making the either positive or negative

Fundamental Factors

- **Coupon specificities**: The coupons payments convention can be different between the CDS and the bond.

Technical Factors

- **Liquidity of the market**: The basis will depend on the relative liquidity in the CDS-market and cash market. The investor who invests in the less liquid market will be compensated.

4.2.4 Summary

	Fundamental factors	Technical Factors
Drivers making the basis positive	- CDS cheapest to deliver option - CDS spread are floored at 0 - CDS restructuring clause (technical default) - Profit realization	- Demand for protection (difficulties in shorting cash bond) - Issuance pattern
Drivers making the basis negative	- Funding issues - Counterparty default risk - Accrued interest differentials on default - Bond trading above par	- Synthetic CDO issuance
Drivers making the basis either negative or positive	- Coupon specificities	- Relative liquidity in segmented market

4.3 Analysis of the Basis

In order to formulate a framework to price the basis we have to understand the characteristics and the behavior of the CDS as an instrument of hedging.

We have two ways to price the credit-risk which are the CDS premium or the cash bond yield. When we compare both, we need to adjust one before comparing to the other,

hence the name "adjusted basis" or "adjusted Z-spread".
To calculate the basis we have two options:

- From the bond yield curve calculate the CDS price which is the price of the CDS on the bound curve and then compare it to the CDS market price
- Using the CDS term structure calculate the cash bond spread which is the price of the bond according to the CDS curve and compare this spread to the bond market spread.

Theoretically the basis should be the same whichever approach is used.

Firstly we need to set the relationship between CDS and bond pricing. We have seen previously that the contract of a CDS is finished when a credit event occurs, with settlement taking place upon payment of the accrued premium.

Denoting by $D(t)$ the duration of an interest-rate swap of maturity t, and by T_i the fixed-rate payment dates she have:

$$D(t) = \sum_i (T_i - T_{i-1}) \times P(T_i)$$

The duration of the CDS contract at the termination date is :

$$D_{\min}(t) = \sum_i (\min(t, T_i) - \min(t, T_{i-1})) \times P(t, T_i)$$

with $P(t)$ the price of a zero-coupon risk-free bond of maturity t
T the maturity date of both CDS and cash bond

The expected value of the duration of the CDS at the termination date gives us:

$$E_\tau[D_{\min}(\tau)] = \int_0^T D_{\min}(t) f_\tau(t) dt + \mathbb{Q}(\tau > T) D(T)$$

with τ the date of the credit event
f_τ the unconditional rate of default
$\mathbb{Q}(\tau > T)$ the probability of $\tau > T$
E_τ the expectations operator at the date of the credit event

Consequently the Net Present Value of the CDS fixed-rate premiums is given by:

$$NPV = S(t) D_{CDS} = S(t) E_\tau[D_{\min}(\tau)] = S(t)[\int_0^T D_{\min}(t) f_\tau(t) dt + \mathbb{Q}(\tau > T) D(T)]$$

with $S(t)$ the credit spread of the CDS with maturity t.

In case of credit event the protection payment has to be paid to the buyer of the CDS. The Net Present Value of the protection payment, which we consider as the CDS floating-rate payment, is given by:

$$E_\tau[(1 - R) P(\tau)_{\tau \leq T}] = (1 - R) \int_0^T P(t) f_\tau(t) dt$$

with R the recovery rate of the underlying involved in the CDS contract.

Pricing the security as an asset-swap, the bond valuation is given by:

$$S(T)D_{ASW}(T) = \sum_i (T_i - T_{i-1}) \times C \times \mathbb{Q}(\tau \leq T_i)P(T_i) + \mathbb{Q}(\tau \leq T)P(T) - \int_0^T R \times P(t) f_\tau(t) dt$$

with D_{ASW} the duration of the interest-rate swap in an asset-swap package
C the coupon of the fixed-rate bond in the asset-swap package

Combining all these equations we get:

$$S_{ASW}(T)D(T) = (C - r) \times (D(T) - D_{CDS}(T)) + S_{CDS}(T)D_{CDS}(T)$$

with r the swap rate corresponding to the maturity date T.

From the equation above we can observe that two significant factors drive the basis, namely:

- The coupon or the range in the asset-swap spread given change in the CDS-spread, the bond coupon and the swap rate
- The convexity which a function of the difference between D_{CDS} and D_{ASW}

4.4 Adjusted Basis Calculation

The basis has been defined as being the difference between the CDS spread and the asset-swap spread. This definition and measure can be regarded as being of sufficient accuracy if the CDS and the bond have the same maturities. Nevertheless coupons are not continuously compounded and most of the time they have different maturities.

Another critical issue is the nature of the construction of the asset-swap structure. To obtain the asset-swap spread the bond has to be priced at or very close to the par. However most of the bonds are traded away from the par, making the par asset-swap price an inaccurate measure of the credit risk. The asset-swap price will overestimate the credit risk if the bond is priced above the par and will underestimate the risk if the bond is priced below the par.

These reasons make this model not practical and provide us an inaccurate measure of the basis.

As we have seen in the previous chapter the Z-spread uses the zero-coupon yield curve to calculate the spread and so seems to be a good alternative to the asset-swap spread. To simplify the Z-spread would be the spread we need to add to the implied spot yield curve in order to have equality between the discounted cash-flows of the bond and its present value. The issue with the use of the Z-spread is the fact that it does not contain the different level of credit risk carried at each cash-flow received over time. It is explained by the use of the zero-coupon rates to discount each bond's coupons in the computation of the Z-spread.

To offset all these problems traders have developed a new approach to compute the basis. This new basis measure named "adjusted basis" used a new CDS spread measure and the Z-spread.

$$\text{adjusted basis} = \text{adjusted CDS spread} - \text{Z-spread}$$

The CDS price is converted to a CDS-equivalent bond spread. It is calculated by applying CDS maker default probabilities to the bond. The price obtained is a function of the recovery rate, the default probabilities implied by the CDS curve and the bond's cash flows.

The method to compute the adjusted CDS spread is the following:

1. From the CDS quotes provided by the market the term structure of credit rates is plotted. To obtain better results a large number of CDS prices has to be used.
2. Using the bootstrapping of the default probabilities the survival curve is constructed from the recovery rate and the cumulative default probability which are obtained from the credit curve.
3. The bond is priced with a binomial tree. This price represents the price of the CDS if it traded as a bond. The bond price obtained will differ from the market price.
4. The adjusted CDS spread is the spread above or below the swap curve that equates the bond's market price to its price calculated above.

The probability of default at each time period is given as p_i
Probability is conditional on survival time at time $i - i$
Discount rate is relevant tenor swap rate for each cash-flow or maturity date swap rate for all cash-flows

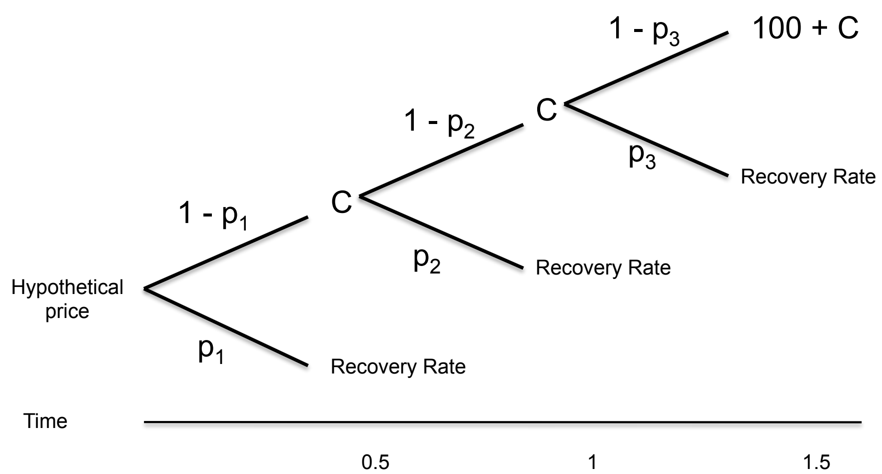


Figure 4.5: Bond hypothetical price using implied default probabilities

Actually with this method we produce an adjusted Z-spread based on the CDS prices and we compare it with the original Z-spread of the bond at its market price. The adjusted CDS spread can be seen as the Z-spread of the bond at its hypothetical price.

Pricing example:

We will consider an hypothetical bond G.E. 6.00%, 9^{th} January 2015 which priced at \$102.47 on the 9^{th} of January 2012. The bond has exactly 3 years to maturity and we discount the cash-flows using only the 3-year rate to simplify the computations. The market price is \$102.47. The recovery rate is equal to 40%. The 3Y swap-rate is equal to 4,90%. The Z-spread of the bond at his market price is 14 bp. We have the following quotes and cash-flows:

CDS Maturity	0.5Y	1Y	1.5Y	2Y	2.5Y	3Y	5Y	10Y
CDS Spread (bp)	5	12	14	17	19	21	30	41

Time	0.5Y	1Y	1.5Y	2Y	2.5Y	3Y	Total
Cash flows	3	3	3	3	3	103	118

To compute the default probability we will use the relation between CDS spread, Default probability and recovery rate:

$$\text{CDS Spread} = \text{Default Probability} \times (1 - \text{Recovery Rate})$$

$$\begin{aligned}
\iff \text{Default Probability} &= \frac{\text{CDS Spread}}{(1 - \text{Recovery Rate})} \\
\iff \text{Default Probability} &= \frac{0.0021}{(1 - 0.40)} \\
\iff \text{Default Probability} &= 0.0035
\end{aligned}$$

To compute the hypothetical price we need the discount factor:

$$\begin{aligned}
\text{Discount Factor} &= \frac{1}{(1 + \text{3Y swap-rate})^3} \\
&= \frac{1}{(1 + 0.049)^3} \\
&= 0.86631041 = \text{DF}
\end{aligned}$$

With all these data we can now obtain the hypothetical price:

$$\begin{aligned}
\text{Hypothetical Price} &= [\text{Bond cash-flow} \times \text{Survival Proba} + \text{Recovery} \times \text{Default Proba}] \times \text{DF} \\
&= [118 \times (1 - 0.0035) + 0.40 \times 0.0035] \times 0.86631041 \\
&= 101.8680556
\end{aligned}$$

To get the adjusted CDS spread we need now to compute the spread above or below the swap-curve that equates the market price.

$$\tilde{r} = \left(\frac{\text{Bond cash-flow} \times \text{Survival Proba} + \text{Recovery} \times \text{Default Proba}}{102.47} \right)^{\frac{1}{3}} - 1 = 4,694\%$$

$$\text{adjusted CDS spread} = 490 - 469.4 = 20.6 \text{ bp}$$

With this value of the adjusted CDS spread we obtain an adjusted basis equals to:

$$\text{adjusted basis} = 20.6 - 14 = 6.6 \text{ bp}$$

We will compare this value with the regular basis:

$$\text{basis} = 21 - 23.12 = -2.12 \text{ bp}$$

So the true basis is positive but the adjusted basis is negative.

4.5 Choice of the basis measure

As we have seen in the previous sections the value of the basis will be different depending on the spread used to perform the measure. The following graphic illustrates these differences. For a pool of corporate names[1] from August 2003 to September 2005 we can observe that the regular basis, the Z-spread basis and the adjusted basis are different.

$$\begin{aligned} \text{Basis} &= \text{CDS Spread} - \text{Asset-Swap Spread} \\ \text{Z-Spread Basis} &= \text{CDS Spread} - \text{Z-Spread} \\ \text{Adjusted Basis} &= \text{Adjusted CDS Spread} - \text{Z-Spread} \end{aligned}$$

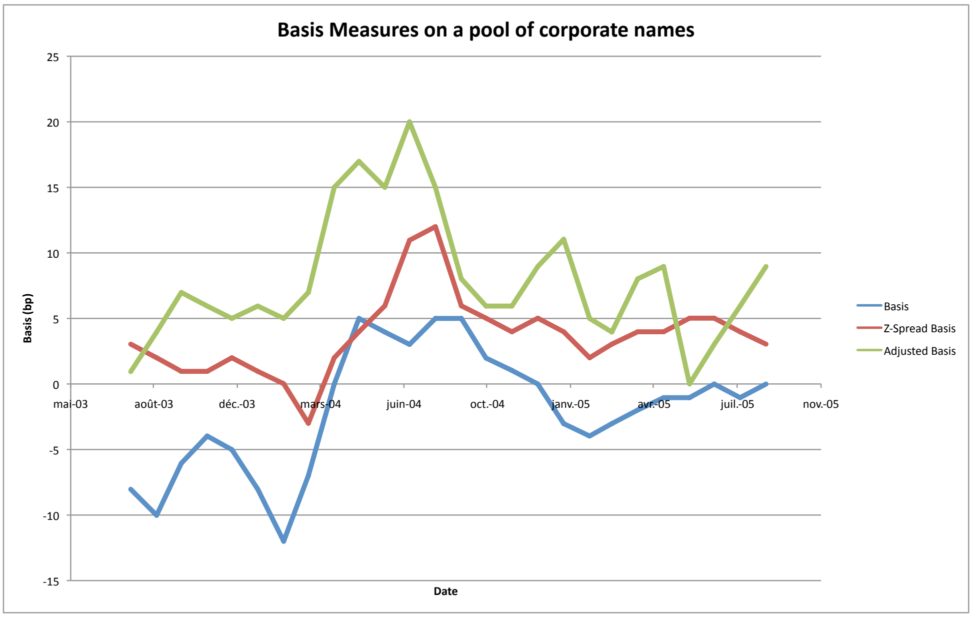


Figure 4.6: Value of the different basis measures

[1] The Credit Default Swap Basis, *Choudhry*, Bloomberg Edition, p.112

Conclusion

While I was a front-office developer I have had the opportunity to work on a negative basis trade pricing spreadsheet. The aim of this mission was to create a new Excel spreadsheet using the new pricer developed by the quantitative research team in order to price on a daily basis the price of products involved in negative basis trades. I discovered Societe Generale had finally lost money on these arbitrage opportunities. Thus, I decided to focus my master thesis on the basis to see if it was as it seems to be at the first glance a good and an easy arbitrage opportunity.

This master thesis report provides a description and explains with details the Credit Default Swap and the Asset-Swap, products involved in the computation of the basis. It is mainly based on what I learned during my eleven months of internship within Societe Generale in London as a commando for an exotic credit derivatives desk. Some parts have been written thanks to what I learned in the books described in the bibliography. The methodology to price these products has been studied and an example of an Excel spreadsheet used to price an Asset-Swap can be obtained in this master thesis.

As it is described in the last section of this report the basis is finally not so easy to compute. Theoretically it would be the difference between the credit default swap spread and the asset swap spread. Nevertheless, for practical reasons traders prefer using the adjusted basis or the Z-spread basis. For the same underlying, the value of the basis can be very different depending on the spread used: Z-spread, adjusted CDS spread or asset-swap spreadÉ In some cases we can get a positive basis, in others a negative one. Consequently a trade seen as an arbitrage opportunity, negative or positive basis trade can be transformed into a disaster, causing a massive loss for the investor.

To conclude we state that the basis can be a profitable arbitrage opportunity if the basis has the same sign whatever the spread used to compute the basis is.

A Appendix A : Bond pricing basics

A.1 Recall on the Bonds

A.1.1 Fixed Rate Bond: Z-Spread

Riskless bond

- $\Pi_B(t)$ risk free price
- T bond maturity
- N_t outstanding balance
- c periodic coupon rate
- r periodic risk free rate

$$\Pi_B(t) = \frac{1}{N_t} \sum_{k=1}^{T} \frac{cN_k + N_k - N_{k+1}}{(1 + r_k)^k}$$

Risky bond

The Z-Spread is an additional spread which shifts all the point of the Zero-Coupon rate curve in order to match the present value of the cashflows to the market price

- $\Pi_B(t)$ risk free price
- T bond maturity
- N_t outstanding balance
- c periodic coupon rate
- r periodic risk free rate
- Z_{spread} (Zero volatility spread) additional spread over the yield curve

$$\Pi_B(t) = \frac{1}{N_t} \sum_{k=1}^{T} \frac{cN_k + N_k - N_{k+1}}{(1 + r_k + Z_{spread})^k}$$

A.1.2 Floating Rate Note: Discount Margin

Risky bond

A floating rate note is a bond that makes periodic coupon payments linked to a variable interest rate index. Typically the bond pays an additional spread that is intended to bring the price of the bond to par on the issue date of the bond. This additional spread is called the par floater spread which is similar to the discount margin.

$$\Pi_{FRN} = \sum_{i=1}^{N}(L(i-1,i)+S)df(0,i) + df(0,N)$$

where $df(0,i)$ is the discount factor from, today to the next coupon payment date, $df(0,i)$ is the discount factor from today to the i^{th} coupon date and $L(i-1,i)$ is the forward LIBOR rate.
Supposing at t the T-maturity par floater spread of the issuer is F, the discount factors are given by:

$$df(0,i) = \frac{df(0,i-1)}{1+L(i-1,i)+F} \text{ where } df(0,0) = 1$$

Substituting $F = S$ in the 2 previous equations we find $\Pi_{FRN} = 1$ meaning that if the par floater spread F is equal to the fixed spread S on a coupon date the floating rate note prices at par.

The Discount Margin is an additional spread which shifts all the point of the flat rate curve in order to match the present value of the cashflows to the market price:

- $\Pi_B(t)$ risk free price
- T bond maturity
- N_t outstanding balance
- X is the quoted margin
- $L(T_i)$ is the LIBOR rate resetting at time T_i
- DM (Discount Margin) additional spread over the flat curve

$$\Pi_B(t) = \frac{1}{N_t}\sum_{k=1}^{T}\frac{(L(T_i)+X)\delta_k N_k + N_k - N_{k+1}}{(1+\delta_k(L(T_i)+DM))^k}$$

A.1.3 Default intensity

Default Free

Assuming no default occurs the bond price at maturity is given by

$$\Pi_B(t,T) = \mathbb{E}^{\mathbb{Q}}[D(t,T)]$$

where $D(t,T)$ is the stochastic discount factor defined as $D(t,T) = e^{\int_t^T r(s)ds}$

With Default

When considering the default of the issuer entity, the bond value is given by

$$\mathbf{1}_{\tau>t}\Pi_B(t,T) = \mathbb{E}^{\mathbb{Q}}[\frac{\mathbf{1}_{\tau>t}D(t,T)}{F_t}]$$

where

- $\mathbf{1}_{condition}$ is the indicator function where $\begin{cases} 1, & \text{if no default} \\ 0, & \text{otherwise} \end{cases}$
- F_t is the information set
- τ is the default time

By including the recovery (R) of the bond, the coupons (c_i) the discount payoff is now defined as

$$\Pi_B(t) = \frac{1}{N_t}\sum_{i=a+1}^{b}[(c_iN_i + N_i - N_{i+1})\delta_i D(t,T_i)\mathbf{1}_{\tau>T_i}] + R\sum_{i=a+1}^{b} D(t,T_i)N_i\mathbf{1}_{\tau\in[T_{i-1},T_i]}$$

Fixed rate coupon

$$\Pi_B(t) = \frac{1}{N_t}\sum_{i=a+1}^{b}[(cN_i + N_i - N_{i+1})\delta_i D(t,T_i)\mathbf{1}_{\tau>T_i}] + R\sum_{i=a+1}^{b} D(t,T_i)N_i\mathbf{1}_{\tau\in[T_{i-1},T_i]}$$

where c the annual fixed rate coupon.

Floating rate coupon

$$\Pi_B(t) = \frac{1}{N_t}\sum_{i=a+1}^{b}[((L(T_{i-1},T_i)+X)N_i+N_i-N_{i+1})\delta_i D(t,T_i)\mathbf{1}_{\tau>T_i}]+R\sum_{i=a+1}^{b} D(t,T_i)N_i\mathbf{1}_{\tau\in[T_{i-1},T_i]}$$

where $L(T_{i-1},T_i)$ the LIBOR rate resetting at T_{i-1} and paying at T_i plus a contractual spread X.

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