On Lapse risk factors in Solvency II

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Abstract

In the wake of the sub-prime crisis of 2008, the European Insurance and Occupational Pensions Authority issued the Solvency II directive, aiming at replacing the obsolete Solvency I framework by 2016. Among the quantitative requirements of Solvency II, a measure for an insurance firms solvency risk, the solvency risk capital, is found. It aims at establishing the amount of equity the company needs to hold to be able to meet its insurance obligations with a probability of 0.995 over the coming year. The SCR of a company is essentially built up by the SCR induced by a set of quantifiable risks. Among these, risks originating from the take up rate of contractual options, lapse risks, are included.

In this thesis, the contractual options of a life insurer have been identified and risk factors aiming at capturing the risks arising are suggested. It has been concluded that a risk factor estimating the size of mass transfer events captures the risk arising through the resulting rescaling of the balance sheet. Further, a risk factor modeling the deviation of the Company's assumption for the yearly transfer rate is introduced to capture the risks induced by the characteristics of traditional life insurance and unit-linked insurance contracts upon transfer. The risk factors are modeled in a manner to introduce co-dependence with equity returns as well as interest rates of various durations and the model parameters are estimated using statistical methods for Norwegian transfer-frequency data obtained from Finans Norge.

The univariate and multivariate properties of the models are investigated in a scenario setting and it is concluded the the suggested models provide predominantly plausible results for the mass-lapse risk factors. However, the performance of the models for the risk factors aiming at capturing deviations in the transfer assumptions are questionable, why two means of increasing its validity have been proposed.

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Contents

1 Introduction

This chapter aims at providing a background to the project, as well as defining its purpose, where the scope of the project ends and providing the reader with a disposition of the report.

No person can accurately plan several decades into the future. Because of this; a life insurer in a competitive market must offer the opportunity to change some terms of a life insurance policy in order to attract customers. But in the same time, financial authorities put hard pressure on life insurers to make sure they are able to honor the obligation they undertake through their policies, especially as life insurers often hold the populations' pension capital. At every point in time, the insurer has to hold assets covering the present value of every payment scheme it has undertaken. Because of the time value of money, the companies have to engage in complex calculations to determine this present value. This complexity makes the present value sensitive to the changes in the policy, which is vital for the insurer to offer its customers.

In order minimize the risk of a European life insurer becoming insolvent, namely unable to meet its obligations, the European Union introduced the Solvency I regulations in the 1970s. By 2009 the EU deemed the framework to be obsolete enough to issue the Solvency II directive, a set of more rigorous regulations as regard to e.g. quantitative requirements, governance and risk management systems and transparency. Among the quantitative requirements the solvency requirement capital (SCR) is found, stating a requirement of how much equity an insurer must hold. For the calculation of this measure, the EU has supplied the insurance industry with a standard formula, but has also left the doors open for each company to use its own internal model. However, before an internal model may be used in an operative setting, it has to go through a rigorous approval process by the financial authorities of the country the company is active in.

As the equity of the company is defined as its assets minus its liabilities, calculations in the internal model requires the company to calculate the value of its assets and obligations. Because the present value of its obligations is sensitive to changes in the contracts but the company still offers options to do so, it must take the expected cost of this into consideration through an assumption of the fraction of contracts being changed per year. The true fraction is naturally not deterministic, it may deviate from the assumed one, which thus exposes the insurer to lapse risk.

In this degree project, a Company in need of a model that captures the size of the lapse risk and its co-variation with other relevant risks has been assisted.

The purpose of this degree project has thus been to first identify possibly material contractual options in the company and provide them as input to a materiality analysis, performed by the Company. Further purpose has been to propose a model capturing the multivariate and marginal properties of lapse risk factors for a life insurer.

The scope of this degree project will be bound to the setting of the Company. By this, it is meant that e.g. assumptions made are not necessarily general, but rather specific.

The questions of issue have been reduced to;

1. Which non-neglectable contractual options do the policyholders of the Company own and in which way is the SCR affected by the exercise of these options? and

2. How could the selected risk factors be modeled to resemble the truth in their multivariate and marginal properties?

Section 2 aims at giving the reader a more thorough introduction to the insurance industry and its terminology. Through its subsections, the section branches of the very general topic of insurance to only include investment schemes within traditional life insurance and unit-linked insurance. Section 3 describes the regulatory requirements under the Solvency II framework and is used to define several key concepts used in the analysis, such as the components of the Solvency II balance sheet. Section 4 is used to describe scenario-based risk management approach in which the lapse-risk modules are found and to review some statistical tools essential for the methodology. Section 5 contains the assessment of the contractual options the Company has sold and how their exercise would affect a Solvency II balance sheet. These results are then run through an internal analysis by the company, returning the final suggested model scope. Section 6 defines the data that will be used in the statistical modeling. Section 7 proposes models for the defined risk factors and then defines the method of how they are estimated. Section 8 contains the results of the risk factor modeling, when it is tested in the framework of the Company. Section 9 makes an assessment of the performance and validity of the statistical properties of the risk factors. Finally, Section 10 concludes the thesis.

2 Insurance - Industry

2.1 Principle & History

The basic principles for insurance rely on the fact that funds from a big amount of entities are pooled and used to cover losses incurred when a predefined event occurs. In this way, each entity pays a fee, the premium, in order to be protected from the risk of suffering loss caused by the event. Since the collective commits to paying possible losses for every entity, the size of the premium will depend on the probability of the event occurring and the expected costs incurred by it and may vary across the participating entities. If the number of participating entities is big enough, the law of large numbers supports that the sum of all expenses will be covered by the premiums, if they have been truly calculated. An insurance company or collective acts as the middleman by collecting the premiums and paying out the benefit to cover for losses.

The concept of distributing the risk of loss over several entities may be tracked back to Chinese and Babylonian traders operating during the 2th and 3rd millennia before Christ. The context of these early insurance schemes was often the marine industry. Merchants would for instance distribute their cargo over several vessels or pay an extra fee when taking a loan for a shipment, freeing them of the debt in the case the shipment was lost. During the 17th century, the concept spread further. E.g. In 1666, the Great Fire of London introduced a great need for insuring property against fire and in the beginning of the 18th century, the first life insurance policies were agreed upon, aiming at supporting the widows and children of deceased men.

2.2 Life & Non-life insurance

The companies on the insurance market are primarily divided into life and non-life insurance companies. Life insurance companies offer contracts where the event that triggers a loss is linked to the life, health or work capacity of a person, while non-life insurers offer liability insurance and contracts that protect against financial loss incurred by damage to or loss of property. In this paper, we will limit ourselves to the life insurance business [8].

2.3 Investment & Protection policies

Life insurance contracts may be divided into two subcategories, namely

- Protection policies, where an agreed benefit is paid upon the occurrence of a specific event. For example: A contract that, upon death of the insured, pays a lump sum or a limited annuity to surviving relatives, the size of which could be either fix or in some way related to the income of the deceased. A policy providing this kind of protection would most likely be paid on a yearly basis with a premium varying with the age and other death-inducing or reducing factors of the insured. Thus, no investment of capital is made in the policy.
- Investment policies, where the objective is to further growth of capital through continuous or single premium payments and interest. For example: A pension plan, where either an individual or his or her employer pays a recurring or single premium to a life insurance policy. The capital invested in the policy then grows through premium payments and compound interest, until the person retires. The contract then enters its pay-out phase and capital is paid out in a predefined frequency over a predefined period.

A specific life insurance policy may consist of only one or a combination of the above. In this project, only investment policies have been be considered.

2.4 Investment policies

On the Swedish life insurance market, there are two main types of investment policies, traditional life insurance policies (traditionell livförsäkring) and unit-linked insurance policies (Fondförsäkring). In Sweden, insurance companies offering investment policies may be of two legal forms [8]; these are closely related to;

- Private/Public limited (Vinstutdelande bolag), from here referred to as type I.
- Non-profit organization (Ömsesidigt bolag) from here referred to as type II.

2.4.1 Traditional life insurance

In traditional life insurance, the policyholder has a claim on the insurance company. This claim equals the sum of all premiums paid compounded with a contract specific yearly guaranteed interest rate, r_g. The insurance company invests the capital and any realized growth that exceeds r_g is distributed between the company's equity and the policyholders' claims, depending on the legal structure of the firm. Since the guaranteed capital is a fixed size debt, the company bears the investment risk in traditional life insurance, making risk management vital.

In type I companies, each policyholder has his or her own claim and a certain percentage of the excess growth is added to the policyholder's claim, while the rest is distributed to the company's shareholders. In type II companies, the company each year decides how much additional interest will be distributed preliminary to the policyholders' insurance capital, forming the total insurance capital. The preliminary distributed part however is not a debt, but rather the policyholders allocated share of the company's equity, making the company's customers its owners. The scope of this project will be limited to companies of type II.

Below, the model traditional life insurance contract used in this thesis is defined and described:

Let the guaranteed yearly interest rate be $r_g = 0.03$, the age of the policyholder when signing the contract to be $y_0 = 35$ years old, the monthly premium be a monthly recurring single premium and equal $\Pi_m = 1000$ SEK, the agreed upon age of the policyholder for retirement be $y_r = 65$ and the agreed upon length of the payout scheme to be $y_p = 10$ years. Further assume that the total realized yearly allocated growth is $r_b = 0.05$. Introduce the contract C_t which is defined by

$$C_t = \{r_g, y_0, \Pi_m, y_r, y_p, r_b\}.$$

Note that r_g is also guaranteed in the payout term. The guaranteed and total insurance capital of C_t over the time, growing with compound interest rate and premium payments, may be seen in Figure 1.

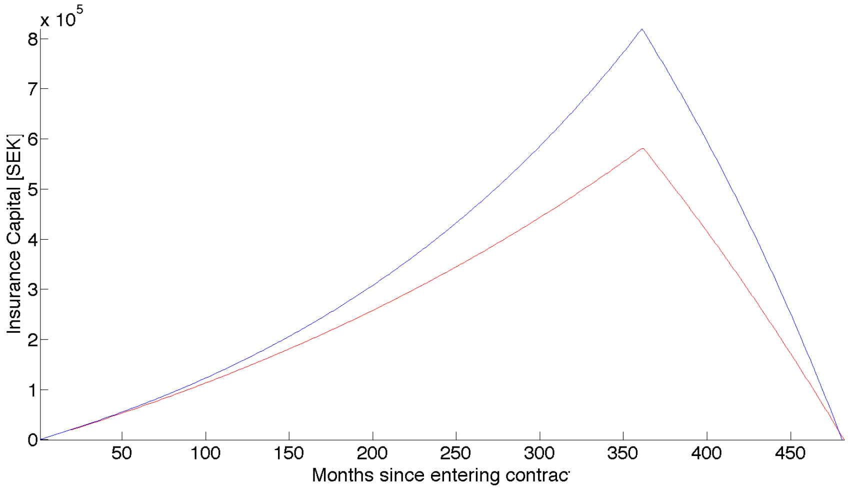

Figure 1: Insurance capital over time for the model traditional life contract, C_t. The blue line is the total insurance capital while the red one is the guaranteed insurance capital.

2.4.2 Unit-linked insurance

In unit-linked contracts, the accumulated capital of each specific policyholder is invested in a mixture of equity and fixed income mutual funds. The company chooses the funds allowed to invest in but the policyholder does the allocation of the capital. In this way, the policyholders' claim follows the value of the decided investment and he or she bears the investment risk. The fees for the company owning the investments for the sake of the policyholder are often built up of a fixed part and a variable part, proportional to the size of the policyholders insurance capital. The fees are used to cover management costs and the profit is transferred to the company's equity and thus its shareholders. For natural reasons, companies offering unit-linked insurance in Sweden may only be of Private/Public limited company form (Vinstutdelande bolag).

The following will be the model contract used for unit linked insurance schemes in this thesis, denoted as C_u. C_u differs from C_t in that no yearly rate is guaranteed, the policyholder does the investment choice. Let r_u^i be the growth rate of the scheme i years after entering the contract and replace r_g and r_b with $\{r_u^i\}_{i=1}^{y_0+y_r+y_p}$ in C_t to get C_u. Note that r_u^j is only known for $i \geq j$. In Figure 2, the insurance capital over time may be seen. The growth of capital in the pay-out phase is approximated as the risk free rate and the r_u^i are simulated from a normal distribution with yearly mean 0.06

and standard deviation 0.05, in order to simplify calculations. Further, in the pay-out phase, a yearly growth of remaining insurance capital with the risk free interest rate is assumed.

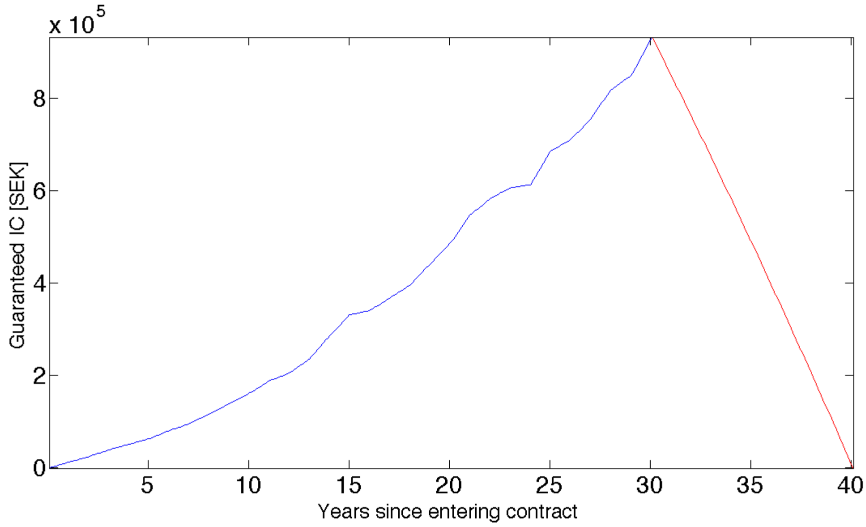

Figure 2: Insurance capital over time for the model unit-linked contract, C_u. The blue part of the line indicates the pay-in phase and the red one the pay-out phase.

2.4.3 Taxation

There are two main legal forms of investment life insurance policies, namely K- and P-taxed contracts.

P-taxed policies, pension insurance policies (pensionsförsäkring), are taxed and regulated in a way to create incentives for the policyholder to save capital for their pension. The premiums paid are up to a certain extent tax deductible and the insurance capital is taxed yearly at a flat rate, rather than taxation on yearly growth. In exchange for this, income tax is paid when the benefits are paid out. Additionally, the regulations stipulate that

- The policy may not enter its pay-out term before the policyholder is 55 years of age,
- the pay-outs must be distributed over at least 5 years and
- the beneficiaries of the contract can only be the policyholder or his or her partner or children.

The contract thus allows the policyholder to transfer income from a high income regime, when in working age, to a low income regime, when a pensioner. Under increasing marginal taxation legalization, this transfer lightens the policyholder's total tax burden, creating incentives for the policyholder to save for his or her old age.

P-insurance policies may, in Sweden, roughly be further divided into two subgroups, namely occupational and private schemes. Occupational P-insurances are schemes where the premium for the policyholder is paid by his or her employer, as a benefit. In Sweden, pension plan benefits are very common and about 85% of the companies in the private sector and 100% of the public sector employ plans specified by branch specific collective agreements (Kollektivavtal). The collective agreements are negotiated between the relevant Worker's and Employer's associations. For companies not affiliated to any of these Employer's associations, occupational pension plans for the employees are negotiated directly with the insurance company. These policies are however seldom as advantageous as the collectively agreed plans. Private P-insurances are regular plans where the policyholder pays the premiums.

K-taxed policies, also known as Endowment policies, are contracts where the length and beginning of the policy's pay-out term is specified when the contract is signed. Apart from companies sometimes requiring a minimum pay-in period of the policy, the pay-out term may commence at any time and be paid out directly or over a given term. Also, no constraints are set as to who may receive the benefits of the policy. As opposed to P-policies, the premiums are not deductible and the pay-outs are tax-free. However, also here the insurance capital is taxed yearly at a flat rate, rather than the yearly growth being taxed. The structure of the K-policies makes them more of a pure-investment form, rather than investment targeted at financing ones pension.

3 Insurance - Regulation

3.1 Balance sheet of insurance firms

As with any company, the assets of both type I and type II insurance companies equals its liabilities plus its equity.

3.1.1 Assets

The assets of the company is the value of everything tangible and intangible the company owns that could be converted to cash. This may include investments in equity, property, bonds as well as patents and goodwill. In the context of a unit-linked insurance company, it makes sense to identify the entity *Present Value of Future Profits* (PVFP) within the assets. The PVFP aims at valuing the Company's current business in terms of the future profits generated by the fees of its policies. As mentioned above, these profits relate, through the varying part of the fees, to the values of the policyholders' portfolios. E.g. a decrease in insurance capital gives a decrease in the fee the policyholders have to pay the company. Since the policyholders choose the investment allocation of their insurance capital themselves, the PVPF is built up by assets exceeding these capital investments.

3.1.2 Liabilities

The liabilities together with the equity describe how the assets of the company are financed. The liabilities is the part financed by debt, which in an insurance context is mainly made up the policyholders' claims on the company.

3.1.3 Equity

The equity is what remains when the value of the liabilities is subtracted from the value of the assets. In a type I insurance firm, the equity is owned by the shareholders of the company and consist partly of the PVFP, as described above. In a type II life firm, a division of the equity into allocated capital and unallocated capital may be made. The allocated capital is that part of the excess capital generated by excess over guaranteed growth which has been preliminarily allocated to the policyholders, through the so called bonus rate (återbäringsränta). The unallocated capital is capital not yet distributed. Since the allocated capital has only been allocated, and not awarded, it bears the status of equity, rather than debt, as is the case in a type I company.

3.2 Solvency II

In the wake of the sub-prime crisis of 2008, the European union, through its supervisory authority the European Insurance and Occupational Pensions Authority (EIOPA, earlier CEIOPS) has issued the Solvency II directive, aiming at replacing the obsolete previous EU insurance regulations, Solvency I. A driving factor has been to adapt the regulations to the fact that more advanced risk management systems have been developed since the initial regulations were introduced. The decomposition of the balance sheet under Solvency II is described below and visualized in Figure 5.

3.2.1 Market Value of Assets (MVA)

In [9] the following may be read about how the valuation of the assets of an insurance firm are is to be made;

Member States shall ensure that, unless otherwise stated, insurance and reinsurance undertakings value assets and liabilities as follows:
(a) assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction

This means that under Solvency II, the assets of the company are market valued.

3.2.2 Market Value of Liabilities (MVL)

In the Solvency II context, the MVL (or technical provisions) consist of all the insurance obligations of the insurance firm. According to [10], the technical provisions are also to be valued with their market value. The following is stated in the document;

The value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking.

It is further stated that;

The value of technical provisions shall be equal to the sum of a best estimate and a risk margin [...],

From which the following decomposition of the technical provisions is made;

- the best estimate and
- the risk margin.

The best estimate is defined as;

The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

in [11].

Thus, the best estimate of liabilities corresponds the sum of the expected value of future cash flows, e.g. pre-specified cash flows undertaken through life insurance policies.

For the model contracts C_t and C_u, the best estimate at each time T would be calculated approximately as follows.

- Traditional life insurance contract, C_t: At the year of retirement, in $y = y_r - y_T$ years, the guaranteed insurance capital will be $C_y = C_T(1 + r_g)^{y_r - y_T}$, where C_T is the insurance capital at time T. From C_y, the yearly benefit C_b is calculated. Now, the best estimate of the liability for this contract may be calculated as

$$BE_t = \sum_{i=1}^{y_p} \left(\frac{1}{1 + r_f^{(i+y)}} \right)^{i+y} C_b$$

  where $1/(1 + r_f^{(i+y)})$ is the price of a risk free zero coupon bond with face value 1 maturing in $i + y$ years, namely the value of 1 unit of cash $i + y$ years from now. In Figure 3, the best estimate of the obligation for the contract, over time, is visualized as the blue line, when the risk free interest rate is assumed to be $r_f = 0.02$ for all durations.

- Unit-linked insurance contract, C_u: At every point in time in the pay-in phase, the policyholder's insurance capital at y_p equals the C_T, as no further growth is guaranteed. From C_T, the monthly benefit C_b is calculated and the best estimate of our model unit-linked contract may be calculated in the same way as for C_u. Figure 3 shows the best estimate of the liabilities associated with contract C_u, under the assumption that $r_f = 0.01$ for all durations.

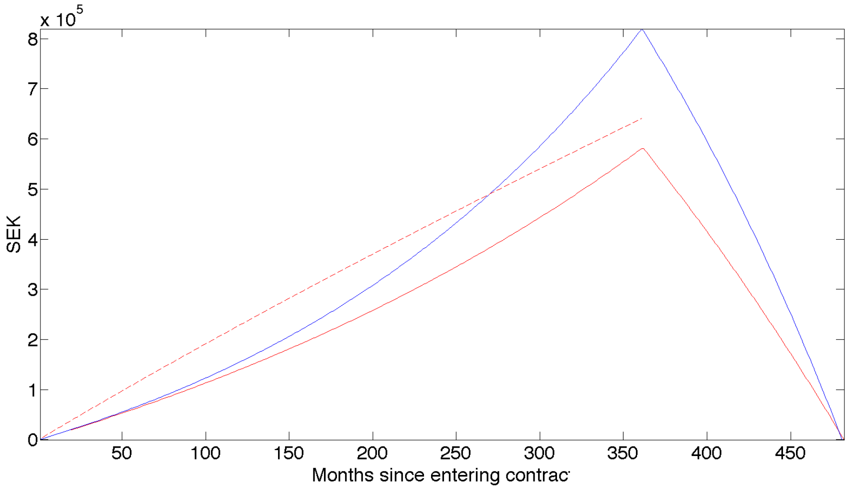

Figure 3: Best estimate of the liability of the model traditional life insurance contract, C_t (Red dashed), together with the previously presented guaranteed insurance capital (red solid line) and the total insurance capital (blue solid line).

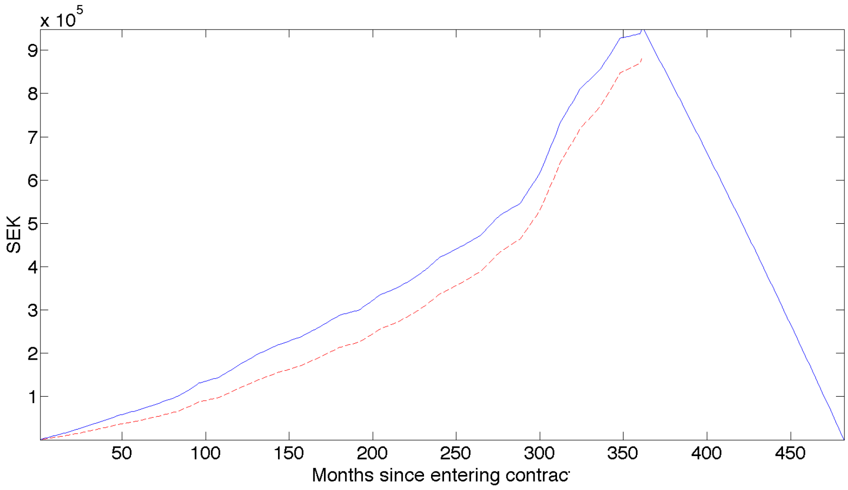

Figure 4: Best estimate of the liability of the model unit linked insurance contract, C_u (red dot-dashed line), together with the already presented insurance capital (blue line).

The risk margin is in [10] defined as follows;

The risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations.

This roughly means that the risk margin corresponds to the capital in excess of the best estimate of the liabilities the company would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking.

The following citation from [11];
When calculating technical provisions, insurance and reinsurance undertakings shall take account of the value of financial guarantees and any ***contractual options*** *included in insurance and reinsurance policies*

is of particular interest to this paper. It indicates that the company needs to include some assumptions about the **contractual options** in its contracts, when calculating the MVL. The definition of an option of this sort will be given at a later stage. A summary of the balance sheets in their accounting and Solvency II forms may be seen in Figure 5.

3.2.3 Solvency Capital Requirement (SCR)

Consider the random variable X, which represents the net asset value a year into the future. Now, define the random variable Y as

$$Y = P_0(X_{BE} - X)$$

where $P_0(x)$ indicates the present value of x and X_{BE} denotes the best estimate of X, and is thus deterministic as seen from the report date. Further, denote $F_Y(y) = \mathbb{P}(Y \leq y)$ as the cumulative distribution function of Y. We recognize Y as the present value of the difference between the best estimate of the value of the own funds and the value of the own funds a year into the future. In the Solvency II framework, the Solvency Capital Requirement (SCR) is defined as that observation of Y, which will only be surpassed with a probability of 0.005, namely once in two hundred years. We denote this as

$$F_Y(SCR) = P(P_0(X_{BE} - X) \leq SCR) = 1 - 0.005 = 0.995.$$

Since it is only the value of the own funds one year into the future that is of interest, $P_0(x)$ simply becomes $1/(1 + r_f)x$, i.e. a linear function of x, where

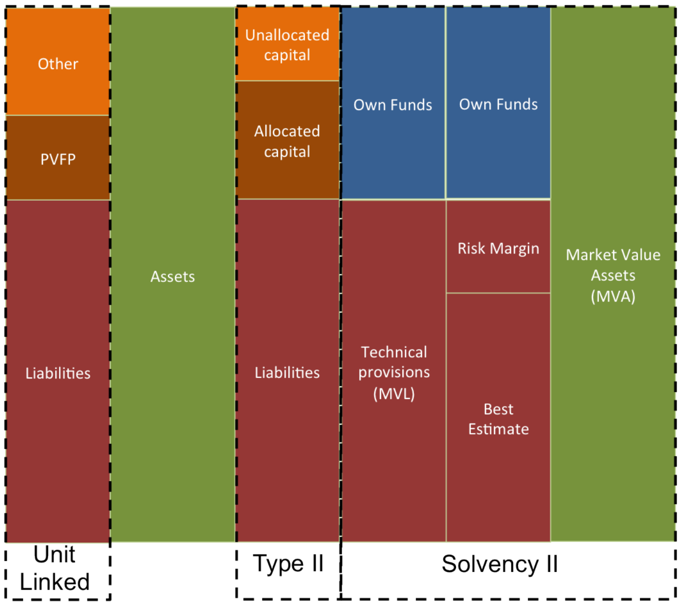

Figure 5: Balance sheet for unit-linked and type II traditional insurance firm in regular and solvency II setting. The first three columns represent the meta-view of the balance sheet while the last three describes the decomposition as prescribed in the Solvency II framework.

r_f is the one-year risk free interest. From this,

$$P(X \geq X_{BE} + (1+r_f)SCR) = 1 - F_X(X_{BE} + (1+r_f)SCR) = 0.995$$

follows. Under the assumption of F_X being and increasing, we get the following explicit expression for the SCR;

$$SCR = \frac{1}{1+r_f}(X_{BE} - F_X^{-1}(0.005)).$$

We denote $F_X^{-1}(0.005))$ as X_{WC} and recognize it as the 0.5% quantile of X or the 0.5% Value-at-Risk (VaR) of the net asset values a year into the future.

EIOPA requires life insurers to hold own funds enough to cover the SCR, namely that

$$X_0 \geq SCR,$$

where X_0 is the net asset value of the report date. This in short translates to requiring the life insurer, when operating in the EU, to hold enough own funds to meet its obligations, and thus stay solvent, over the coming twelve months with a probability of 0.995.

The structure of the balance sheet indicates that the changes in own funds directly originates from changes in the Market Value of Assets (MVA) and/or the Market Value of Liabilities (MVL), and Y may thus also be regarded as the deviation of the net change of MVA and MVL from the best estimate of this net change. The SCR is then calculated as that combination of net change of MVA and MVL, which's sum is exceeded with probability 0.995. A principle figure describing the balance sheet a year in the future, in the case of the best estimate being realized and in the case of a one-in-two-hundred-years year being realized, and how SCR is calculated from this, is seen in Figure 6.

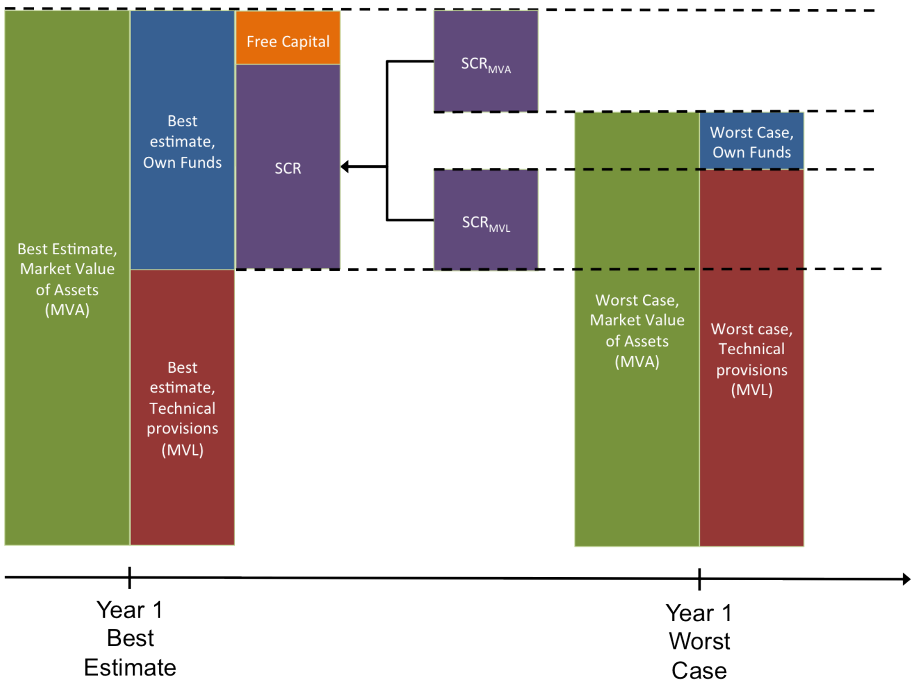

Figure 6: Principle sketch of SCR. The deviation from the best estimate, in one year, of own funds for a year that occurs once in two hundred years. Namely the 0.05% quantile of the distribution for the own funds a year into the future. The net deviation in MVA (denoted SCR_{MVA}) and the net deviation in MVL (denoted SCR_{MVL}) are marked to describe how the SCR is built up.

3.2.4 SCR Calculation

EIOPA allows life insurance companies to calculate the SCR in one of three ways, namely through;

- The standard formula, from here denoted as SF,
- an internal model, from here denoted as IM and
- a partial internal model, from here denoted as PIM.

The standard formula is provided by the regulators and the calculation is decomposed to several submodules, each representing a specific risk type, see Figure 7. In principle, the SCRs of the modules are calculated as the change in own funds when stressing the properties of each module in isolation, where the stresses are specified by EIOPA. The SCRs are then aggregated in a manner allowing for diversification effects from pre specified intra-dependence structures.

An internal model allows the companies to either develop their own process for the calculation of SCR or buy an external solution. This allows each company to tune the SCR calculation to the company's specific properties and operational environment. Before allowing the company to use their internal model however, it must be sanctioned by the local financial authorities, in Sweden this is *Finansinspektionen*.

A partial internal model indicates the combination of the two approaches described above. Also here, the financial authorities need to sanction the modules modeled by the company and their aggregation with the standard formula results.

3.2.5 Contractual option

In [11] the following definition for a contractual option may be read;

3.122.A contractual option is defined as a right to change the benefits, to be taken at the choice of its holder (generally the policyholder), on terms that are established in advance. Thus, in order to trigger an option, a deliberate decision of its holder is necessary.

The contractual options thus originate in the fact that in many insurance contracts, the policyholder has the right to change some of the contract terms, at his or her own discretion. This right may be described as the

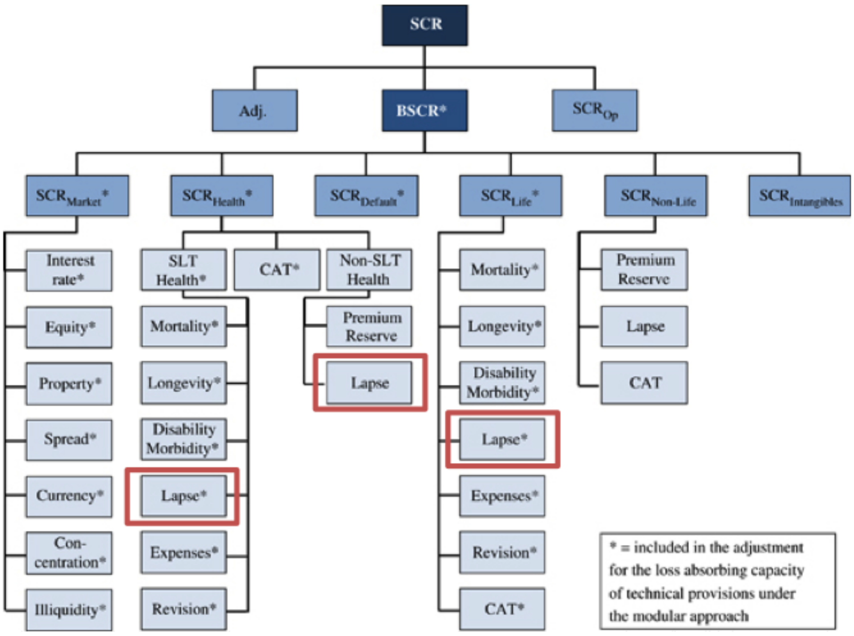

Figure 7: The tree structure of the risk types specified in the standard formula and the modules marked by red boxes are the modules describing the risks assessed in this project.

policyholder owning a set of options, each giving him or her the right but not the obligation to change a certain term of the contract. It is obvious that the exercise of a contractual option may cause the best estimate of the liability for a policy to change. As already mentioned in the technical provisions subsection, the company has to include assumptions as regards to the rate with which the options are exercised when calculating its MVL.

3.2.6 Lapse risk

One of the risks that the company is exposed to is the lapse risk. For the standard formula, [12] states the following regarding this risk type;

Lapse risk was understood to arise from unanticipated (higher or lower) rate of policy lapses, terminations, changes to paid-up status (cessation of premium payment) and surrenders.

In the standard formula, this means that the company is exposed to lapse risk by the fact that the actual exercise rate of the contractual options listed in the quote deviates from the exercise rate assumed when calculating the

the MVL. In this project, the definition is revised to;

The risk arising from the policyholder's actual take-up rate of any contractual option deviating from the take-up rate assumed when calculating the best estimate of liabilities.

4 Modeling & Statistics

4.1 Scenario modeling

One possible approach for SCR calculation could be scenario based, where the principle of the approach could be;

1. Identify the risk factors needed and model the marginal and multivariate properties of these, using various methods.
2. Using these models to simulate a large amount of one-year scenarios.
3. Calculating the resulting value of the Company's own funds in each scenario.
4. Identifying the SCR as the discounted value of the difference between the best estimate of the own funds and the 0.005 quantile of the empirical distribution of the value of own funds, one year into the future, as caused by the risk factors.

Among the risk factors modeled, several factors describing the exercise rate of contractual options may be found. In the standard formula framework the following factors with predefined stressed scenarios are specified in [13]

- A permanent increase of lapse rates with 50%.
- a permanent decrease of lapse rates with 50%.
- a mass lapse event of 40%, for retail businesses.

Thus, viable Lapse risk factors should reflect the same risks as the stressed scenarios above would.

4.2 Statistical tools

For the benefit of the reader, and as support for this thesis, a few statistical tools that are used in the modeling sections of this thesis are briefly reviewed below.

4.2.1 Distribution fitting

In the modeling process, there are several steps where a random variable, say Q, is defined to represent a certain phenomenon. Data that is believed to represent or well approximate this phenomenon is then used to fit a distribution to Q. This process may be divided into two main parts, namely the selection of a candidate distribution and the calibration of the parameters of this distribution. In this thesis, the order has primarily been to fit a distribution using maximum likelihood estimation and then assessing the fit with ecdf and QQ-plots.

4.2.1.1 Empirical cdf plot When the parameters of a candidate distribution have been estimated with MLE, its cumulative distribution function is plotted together with the empirical cumulative distribution function of the sample. The power of the ecdf lies in the fact that it, in the limit, approaches the true distribution of, if the sample is i.i.d. [5]. If the functions resemble each other a great deal, a more rigorous investigation of how well their quantiles fit, using the quantile-quantile-plot (QQ-plot) is done. In the thesis, the function *ecdf* in the statistical toolbox of MATLAB has been used for generation of ecdf-plots.

4.2.1.2 QQ-plot In the plot, the theoretical quantiles of the fitted candidate distribution F are plotted against the empirical quantiles of the sample. In the case where the sample indeed is a set of realizations from F, the value for each empirical quantile should be close to equal to the value of the same theoretical quantile, and the points in the plot should thus form a straight line. In the tails of the distributions, namely at low and/or high quantiles, there will, by definition, be few observations in the sample, making the empirical estimate of the quantile less accurate. Thus, the lower few and upper few points are likely to deviate from the line, even if the sample actually is from F. Further properties of the tool may be read about in [1]. The function *qqplot* in MATLAB's statistical toolbox is used when generating QQ-plots.

4.2.1.3 MLE confidence intervals When calculating the confidence intervals for the MLE parameters, two different methods have been used, depending on the distribution that has been fitted. When fitting a normal distribution, the MLE-estimates are known beforehand, allowing for parametric bootstrap [6] to be used. For the calculations the function *ciboot* has been used to this end. For the beta distribution however, the normality assumption of the maximum likelihood [14] estimator has been used through the function *betafit* in MATLAB's statistical toolbox.

4.2.2 Iteratively reweighted least squares

Because of the impending risk of the OLS-assumption of homoscedasticity being violated, the regular OLS-regression has been replaced by a robust regression method, namely iteratively reweighthed least squares (IRLS). The method, as it name indicates, performs a weighted linear regression, the result of which it uses to reweigh the errors for the following iteration. A somewhat more thorough summary of the IRLS, as described in [2] follows.

To begin with, consider the the matrix form of the regular linear model

$$\boldsymbol{y} = \boldsymbol{x\beta} + \boldsymbol{e}$$

For the best possible fit, one wants to minimize

$$\sum_{i=1}^{n} \rho\left(\frac{y_i - x_i\boldsymbol{\beta}}{\sigma}\right),$$

with respect to $\boldsymbol{\beta}$, for some robust loss function ρ and error standard deviation σ, x_i is row i of $\boldsymbol{x}$. Note that in the OLS case, we have that

$$\rho\left(\frac{y_i - x_i\boldsymbol{\beta}}{\sigma}\right) = (y_i - x_i\boldsymbol{\beta}).$$

A necessary condition for optimality is that the normal equations are satisfied, namely that

$$\sum_{i=1}^{n} x_{ij}\rho'\left(\frac{y_i - x_i\boldsymbol{\beta}}{\sigma}\right) = 0, \quad j = 1, \dots k. \tag{1}$$

The notation $rw(r) = \rho'(r)$ is introduced and $\left(\frac{y_i - x_i\boldsymbol{\beta}}{\sigma}\right)$ is denoted as r_i, giving (1) the form

$$\sum_{i=1}^{n} w(r)x_{ij}r_i = 0, \tag{2}$$

which is recognized as the normal equations for the weighted least squares problem with variable weights $w(r)$, which has the solution

$$(\boldsymbol{x^T W(r) x})^{-1}\boldsymbol{x^T W(r) y}. \tag{3}$$

Above, $W(r)$ is a matrix with $w(r_i)$ on its diagonal. One robust weight function $w(r)$, suggested in [3], is

$$w^W(r) = exp[-(r/W)^2],$$

where $W = 2.985$ and is a constant specified by the author of [3]. Because of the non-linearity of (2), an iterative method to find the IRLS-estimates $\hat{\boldsymbol{\beta}}$ is required, which in the MATLAB statistical toolbox is implemented as follows. Initially, $\boldsymbol{W(r)}$ is set to the unit matrix, $\boldsymbol{W_0(r)} = \boldsymbol{I}$, and $\hat{\boldsymbol{\beta}}_{\mathbf{0}}$ is calculated using (3). Then:

1. Using $\boldsymbol{y} - \boldsymbol{x}\hat{\boldsymbol{\beta}}_{\mathbf{0}}$, the residuals $\boldsymbol{e}$ are obtained.
2. The residuals are first adjusted to $e_i^{adj} = e_i/\sqrt{1-h_i}$, where h_i is the leverage of point i. Further, they are normalized to $r_i = e_i^{adj}/s$, where s is the robust variance, calculated as the median absolute deviation of the residuals divided by 0.6745.
3. Using r_i, the robust weights are obtained through $w_i = w^W(r_i)$ and the new robust weight matrix $\boldsymbol{W_1(r)}$ is formed.
4. $\boldsymbol{W_1(r)}$ replaces $\boldsymbol{W_0(r)}$ in (3) and $\hat{\boldsymbol{\beta}}_{\mathbf{1}}$ is calculated.

The procedure then restarts from 1. until $\hat{\boldsymbol{\beta}}$ converges, producing the IRLS-estimate.

4.2.3 Tail dependence

In the analysis of the dependence behavior of the modeled risk factors, the concept tail dependence will be used to asses the strength of this behavior in the tails of two variables. Definitions for upper and lower tail dependence have been collected from [4] and are stated below. The lower tail dependence between X_1 and X_2 is defined as;

$$\lambda_l = \lim_{q\to 0} P(X_2 \le F_2^{-1}(q) | X_1 \le F_1^{-1}(q)) \tag{4}$$

and the upper as

$$\lambda_u = \lim_{q\to 1} P(X_2 > F_2^{-1}(q) | X_1 > F_1^{-1}(q)), \tag{5}$$

where independence would yield $\lambda_l^{ID} = \lim_{q\to 0} F_2^{-1}(q) F_1^{-1}(q)$ and $\lambda_u^{ID} = \lim_{q\to 1}(1 - F_2^{-1}(q))(1 - F_1^{-1}(q))$.

To investigate the tail dependencies between a multivariate sample of X_1 and X_2, one may replace F_1^{-1} and F_2^{-1} with the empirical distribution functions of samples of X_1 and X_2. Then plot $P(X_2 \le F_{2E}^{-1}(q) | X_1 \le F_{1E}^{-1}(q))$ and $P(X_2 > F_{2E}^{-1}(q) | X_1 > F_{1E}^{-1}(q))$ together with $F_{2E}^{-1}(q) F_{1E}^{-1}(q)$ and $(1 - F_{2E}^{-1}(q))(1 - F_{1E}^{-1}(q))$ for decreasing and increasing q respectively. If the tail-dependencies deviate from the those that would be observed when independent, presence of tail-dependence is plausible.

5 Contractual options & Model scope

5.1 Review of internal & external documentation

The following types of documents have been reviewed in order to assess the contractual options found in the contracts of the Company;

- Insurance contract terms for active products within;
  - Occupational pension schemes
  - Private pension policies
  - Private endowment policies
- Required conditions for the pension schemes within the collective agreements, for those plans the Company has decided to offer.
- Earlier internal analyses for contractual options.

5.2 Contractual options in the company

In the above mentioned analysis five contractual options, such that an exercise is likely to influence the value of Own funds, were found. These include;

- The transfer option (Flytträtt)
- The surrender option (Återköp)
- The change to paid-up status option (Fribrev)
- The change of pay-out term option (Ändring av utbetalningsperiod)
- The change of pay-out age option (Ändring av tid för utbetalningsstart)

5.2.1 Transfer option

Depending on the nature of the contract, the policyholder may or may not own the right to transfer his or her total insurance capital to another insurance firm. Upon exercise the transferrable capital is calculated as the insurance capital minus fees aiming to keep the remaining insurance collective unharmed. In the case where the MVL and allocated own funds together exceed MVA, a further decrease of the capital is made by a factor of MVA/(MVL+allocated own funds). Since a large fraction of the Company's policies are occupational pensions schemes, where the rights to transfer vary, the distribution and specific setting of this right across the Company's investment policies is of interest;

- Occupational p-policies
  - Right to transfer entire insurance capital
    - Some collectively agreed plans, namely ITP 1/2, FTP1/2, SAFLO, Gamla PA-KFS09.
    - Plans signed directly between the company and the employer.
  - Right to transfer entire insurance capital accumulated after 2006-01-01;
    Some collectively agreed plans, namely PA-KFS09, PA03 and a few ITP-like plans.
  - No right to transfer any insurance capital;
    ITP-PP, the plan for workers within media.
- Private p-policies;
  Right to transfer the entire insurance capital.
- Endowment policies;
  No right to transfer any insurance capital

The impact of the exercise of the transfer options on the reserve for the model policies C_t and C_u may be seen in Figure 8. For C_t we get a positive net change of own funds and for C_u a negative one.

5.2.2 Surrender

Upon exercise of the surrender option, the policyholder is repaid the allocated insurance capital. The surrender value of the contract is calculated in a manner that closely resembles that of the transferable value, discussed above. As mentioned in the theoretical framework, p-taxed insurance policies, are taxed in a way to incentivize pension savings. Thus, in order to keep the policyholders from accessing the capital invested in their policies, the law prohibits the surrender of p-schemes. Endowment policies however are not taxed to promote pension savings per se. The Company's endowment policyholders thus own the option to surrender their policies. The impact of the exercise of the surrender options on the reserve the model policies is the same as that described the transfer option, see Figure 8.

5.2.3 Change to paid-up status

All the Company's investment policyholders own the option to, at any point, cease with premium payments and convert the policy to paid-up status. The meaning of this is that the insurance capital of the policy from that point only

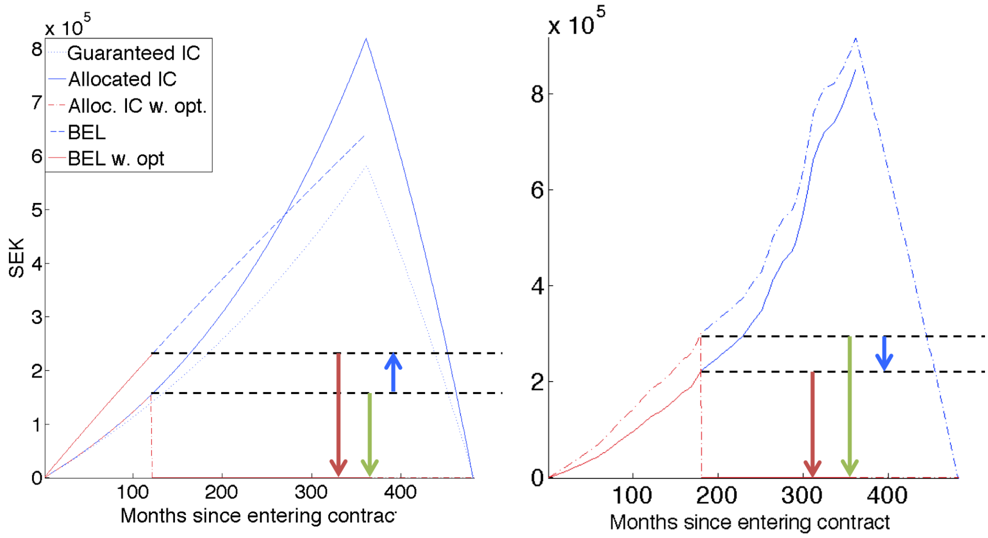

Figure 8: Insurance capital and best estimate of the liability (BEL) for the model contracts C_t (left) and C_u (right), marked in dashed and solid blue when no transfer option is exercised and in dashed and solid red when the transfer option is exercised. The green arrow shows the change in the Company's assets, the red the change in its liabilities and the blue the net change and direction of the own funds. The option is in this example exercised ten years into the contract.

increases by the guaranteed and bonus rates for traditional life and with the interest rate realized from the investment choices of the policyholder for unit linked insurance. The impact of the exercise of the paid-up status option on the reserve for the model policies C_t and C_u may be seen in Figure 9. For a traditional life scheme, we get a negative change in own funds while nothing happens in the unit-linked case.

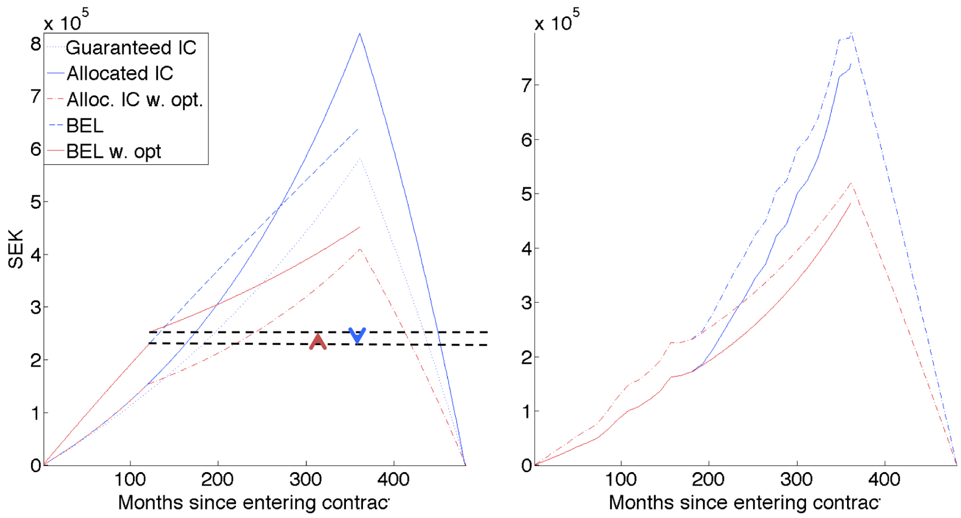

Figure 9: Insurance capital and best estimate of the liability over for the model contracts C_t (left) and C_u (right), marked in dashed and solid blue when the change-to-paid up status transfer option is not exercised and in dashed and solid red when it is. The green arrow shows the change in the Company's assets, the red the change in its liabilities and the blue the net change and direction of the own funds. The option is in this example exercised ten years into the contract.

5.2.4 Change of payout-term

This option entitles the policyholder to change the length of the pay-out term of the benefit. For occupational and private pension schemes, the policyholder may at any point apply to change the pay-out term of the policy, but may be subject to a health check to avoid anti-selection. However, when exercising the option in connection to the first benefit payment, no health check is required. For some of the collectively agreed plans, other rules regarding the pay-out term apply. These regulations are senior to those of the Company. The endowment policyholders receive their insurance capital in a lump sum upon retirement of the contract. However, they may also apply to obtain the benefit through a limited annuity. The impact on the best estimate of the the own funds of increasing the pay-out term from 10 to 20 years for the model policies C_t and C_u may be seen in Figure 10.

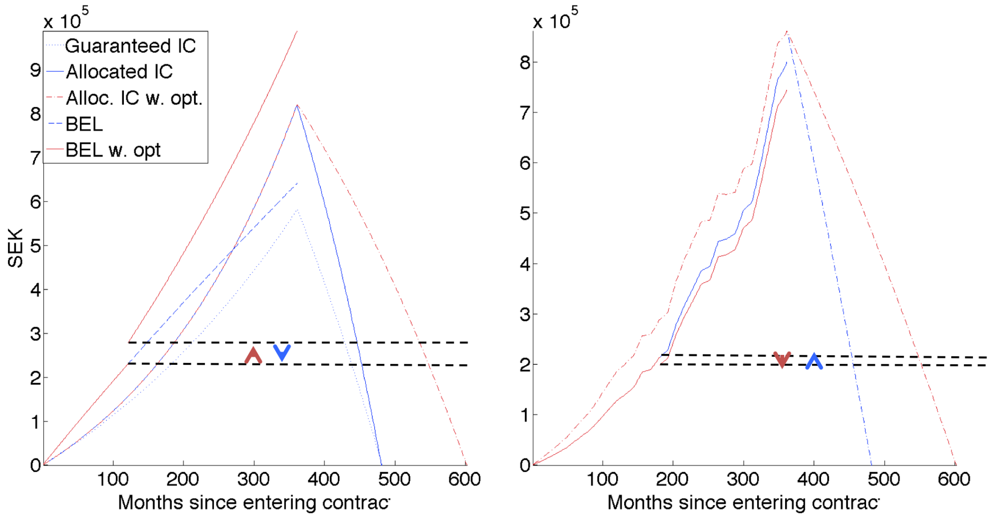

Figure 10: Insurance capital and best estimate of the liability for the model contracts C_t (left) and C_u (right), marked in dashed and solid blue no option is used and in dashed and solid red when the option is exercised, increasing the payout term from 10 to 20 years. The green arrow shows the change in the Company's assets, the red the change in its liabilities and the blue the net change and direction of the own funds. The option is in this example exercised ten years into the contract.

5.2.5 Change of start of payout-term

In some policies, the option to change the time at which the contract enters its pay-out state is included. For the occupational p-contracts, the exact settings of this option vary from plan to plan. In the private p-schemes the policyholders usually only have the right to apply for a change of time for retirement, and may be subject to a health check for the request to be approved. For endowment schemes, the policyholder needs to surrender the contract in order to exit the contract earlier than initially agreed. The impact of the exercise of the change of pay-out start option on the reserve for the model policies C_t and C_u may be seen in Figure 11.

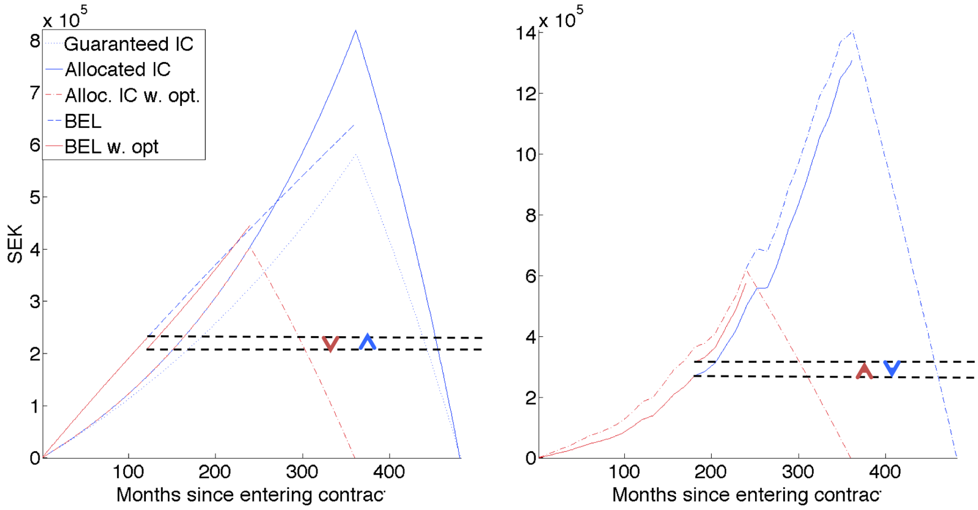

Figure 11: Insurance capital and best estimate of the liability for the model contracts C_t (left) and C_u (right), marked in dashed and solid blue when no option is used and in dashed and solid red when the option is exercised, moving the retirement date back with 10 years. The green arrow shows the change in the Company's assets, the red the change in its liabilities and the blue the net change and direction of the own funds. The option is in this example exercised ten years into the contract.

5.3 Model scope

5.3.1 Materiality Analysis & Customer behavior proxy

The Company, using a combination of expert judgment as well and stress testing of contract pools, would do the assessment as to which of the above-mentioned options are material in the Company's specific contract pool. The specific materiality analysis is not in the scope of this thesis. Further, the Company deems it would be able to assume that the exercise frequency of the transfer option can be used as an approximation to all the customer behavior risks arising. This means that the probabilistic properties and dependence structures of the risk factors describing the risk originating in the other material options would be based on those modeled for the transfer option exercise rate, most likely through an affine (shift-scale) transformation. The assumption limits the statistical modeling needs to only the risk factors related to the exercise frequency of policy transfers.

5.3.2 Risk factors

Much in accordance with the scenarios required for lapse risks in the standard formula, the proposed choice of risk factors has become;

(A) The transfer exercise frequency for the coming year, also know as mass transfer risk factor for traditional life business.

(B) The transfer exercise frequency for the coming year, also know as mass transfer risk factor for unit linked business.

(C) The deviation from the assumed future yearly exercise frequency for contracts with negative surrender strain.

(D) The deviation from the assumed future yearly exercise frequency for contracts with positive surrender strain.

Risk factors A & B These risk factors are random variables describing the rate with which the policyholders exercise their transfer option over the coming 12 months. In the SCR calculation setting however, all exercises over the year are assumed to be done instantly after the report date, creating a mass-transfer event. By this, the risk factor aims at assessing the change in Own funds from a sudden downscaling of the balance sheet, caused by a substantial amount of policies being transferred.

To describe the effects on the own funds of a mass transfer event a few simplifications are made. Let us primarily assume that all the contracts policyholders have full transfer rights, meaning that every policy may be transferred at any time. For a traditional life company, an instant mass-transfer where the fraction X of the insurance capital of the company is transferred would, if we neglect non-policy based technical provisions, decrease the technical provisions to $(1-X)(MVL)$. Because of the allocated part of the own funds also being subject for transfer, the value of the own funds after the event would be $(1-X)Y(OF)$, where Y denotes the fraction of the own funds that is allocated to the policyholders. Now, we see that

$$(1-X)Y(OF) \leq (OF),$$

meaning that the value of the own funds decrease. The principle of this may be seen in Figure 12. It indicates that a life insurance company is worse off after a mass-lapse event. For a unit linked firm, none of the own funds are subject for transfer. However, when $(1-X)$ of the policyholders transfer their capital from the Company, the PVFP from the fees of the policies is also decreased to $(1 - X)PVFP$. By this reasoning, if we let Y denote the fraction of the own funds built up by the PVFP, the above reasoning and Figure 12 is valid in the unit-linked case as well.

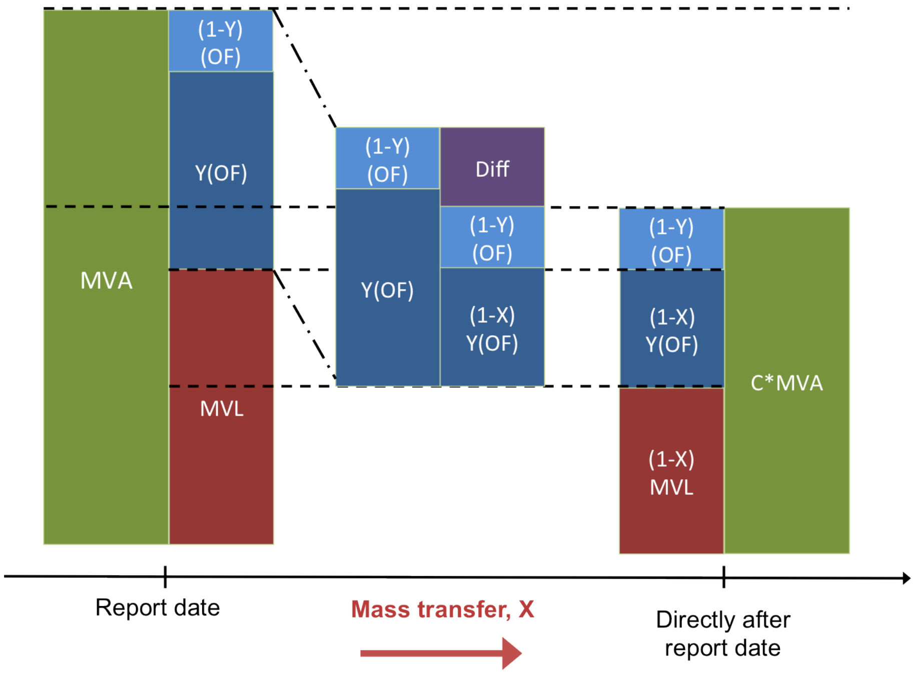

Figure 12: Principle figure of how the Solvency II balance sheet is rescaled when exposed to a mass transfer event where the fraction X of the transferable capital is transferred, when the allocated funds (traditional life) or the PVFP (unit-linked) amounts to the fraction Y of the total amount of own funds.

Risk factors C & D As stated when describing the best estimate part, in the calculation of MVL, the Company makes assumptions regarding what the average yearly transfer frequency will be in the future. We denote this assumption L_N. One year into the future, events may have happened that will cause the Company to change this assumption to a new one, denoted L_{N+1}. This deviation from the initially assumed long-term yearly transfer rate will cause changes in the MVA and the MVL, which induces changes in the own funds. An example of an event triggering a change of the assumption could be that the prospect for the introduction of full transfer rights by law has increased. The net effect of the change in own funds however differs depending on if the transfers are done from contracts with positive or negative surrender strain.

The following definition for surrender strain may be found in *CEIOPS' Advice for Level 2 Implementing Measures on Solvency II;*

The surrender strain of a policy is defined as the difference between the amount currently payable on surrender and the best estimate provision held.

Translated to the language of this thesis, the surrender strain of a policy is the difference between the capital received by the policyholder when surrendering the policy or transferring the insurance capital and the best estimate of the obligations of the contract.

A problem arises with the fact that the data available for transfer frequencies cannot be sorted with respect to their surrender strain. Instead, the following approximations are made;

1. The long-term transfer frequency for contracts with negative surrender strain (NSS) is approximated as the long-term transfer frequency for traditional life contracts.

2. The long-term transfer frequency for contracts with positive surrender strain (PSS) is approximated as the long-term transfer frequency for unit-linked contracts.

By observing the behavior of the contracts C_t and C_u in figures 3 and 4, an assessment of the validity of the approximations may be made.

For unit linked insurance, Figure 4 clearly indicates that the insurance capital of the policy exceeds the best estimate of the contract liabilities at every time, making it, by the definition above, a contract with positive surrender strain. Even though the contract in the figure is a sample contract, the fact that the future cash flows may be discounted with the risk free rate, while no further growth of the insurance capital is guaranteed, supports the assumption. Figure 13 shows the principle of this; at present day, the contract has insurance capital IC_0. At maturity, the insurance capital is $IC_M = IC_0$. The benefit is simplified into two payouts, B_1 and B_2 with the first occurring directly at maturity and the second somewhat later. As seen, the benefit cash flows are discounted with r_f while no growth of IC is guaranteed. When bundling the present value of B_1 and B_2, it thus amounts to less than the insurance capital, and the surrender strain is the difference between the two and is positive. Note that the visualization of growth in the figure is made linearly, which is not the case with compounded interest rates. However, the figure proves the point.

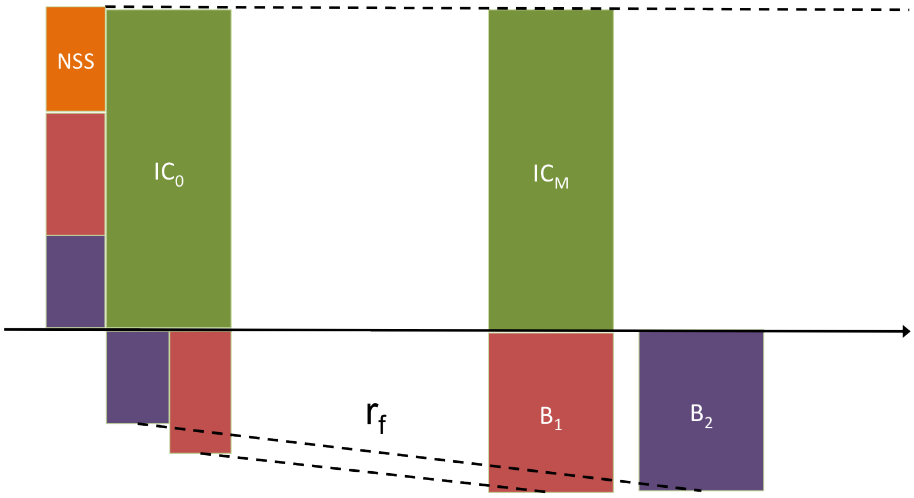

Figure 13: Principle of why unit-linked contracts are associated with positive surrender strain. The green bars indicate the insurance capital at time zero and at maturity of the contract, while the red and purple bars are the present values of the benefits and their value when paid out. The orange box is the resulting surrender strain.

For traditional life insurance schemes however, the approximation may be somewhat more questioned. As seen in Figure 3, the sign of the surrender strain shifts about 270 months into the contract, from negative to positive. The picture in Figure 14 shows the principle of how the surrender strain may be assessed in a very simple contract, where, as in the unit linked case, the initial capital at current date is IC_0, at maturity IC_M and the benefits are B_1 and B_2. In this case the company guarantees a growth from time 0 to maturity, making $IC_M > IC_0$. The company also guarantees growth r_g of the insurance capital still in the company in the pay-out phase, which is seen by the fact that $B_2 > B_1$ in the picture. Now, if the risk free interest rate is $r_{g,1}$, the present value of B_1 and B_2 sums up to the pillar left or IC_0, making the surrender strain negative. However, if the risk free interest rate is e.g. $r_{g,2} > r_{g,1}$, the present value of the benefits sum up to the dashed pillars seen to the right of IC_0, making the surrender strain positive. Again, the discount and growth effects are represented linearly in the figure, distorting the result. In essence, what has been discussed above shows that the sign of the surrender strain of a traditional life contract partly depends on how r_g and the risk free rate r_f relate to each other, but partly also on the properties of its payout phase. Since the investment in an insurance policy at a private Company is not entirely risk free, it is plausible that it is often true that $r_g > r_f$, which would indicate that the approximation is at least not heroic.

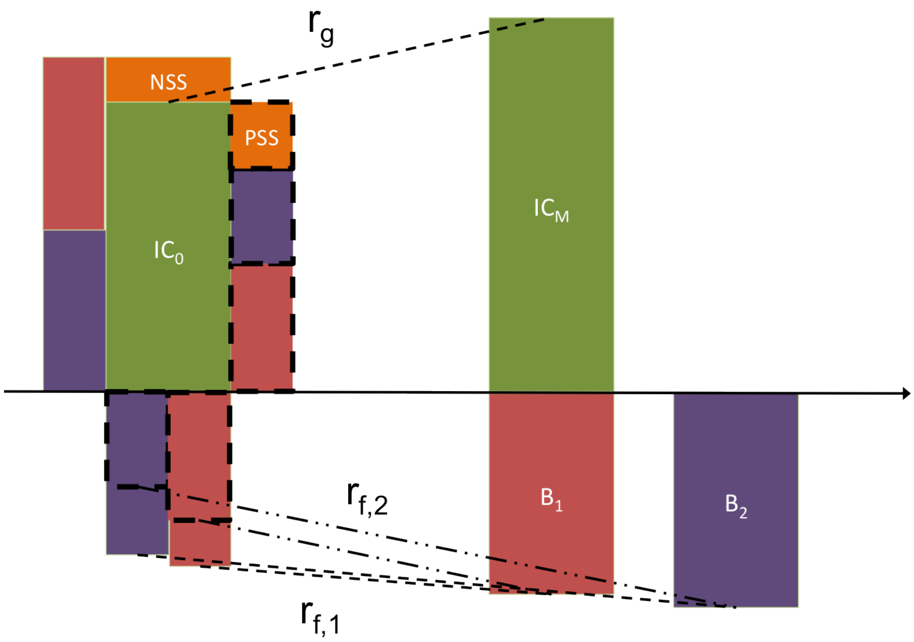

Figure 14: Principle of the calculation of surrender strain for a traditional life insurance contract. The green bars indicate the insurance capital at time zero and at maturity of the contract, while the red and purple bars are the present value of the benefits, assuming $r_f = r_{f,1}$, and their value when paid out. Red, purple and orange boxes with dashed borders are the same entities, when $r_f = r_{f,2} > r_{f,1}$. The orange box is the resulting surrender strain.

When a policy is transferred out of the company, in essence what happens is that the Company's MVA decreases with the insurance capital (IC) of the policy and the MVL decreases with the best estimate of the value of the liabilities (BEL). By this we get that;

1. For PSS contracts, where $IC > BEL$, the value of the Own funds decrease when a contract is transferred. This in turn means that an increase in SCR occurs when the long-term assumption a year into the future, L_{N+1} is *higher* than the current assumption, L_N.

2. For NSS contracts, where $IC < BEL$, the value of the Own funds

increase when a contract is transferred, which means that the increase in SCR occurs when the assumption L_{N+1} is *lower* than L_N.

5.3.3 Dependencies

In each scenario of the simulation, described in the theoretical framework, there is one observation of each of the risk factors listed above, together with observations for the remaining risk factors. In order for each scenario to reflect the real world, the dependence structure between our risk factors and the other simulated risk factors must be assessed.

The assumption has been made that the risk factors representing the transfer frequencies are independent conditioned on their dependence to the equity, 3-month and 10-year risk free interest rate risk factors. This means that we believe that the dependence structure between transfer frequencies and all other risk factors is captured implicitly through the dependence structure with equity returns and interest rate levels. Through this assumption, the extent of the dependence structure to be modeled is reduced.

Since the Company has some of its assets invested in equity, a decrease in the value of these investments, namely a negative equity return, would decrease the MVA. For its traditional life insurance arm, the change in MVL however would be essentially unaffected, resulting in a decrease of the net value of Own funds. For its unit-linked arm, the BEL is directly related to the value of the assets specified by each policyholder, meaning that a negative equity return would result in an equal decrease in the ICs and BELs. However, a negative equity return would also decrease the absolute amount of insurance capital in the company, thus decreasing the value of the PVFP since the varying part of the fee is decreased. This decrease induces a decrease in the own funds. By the above, a decrease in the equity return is negative for both the traditional life arm and the unit linked arm.

Regarding the risk free interest rates, a decrease in r_f gives an increase of the yearly discount factor, as it equals $1/(1 + r_f)$. From this follows that the present value of future cash flows increase, and with it the BEL for both unit-linked and traditional life firms. It is noteworthy that an increased discount factor induces an increase in the PVFP as well, since it is made up by discounted future profits. However, it is reasonable to believe that the value of the BEL exceeds that of the PVFP, making the net effect of the risk free rate increase negative in both cases.

From the above reasoning, the following worst case co-movements with the equity returns and interest rates have been identified for the risk factors;

(A) A large traditional life mass-transfer frequency and a low equity return and/or high risk free interest rate.

(B) A large unit-linked mass-transfer frequency and a low equity return and/or a high risk free interest rate.

(C) A long term assumption one year in the future lower than the the one made at current time together with a low equity return and/or an high risk free interest rate, for negative surrender strain contracts.

(D) A long term assumption one year in the future higher than the the one made at current time together with a low equity return and/or an a high risk free interest rate, for positive surrender strain contracts.

5.4 Discussion

In the analysis of the Company's contractual options and the definition of the model scope, several sources of error may be identified. These are presented below together with an assessment to their severity and/or suggestibility in the scope of this thesis.

To begin with, as the materiality analysis would be conducted by the company and thus works purely as an input into this study, the risk of errors in this analysis exists. However, because the materiality analysis process is out of the reach of this thesis, the suggestibility of this factor is close to none. When it comes to the severity, the fact that the Company has several lines of control to ensure quality in their work is believed to mitigate this. The capability of the selected risk factors, in terms of modeling the risks associated with customer behavior, is also a question of issue. The assessment of this would however, just as in the case with the materiality analysis, be made by the Company, and the same assumption as to severity and suggestibility is made.

Further, an assumed proxy of all customer behavior risks with the transfer frequency risks may be faulty. The suggestibility is limited due to the fact that public data of sufficient quality and quantity is hard to find for the remainder of the contractual options. However, as is seen in section 5.2, the transfer option gives by far the biggest change in own funds upon exercise, for the model contracts. With this in mind, the assumption should be of conservative nature and the severity a measure of its grade of conservatism.

The approximations of negative and positive surrender strain contracts as traditional life insurance and unit-linked policies is another possible source of error. The validity of this assumption has however already been assessed in conjunction with their statement.

6 Data

6.1 Variables

Note that every data point is collected from the end of December in the year it is indicated. A data point for any variable in 2012 thus indicates what has been observed during the year 2012.

6.1.1 Transfer frequency

Swedish & Company data
For the period 2006-2013 for traditional life insurance and 2008-2013 for unit-linked, the observed transfer frequencies for the Company have been provided. We denote $f^i_{C,k}$ as the observed transfer frequency of the Company in year i for business line k. However, because the landscape for the transfer rights in Sweden is uncertain and has varied substantially in the past years, this data is not deemed to in itself represent the transfer behavior the Company is exposed to. For instance, it would not take into consideration the effects of the introduction of legally regulated right to move all capital in any p-insurance scheme, which is a plausible event,e specially in the long term.

Norwegian data
In Norway, full legal rights for transfer of insurance capital have been in action since 1992 [15]. Additionally, the Norwegian life insurance market is similar to the Swedish in terms of products and sales channels. These two factors make observed transfer frequencies from Norwegian companies for years after 1992 a better sample, in regards to representing customer behavior in an insurance market with full legal transfer rights.

The data for transfer frequencies has primarily been collected from the yearly report *Markeds- og regnskapsstatistikk*, published by Finans Norge (FNO). Among other information, the reports contain sheets where the market share of the different life insurers in Norway is stated. Among the data, the following two entities are found for each company;

- Forsikringsforplikteleser/Forsikringsfond
- Overforte reserver til andre

The *Forsikringsfond* would directly translated refer to what we have specified as the technical provisions. However, the relative size of the entity compared to the total assets of the company indicates that the unit rather corresponds to the sum of the technical provisions together with the allocated capital. I.e. the sum of best estimate of the liabilities for the entire insurance capital of each policy, not just the guaranteed part. The *Overforte reserver til andre* is interpreted as the capital flow out of the Company through transfers, for each year. Thus, the quota

$$f_{Norway}^{obs} = \frac{Overfote\ reserver\ til\ andre}{Forsikringsfond}$$

states the percentage of the capital subject to transfer that has actually been transferred. The ratio is therefore used as the proxy for the yearly exercise rate of the transfer option.

In order to get observations of the transfer frequency for firms that as close as possible resemble the well established and large firm the Company is, a few decisional rules when selecting companies to extract data points for were applied, namely;

1. Both the units *Forsikringsfond* and *Overforte reserver til andre* need to exist for the company and year.
2. The size of *Forsikringsfond* has to, in 2013, be at least 5% of the equivalent of the Company, for its traditional and unit-link business respectively.
3. Observations for all years for the companies fulfilling 2, given that no unreasonably big expansion of *Forsikringsfond* has been observed between any two years.

Using the decision rules stated above, the companies and years from which the data points originate are specified, which may be seen in Table 1 for traditional life insurance and in Table 2 for unit-linked insurance.

Table 1: Companies and years selected as data points for transfer frequencies for traditional life insurance.

Company	Years
KLP	1995-2013
Sparebank 1 Liv	1995-2013
Storebrand	1995-2013
Nordea Liv	1995-2013
Vital Liv	1995-2006
Vital Forsikring	2007-2010
DnB Livsforsikring	2011-2013
Gjensidige NOR Spareforsikring	1995-2003

Table 2: Companies and years selected as data points for transfer frequencies for unit-linked life insurance.

Company	Years
Danica Fondforsikring	2003-2013
Gjensidige Pensjon	2009-2013
Nordea Link	2001-2013
Sparebank 1 Fondforsikring	2003-2013
Storebrand	2003-2013
Vital Link/DnB	2000-2006

Risk factors A & B When estimating the properties of risk factors A and B, the data points above have been used directly for traditional life and unit-linked respectively. The observation for company k, year i and business line j is denoted as $f^i_{j,k}$. For traditional life insurance, a total of $n^t_f = 104$ observations have been selected and for unit-linked $n^u_f = 68$. Note that each years observation, for all companies, has been assumed to come from the same underlying distribution and be independent of every other observation. I.e. the sample is assumed to be independent and identically distributed. Figures 15 and 16 show the time series for each company for the traditional life and unit-linked business respectively. The lack of obvious trend within each company and dependencies across the companies allows for the i.i.d. assumption.

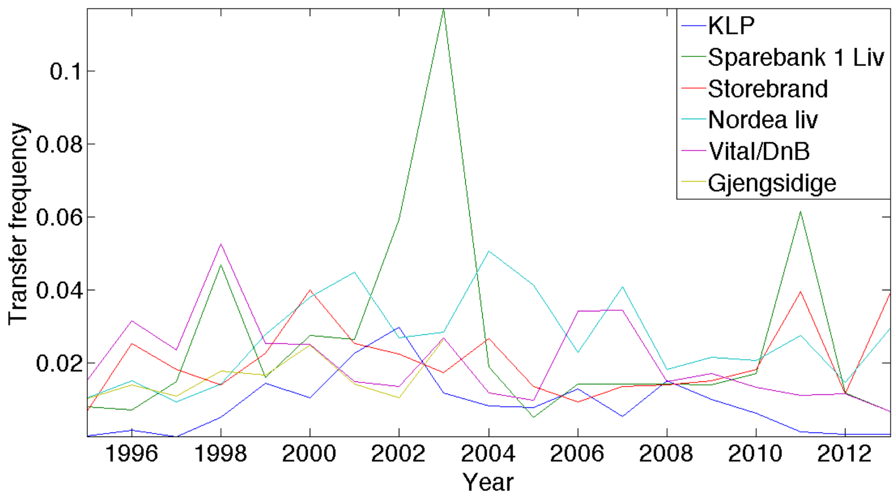

Figure 15: Time series for transfer frequency for the selected Norwegian companies and years, for traditional life insurance

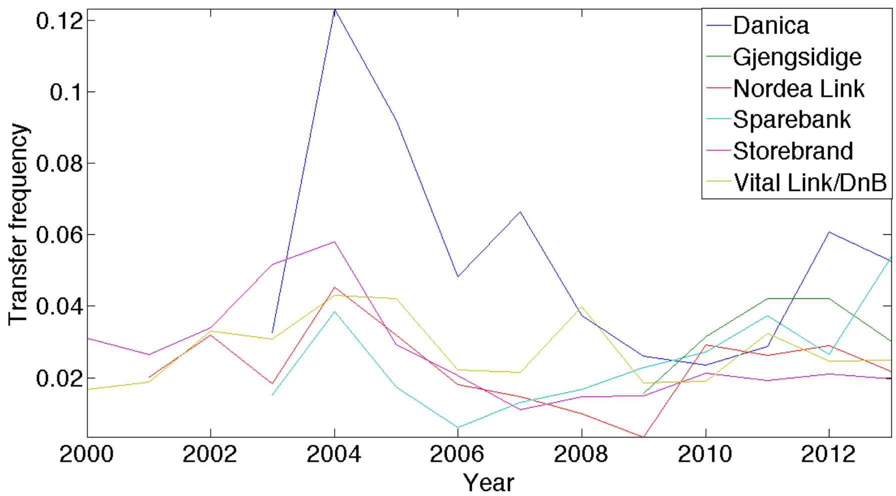

Figure 16: Time series for transfer frequency for the selected Norwegian companies and years, for unit linked insurance

6.1.1.1 Risk factors C & D In the estimation of the properties for the long-term risk factor the observation sets S_t and S_u have been used. These observation sets aim to approximate the difference in transfer frequency for a year between a company operating in the Norwegian full transfer rights market and the Company operating in the Swedish market. The sets are therefore built up of the observations obtained by;

$$s_t^{i,j} = f_{t,k}^i - f_{C,t}^i$$

and

$$s_u^{i,j} = f_{u,k}^i - f_{C,u}^i,$$

respectively. When forming S_t and S_u, the panel data of $s_t^{i,j}$ is reformed to two sets assumed to come from the same underlying distributions, with n_s^t observations and n_s^u, under the assumption of both data sets being i.i.d.

6.1.2 Equity returns

The yearly equity returns, r_e used are the same the Company use in their calibrations. By using this data, we enhance the compatibility of the transfer risk factor models with that developed for remaining risk factors. The data was provided in decimal form and in Figure 17 a blinded time series for the development of the equity returns may be seen.

6.1.3 Risk free interest returns

As with equity returns, the same data points for 3-month interest rates, $r_{\frac{1}{4}}$ and 10-year interest rates, r_{10} that were when modeling the remainder of risk factors have been used. The data was provided in percentage form, namely, a value of X indicates X per cent. In order to fit the data as received from the simulated scenarios, the variables have been transformed to decimal form. In Figure 17, a blinded time series for the development of the two rates of interest may be seen.

6.1.4 Scenario data

In order to investigate how the modeled transfer rate risk factors behave in the setting of the Company's, the observations for the variables used in the functional form were provided for one set of a large amount of scenarios. The scenarios have been simulated and the variables provided and their initial form may be seen in Table 3.

6.2 Discussion

The sources of error in the Data section may be divided into the error introduced through errors in input data and through error in selection and

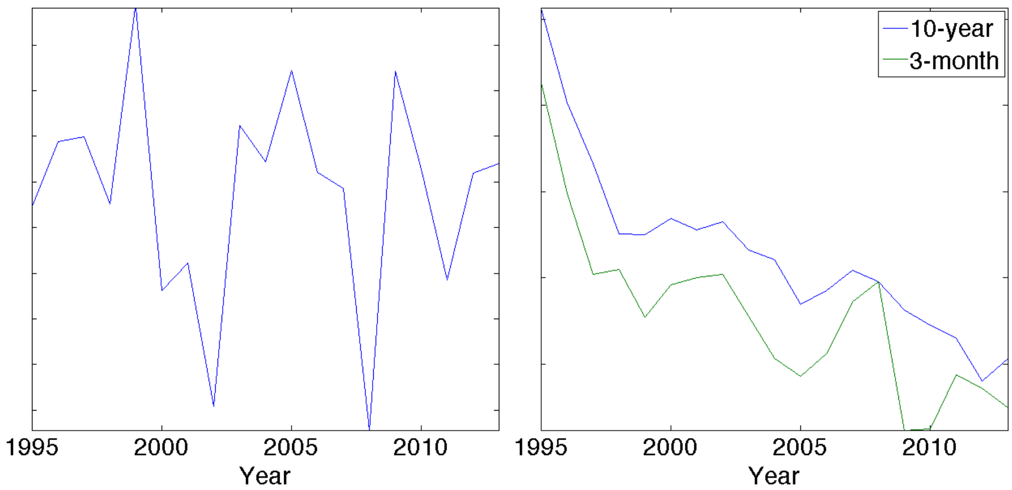

Figure 17: Blinded time series of the equity returns (left) and interest rates (right) used in modeling process.

Table 3: Format in which the variables are provided from the scenario generation.

Variable	Format
Equity returns, r_e	Decimal
Ten-year interest rate, r_{10}	Decimal
Three-month interest rate, $r_{\frac{1}{4}}$	Decimal

forming of new variables. Below these sub-modules are assessed and their severity and suggestibility investigated.

6.2.1 Input data

The only external data that has been used is the *Forsikringsfond* and *Overforte till andre* entities. As these originate from annual reports from one of Norway's biggest employer's associations within the Finance industry, the validity of the numbers are deemed reliable, which indicates the severity of possible errors to be limited. The suggestibility is, as with other input, close to none.

6.2.2 Variable selection and forming

The definition of transfer frequencies as *Overforte till andre* over *Forsikringsfond* could be questioned. However, the observed values of the variable do indicate to be of plausible magnitude, especially for the companies selected in the stated selection process. By the fact that the relation between *Forsikringsfond* and the total turnover, for most of the companies, does indicate that *Forsikringsfond* is indeed the technical provisions together with the allocated capital, the definition is strengthened.

In the selection process, there is an obvious threat for survivorship bias, as only the bigger, well-established companies are included in the final sample. However, the fact that the thesis does not aim at modeling a generic transfer frequency, but rather one that fits the behavior of the Company's customers, defends the selection of only these Norwegian companies.

When reshaping from the panel data form of $f^i_{j,k}$ to a single data set, assumed to have been drawn from the same underlying distribution and used in a regression setting, the assumption of the sample being independent and identically distributed is made. The assumption is, as above stated, based primarily on the fact that the respective time series of the chosen companies don't indicate any obvious structure, and thus no sign of autocorrelation, which would invalidate the i.i.d. assumption. As to the suggestibility, further control could be made of the autocorrelation in the panel-formed sample as well as in the reshaped sample. This would allow for a more thorough assessment of the assumption.

7 Lapse rate risk factor modeling

7.1 Risk factors A & B

7.1.1 Model form

The approach for modeling the univariate and multivariate properties of the mass-transfer rates for traditional life, denoted f_t, and unit linked, denoted f_u, respectively has been regression analysis. By this, the mass transfer frequencies may be linked to the equity return, r_e, the ten-year interest rate, r_{10}, and the three-month interest rate $r_{1/4}$. Let us introduce $\boldsymbol{\beta}_t$ as the vector of parameters needed when linking the four through a functional in the traditional life case. Further, let X_t be the exogenous part of f_t, i.e. the part that the three returns selected cannot describe. Finally, introduce the notation $P_t(x)$ for the functional linking the variables together. This would yield the following relationship

$$f_t = P_t(r_e, r_{10}, r_{\frac{1}{4}}, \boldsymbol{\beta}_t, X_t) \tag{6}$$

for the traditional life case. In the unit-linked case, the notation is the same apart from all subscripts marked as t being replaced with u, yielding

$$f_u = P_u(r_e, r_{10}, r_{\frac{1}{4}}, \boldsymbol{\beta}_u, X_u). \tag{7}$$

For simplicity, it is initially suggested that P_t and P_u are modeled as linear and separable functions, reducing the process to linear regression, namely

$$f_t = \beta_{t,1} + \beta_{t,2}F_{t,1}(r_e) + \beta_{t,3}F_{t,2}(r_{10}) + \beta_{t,4}F_{t,3}(r_{\frac{1}{4}}) + X_t$$

and

$$f_u = \beta_{u,1} + \beta_{u,2}F_{u,1}(r_e) + \beta_{u,3}F_{u,2}(r_{10}) + \beta_{u,4}F_{u,3}(r_{\frac{1}{4}}) + X_u,$$

where $\beta_{j,i}$ is the $i:th$ component of $\boldsymbol{\beta}_j$ and $F_{j,i}(x)$ is a transform aiming at increasing the linearity between f_j and x.

However, in this thesis, we wish to model the mass-frequency for transfers out of the company, while disregarding from any capital moved into the Company. From this, we require of f_t and f_u to be in the interval [0,1], allowing us to also interpret the f_t and f_u as the probability of a contract being transferred during one year. However, the right hand sides of the equations above are unbounded, meaning that they could sum up to be both less than zero and greater than one. This may be resolved by using either probistic or logistic regression, where the unbounded right hand side of the linear regression is mapped to the interval (0, 1) through the so called link function, which we denote as H. The fact that H maps to $(0, 1)$ rather than $[0, 1]$ has no practical meaning.

In the probistic case, we have that $H_{prob}(x) = \Phi^{-1}(x)$ and in the logistic $H_{log}(x) = exp(x)/(1+exp(x))$. As stated in [7], the choice of H is essentially a matter of taste, why we resort to choosing the one with best empirical fit to the data. Further, since the equity return is in the range $[-1, \infty]$, it does not allow for e.g. positive only $F_{j,1}(x)$. To resolve this, the percentage of return, r_e, is replaced by the return factor, namely $(1 + r_e)$ in the equations above. The regression models are thus transformed to;

$$f_t = H_t(\beta_{t,1} + \beta_{t,2}F_{t,1}(1 + r_e) + \beta_{t,3}F_{t,2}(r_{10}) + \beta_{t,4}F_{t,3}(r_{\frac{1}{4}}) + X_t) \tag{8}$$

and

$$f_u = H_u(\beta_{u,1} + \beta_{u,2}F_{u,1}(1 + r_e) + \beta_{u,3}F_{u,2}(r_{10}) + \beta_{u,4}F_{u,3}(r_{\frac{1}{4}}) + X_u) \quad (9)$$

where H_t is the link function in the traditional life case and H_u in the unit linked case. It is noteworthy that logistic and probistic regression is usually used in a binary context. The notion that both f_u and f_t originate from a process where a contract is either transferred or not, namely a binary process, therefore supports the choice of link functions.

7.1.2 Model estimation

In the estimation of parameters $\boldsymbol{\beta}_j$ and selection of functionals $F_{j,i}$ and H_i, as specified in 8 and 9, a regression method has been used. In order to allow for heteroscedasticity and reduce the sensitivity for outliers, the method decided to be used was MATLABs implementation of iteratively reweighed least squares (IRLS).

To begin with, a number of candidate transformations of the dependent variables, $F_j, i(x)$ are suggested. The functionals have been chosen based on their properties and the believed properties of $(1 + r_e)$, r_{10} and $r_{1/4}$. Below, a list of these may be found together with a motivation of their choice.

1. No transform, $\mathrm{F}_{i,j}(x) = x$. Obviously, the degree of linearity between the original variables and the transfer frequencies must be investigated. The effect of the transformation may be seen as the blue line in Figure 18.

2. Squared, $F_{i,j}(x) = x^2$. For the long and short interest rates, with values of magnitude on the first fourth of the unit interval, x^2 allows for a non-linear monotone compression of the observation spectra. This compressions results in possible outliers being more pronounced, and thus given less weight in the robust regression. For the equity return factor, ranging roughly from 0.5 to 1.5, x^2 instead provides a monotone non-linear decompression of the space. The decompression is greater for values over 1, which thus also accounts for the fact that the equity returns are most often modeled to have a positive expected value. It also results in outliers becoming less pronounced and thus given more weight in the robust regression. The effect is further described by the red line in Figure 18.

3. Logarithm, $F_{i,j}(x) = log(x)$. The effect of taking the logarithm on the independent variables is the opposite of that of taking the square. Instead of compression we get decompression for the interest rates and instead of decompression we get a slight compression of the equity return factors, as may be seen with the green line in Figure 18.

4. The discount factor, $F_{i,j}(x) = 1/(1 + x)$. This transform has been included because it has an economic interpretation as the discount factor, namely the factor with which future cash-flows is multiplied with to assess its value at present day. The black line in Figure 18 indicates that the transformation provides an inverse slight compression for both equity returns and the equity return factor.

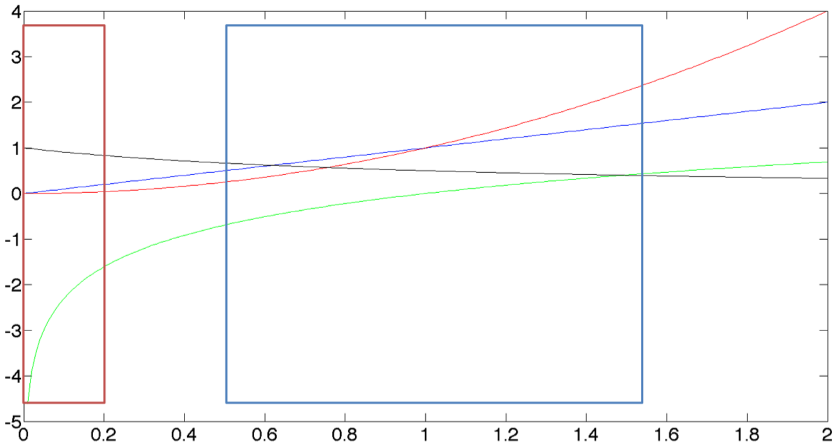

Figure 18: The effect of applying the candidate transforms. The blue line is $F(x) = x$, the red $F(x) = x^2$, the green $F(x) = log(x)$ and the black line is $F(x) = 1/(1 + x)$. The red box indicates the area of the values for r_{10} and $r_{1/4}$ and the blue box the area of the values of $1 + r_e$.

The final selection of which $F_{i,j}$ to use for which independent variable and whether to perform logistic or probistic regression has been done based on the empirical fit of all combinations of the above candidates. The selection process begins with that the n_i data points for $f_j, 1+r_e, r_{10}$ and $r_{1/4}$ are used in the regressions presented in 8 and 9. The regressions are run for every combination of the link functions and independent variable transforms, yielding a total of 128 linear models. When selecting among these 128 candidate models, the following decision process is used;

1. The model with largest value of the measure R^2 is selected. This is motivated by the fact that the models will be used to predict future values of the mass transfer frequencies, given the future observations of dependent variables. The selection corresponds to maximizing how well the linear combination of the functionals describe the variation in the transfer frequencies by varying how the included variables are transformed.

2. The residuals of the selected model are analyzed and it is investigated how well the errors could be modeled as normally distributed with mean equal or close to zero. If this is not the case, the model with next highest R^2 is investigated, and so on.

The final models fitted will be denoted as M_t and M_u for traditional life insurance and unit linked respectively. From the process above we get the selected functional forms $\hat{F}_{i,j}$ and $\hat{H}_j$ and the estimated coefficients $\hat{\boldsymbol{\beta}}_{\boldsymbol{j}}$, leaving only the exogenous parameters X_t and X_u to calibrate. As M_t and M_u have partly been selected on the basis of their respective residuals similarity to a normally distributed sample, fitting normal distributions to their residuals is deemed to work as a good approximation of X_t and X_u. The residuals specified by

$$\hat{r}_t = \hat{H}_t^{-1}(f_t) - [\hat{\beta}_{t,1} + \hat{\beta}_{t,2}\hat{F}_{t,1}(1+r_e) + \hat{\beta}_{t,3}\hat{F}_{t,2}(r_{10}) + \hat{\beta}_{t,4}F\hat{F}_{t,3}(r_{\frac{1}{4}})] \quad (10)$$

and

$$\hat{r}_u = \hat{H}_u^{-1}(f_u) - [\hat{\beta}_{u,1} + \hat{\beta}_{u,2}\hat{F}_{u,1}(1+r_e) + \hat{\beta}_{u,3}\hat{F}_{u,2}(r_{10}) + \hat{\beta}_{u,4}\hat{F}_{u,3}(r_{\frac{1}{4}})] \quad (11)$$

are calculated and normal distributions are fitted to the sets, using maximum likelihood estimation. From the MLE process, parameters as well as their 99.5% confidence intervals for the fitted distributions are extracted, these include

1. The fitted mean for the traditional life insurance case, $\hat{\mu_t}$ and its 99.5% confidence interval $[\hat{\mu}_t^l, \hat{\mu}_t^u]$.

2. The fitted mean for unit-linked insurance case, $\hat{\mu}_u$ and its 99.5% confidence interval $[\hat{\mu}_u^l, \hat{\mu}_u^u]$.

3. The fitted standard deviation for the traditional life insurance case, $\hat{\sigma}_t$ and its 99.5% confidence interval $[\hat{\sigma}_t^l, \hat{\sigma}_t^u]$.

4. The fitted standard deviation for unit-linked insurance case, $\hat{\sigma}_u$ and its 99.5% confidence interval $[\hat{\sigma}_u^l, \hat{\sigma}_u^u]$.

In the Results section, the univariate behavior of f_t and f_u in the simulation setting is investigated for the parameter combinations $[\hat{\mu}_i, \hat{\sigma}_i]$, $[\hat{\mu}_i^l, \hat{\sigma}_i^l]$, $[\hat{\mu}_i^l, \hat{\sigma}_i^u]$, $[\hat{\mu}_i^u, \hat{\sigma}_i^l]$ and $[\hat{\mu}_i^u, \hat{\sigma}_i^u]$, for $i = t, u$. The combinations that generate the heaviest tail of the simulated empirical distributions are then used in further result generation. These worst case parameter pairs are denoted $(\hat{\hat{\mu}}_i, \hat{\hat{\sigma}}_i)$ and the the final result form of the model for the exogenous variables is given by

$$X_t \in N(\hat{\hat{\mu}}_t, \hat{\hat{\sigma}}_t^2)$$

and

$$X_u \in N(\hat{\hat{\mu}}_u, \hat{\hat{\sigma}}_u^2).$$

A visual overview of the model estimation process may be seen in Figure 19, where it is also indicated if the data used originates from external or internal sources.

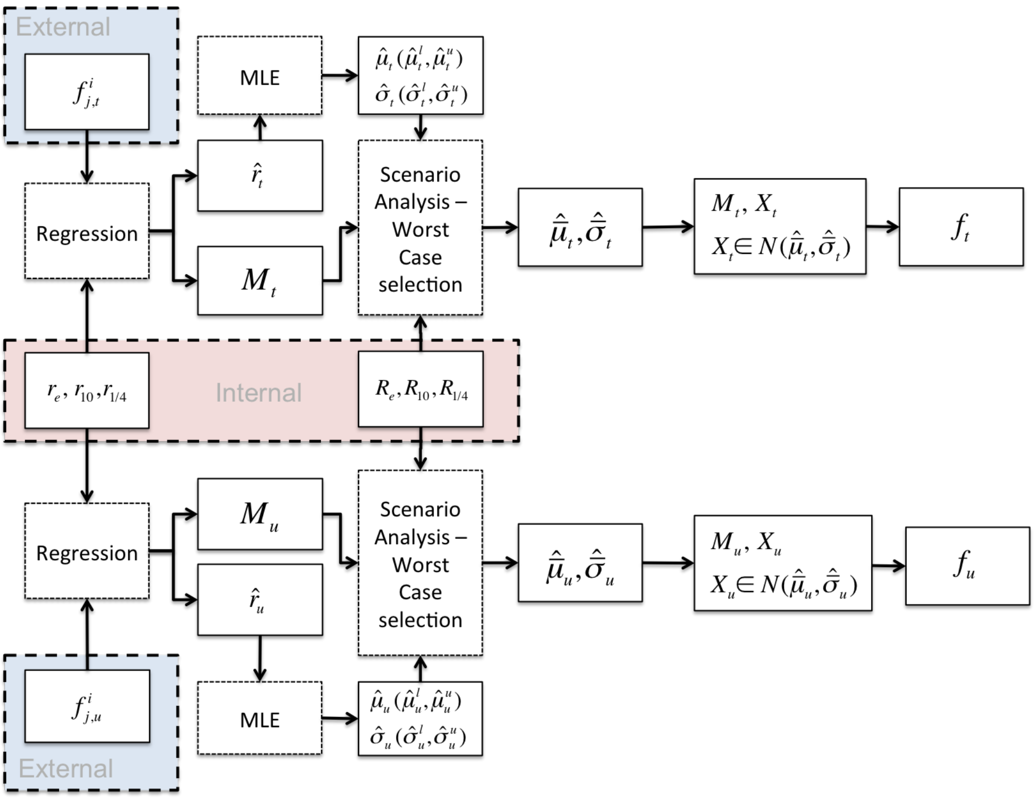

Figure 19: Process chart of the modeling of mass transfer frequency risk factors. White boxes with dashed borders are calculations and white boxes with solid borders are results.

7.1.3 Univariate and dependence behavior

Using the simulated data sets $R_e = \{r^l_{e,sim}\}^L_{l=1}$, $R_{10} = \{r^l_{10,sim}\}^L_{l=1}$ and $R_{\frac{1}{4}} = \{r^l_{\frac{1}{4},sim}\}^L_{l=1}$, the distributional and dependence behavior the model for f_t and f_u may be investigated. This is done by first simulating L samples of X_t and X_u and then using 8 and 9 to generate the sample sets; $F_t = \{f^l_{t,sim}\}^L_{l=1}$ and $F_u = \{f^l_{u,sim}\}^L_{l=1}$. Further, the F_t and F_u which are generated with the worst case parameters for X_t and X_u are denoted F^{WC}_t and F^{WC}_u. To assess the univariate distributional behavior histograms as well as empirical cdf plots have been used. Further, the dependence structures between the pairs $(F^{WC}_t, R_e), (F^{WC}_u, R_e), (F^{WC}_t, R_{10})$ and (F^{WC}_u, R_{10}) are investigated using scatter plots and plots of tail dependencies.

7.2 Risk factors C & D

7.2.1 Model form

Risk factors C and D aim to describe the change over the coming year in the assumptions for the average yearly future transfer rates for contracts with negative and positive surrender strain respectively. Note that the negative and positive surrender strain contracts are approximated by traditional life insurance and unit linked contracts. The Company has a current decision process for putting a number on the mentioned assumptions. Because of this, the risk assessed with these risk factors will be the variability in the result of this decision process.

The process briefly follows the steps

1. The arithmetic average of the observed transfer frequencies for the Company from the K last years is calculated. This may be stated as
$$\bar{f}^i_N = \frac{1}{K}\sum_{j=0}^{K-1} f^{obs,i}_{N-j},$$
where $f^{obs,i}_{N-j}$ is the observed transfer frequency for j years before N and N is the latest available observation, for business line i. When this paper was written, $N = 2013$, while K differs for traditional life and unit-linked insurance.

2. The company assumes that an economy with full transfer rights has a higher rate of transfers than those the Company experience when acting in the Swedish market, with limited transfer rights. On the

basis of this assumption $\bar{f}_N$ is adjusted upwards with s_N^{obs}, in order to account for the prospect of full transfer rights being introduced into future Swedish legalization.

3. The final assumption for future transfer rates may then be expressed as
$$L_N = \bar{f}_N^i + Z,$$
where Z is a positive random variable. Z aims at describing the difference over a year between a market with full transfer rights, such as Norway's, and the Company when acting with limited transfer rights in Sweden.

Based on the above described process, the following model for calculation of average future yearly rate, one year into the future, has been proposed;

$$L_{N+1} = \bar{f}_{N+1} + w_1 Z + w_2 0. \tag{12}$$

In 12, $\bar{f}_{N+1}$ is the K-year arithmetic mean, as calculated one year into the future. Since there exists no observation for the transfer frequency one year into the future, f_{N+1}^i, the frequency is approximated by the one-year mass transfer model described above, namely $f_j, j \in \{t, u\}$. The parameters w_1 and w_2 are included to account for the fact that f_j already is an observation from an economy with full transfer rights, eliminating the need for the Z-adjustment. The first $K-1$ elements in $\bar{f}_{N+1}$ however are observations from the Company when operating in Sweden, thus requiring the Z-adjustment. By this, w_1 is set to $\frac{K-1}{K}$ and w_2 to $\frac{1}{K}$, giving 12 its final form of

$$L_{N+1}^i = \frac{1}{K}(\sum_{j=0}^{K-2} f_{N-j}^{obs,i} + f_{N+1}^i) + \frac{K-1}{K} Z. \tag{13}$$

In the setting of 13, the estimation process is limited to estimate Z for traditional life insurance, Z_t, and unit-linked insurance Z_u respectively.

It is worth reflecting over one further aspect. If the transfer frequency for a specific year is high, it may be the case that those policyholders most prone to transfer their insurance capital have left the policyholder pool. This fact could be represented in the estimation of L_{N+1}^i by introducing the weight $\gamma_i(L_N^i, f_{N+1}^i, K)$ for f_{N+1}^i in its model. It should then be made sure that γ_i equals one when $f^i \in (0, L_N^i]$ and is decreasing for $f^i \in (L_N^i, 1)$. Then, γ_i indicates that the more f_{N+1}^i deviates from the Company's present assumption for future yearly transfer rate, the less is f_{N+1}^i weighed when estimating L_{N+1}^i. On the basis of this, the alternate model

$$L^i_{N+1} = \frac{1}{K}\sum_{j=0}^{K-2} f^{obs,i}_{N-j} + \gamma(L^i_N, f^i_{N+1}, K) f^i_{N+1} + \frac{K-1}{K} Z \tag{14}$$

would be formed, in which case the estimation required is extended to estimation of Z_t, Z_u and γ_i. In this thesis, the estimation process is limited to that of 13, but the effect of a possibly suitable γ_i, namely

$$\gamma_1(L^i_N, f^i_{N+1}, K) = \frac{(1 - f^i_{N+1})^2 + L^i_N}{K}$$

is investigated and presented.

7.2.2 Model estimation

The estimation of the distributional properties of Z_t and Z_u has been made using the observation sets $S_t = \{s^l_t\}^{n_t}_{l=1}$ and $S_u = \{s^l_u\}^{n_u}_{l=1}$, described in the data subsection of the thesis. When selecting which distributions to fit, a few wished properties of Z_t and Z_u have been taken into consideration, these are presented below.

1. Since the Company assumes that the transfer frequency is always higher for full transfer rights than for limited ones, $Z_t > 0$ and $Z_u > 0$ is desired.

2. Additionally, Z_t and Z_u represent a difference between two entities that are bounded by $(0, 1)$, by which, together with the above, requires that $Z_t \in (0, 1)$ and $Z_u \in (0, 1)$.

By inspection of histograms and empirical distribution functions of S_t and S_u, the shape of the beta distribution is deemed to fit both samples. However, as S_t and S_u both contain negative values, the sets $S'_t = S_t + (1+\varepsilon)|min(S_t)|$ and $S'_u = S_u + (1 + \varepsilon)|min(S_u)|$ are introduced for some small ε. This transformation is to ensure that $S'_t > 0$ and $S'_u > 0$ which together with investigating that $S'_t < 1$ and $S'_u < 1$ allows for fitting a beta distribution. Using MLE, the random variables

$$N_t \in Beta(\hat{\gamma}_t, \hat{\kappa}_t) \text{ and } N_u \in Beta(\hat{\gamma}_u, \hat{\kappa}_u)$$

are fitted to S'_t and S'_u. N_t and N_u are then shifted back to the initial positions of the data they were fitted after and, to ensure Z_t and $Z_u \geq 0$, truncated at zero, giving

$$Z_t = max(N_t - |min(S_t)|, 0) \tag{15}$$

and

$$Z_u = max(N_u - |min(S_u)|, 0) \tag{16}$$

Practically, the simulation from Z_i is done as

1. An observation u from $unif(0,1)$ is generated.

2. Using the quantile transform, an observation from N_i, n_i is generated as $n_i = F_{N_i}^{-1}(u)$.

3. Finally, the observation is transformed to an observation from Z_i as $z_i = min(n_i - |min(S_i)|, 0)$.

As in the model estimation of risk factors A and B, the parameters from the MLE estimation and their respective 99.5 % confidence intervals are extracted from the fitting process. The MLE estimates are used in the model and the confidence intervals attained when estimating the parameters of the beta distributions fitted to N_t and N_u, namely $(\hat{\gamma}_i^l, \hat{\gamma}_i^u)$, $(\hat{\gamma}_i^l, \hat{\gamma}_i^u)$, $(\hat{\kappa}_i^l, \hat{\kappa}_i^u)$ and $(\hat{\kappa}_i^l, \hat{\kappa}_i^u)$, $i = \{t, u\}$, are used in the investigation of the models robustness to the beta-estimation.

The final form of the model for the assumption of the future average yearly transfer frequency, made one year into the future is, for traditional life insurance and unit-linked respectively,

$$L_{N+1}^t = \frac{1}{K_t}(\sum_{j=0}^{K_u-2} f_{N-j}^{obs,t} + f_{N+1}^t) + \frac{K_t - 1}{K_t} Z_t \tag{17}$$

and

$$L_{N+1}^u = \frac{1}{K_u}(\sum_{j=0}^{K_u-2} f_{N-j}^{obs,u} + f_{N+1}^u) + \frac{K_u - 1}{K_u} Z_u. \tag{18}$$

In the event that the data sets S_t and S_u, and subsequently S_t' and S_u', are deemed quantitatively or qualitatively insufficient for reliable statistical estimation, the introduction of expert judgment to some degree may be in order. In this thesis, the following, very briefly investigated weighting for this is suggested;

$$Z_i^{adj} = \alpha s_{obs}^i + (1 - \alpha) Z_i, \alpha \in [0, 1], \tag{19}$$

where s_{obs}^i is the Company's latest realized upwards adjustment of $\bar{f}_N$, when L_N is calculated. By choosing α, the weight of the actual adjustment level of $\bar{f}_N^i$, as determined by expert judgement, may be adjusted. In the results subsection, the effect on L^i_{N+1} when $\alpha = 2$, together with when γ_1 is used, is presented. Note that since s^u_{obs} is missing, it has been approximated by s^t_{obs}.

A visual overview of the model estimation process may be seen in Figure 20, where it is also indicated if the data used originates from external or internal sources.

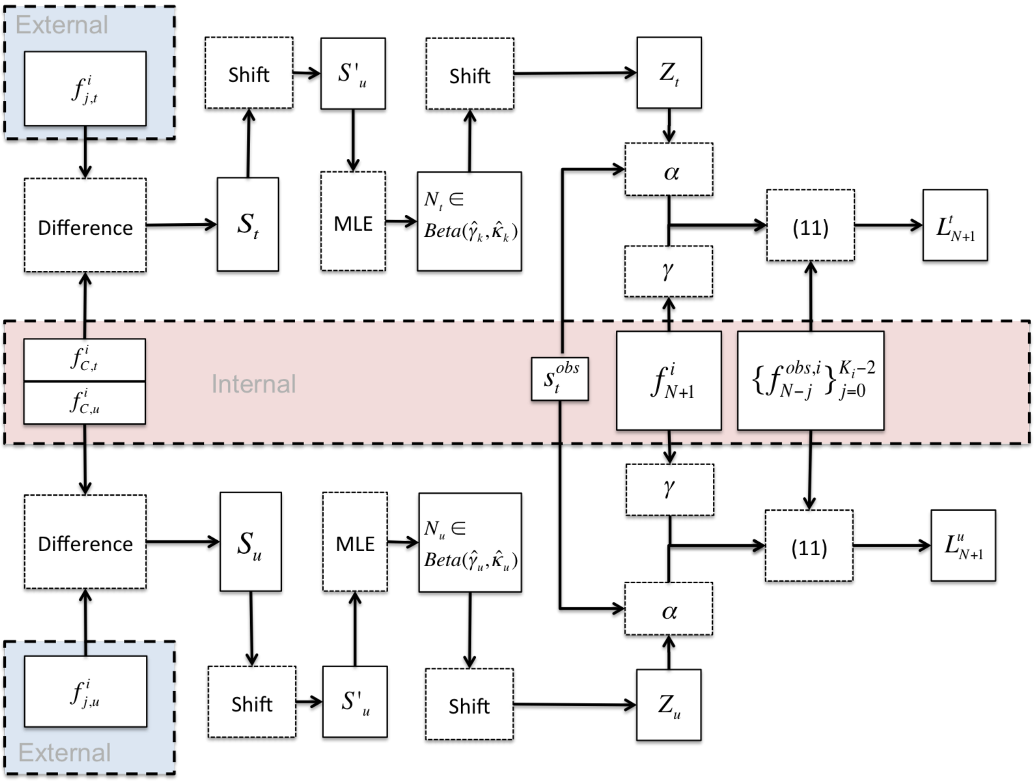

Figure 20: Process chart of the modeling of long term assumption for yearly frequencies. White boxes with dashed borders are calculations and white boxes with solid borders are results.

7.2.3 Univariate and dependence behavior

The sets F^{WC}_t and F^{WC}_u are used, together with L simulations of Z_t and Z_u, to generate the scenario sample sets $L^{sim}_t = \{L^{t,l}_{N+1,sim}\}^L_{l=1}$ and $L^{sim}_u = \{L^{u,l}_{N+1,sim}\}^L_{l=1}$. The same methods that have been mentioned in subsection 7.1.3 are used in the analysis of the models univariate and dependence behavior.

7.3 Discussion

The decision of using a linear model for P_t and P_u is made on the initial assumption that the description power possible to extract from r_e,r_{10} and $r_{1/4}$, given the low number of data points, is limited to begin with, and that marginal behavior as well as validity is easier to assess for a linear model than for a non-linear model.

Further, the fact that the number of investigated transforms is limited may induce some room for improvement of the linear fit. However, it may also be seen in the Results section that several combinations of transforms generate R^2 of close to equal magnitude, making the exact choice of transforms less important. Additionally, the notion of using R^2 as measure for the goodness of the model may be questioned. However, since the number of independent variables is held constant, a higher R^2 is not caused by an increase in the number of independent variables, which is a common problem with the measure.

The assumption that the set S_i consists of n_i^s copies from the same random variable, describing the size of the adjustment when forming L_N, is identified as somewhat bold. Firstly, the adjustment of $\bar{f}$ to L_N is not entirely based on possible future introduction of full transfer rights, but also of other, unobserved or unobservable factors. This might result in the fact that the S_i is not a good enough proxy for the adjustment, as it is exactly the difference between a market with full transfer rights and one without. Secondly, even if S_i is a good enough proxy of the adjustment, the fact that the Swedish legalization on transfer rights has varied over the different yearly observations while the Norwegian legalization has been fix, introduces the risk that S_i are not from the same random variable. The remedy for this is in this thesis, as suggested, to include a certain weight of the latest observed adjustment, s_N^{obs}, i.e. an expert judgment, to tone down the weight of the somewhat uncertain statistical estimation.

8 Results

8.1 Risk factors A & B -

8.1.1 Random variable estimation

In 21, the R^2 for all 128 candidate models for M_t are marked together with the chosen M_t. Table 4 lists the resulting transforms for M_t.

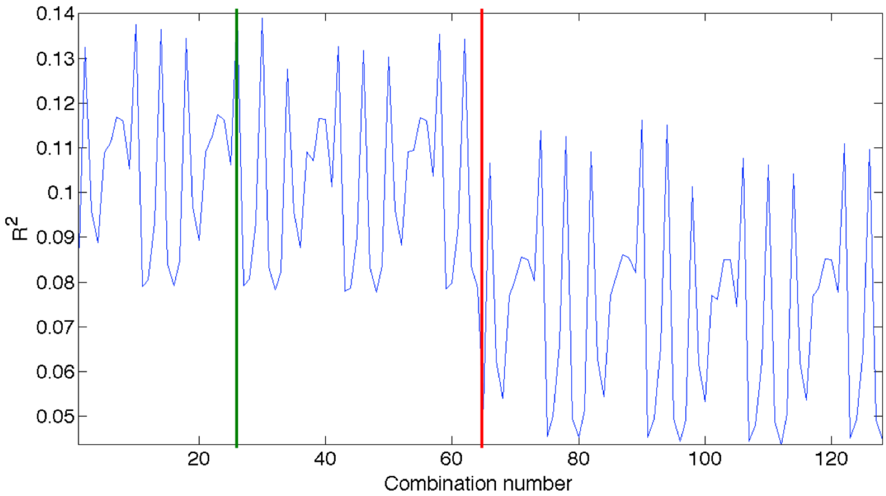

Figure 21: R^2 coefficient for the 128 regression combinations, for traditional life insurance. The green line shows the chosen model, M_t and the red line separates regressions where $\hat{H}_t$ is the logit (left of red line) and probit (right of red line).

Table 4: Resulting transforms the transfer frequencies, equity returns and interest rates for traditional life insurance.

Variable	Transform
Transfer frequency, f_t	Logit, $\hat{H}_t(x) = log(\frac{x}{1-x})$
Equity returns, r_e	Squared, $\hat{F}_{t,1}(x) = x^2$
10-year interest rate, r_{10}	Logarithm, $\hat{F}_{t,2}(x) = log(x)$
3-month interest rate, $r_{\frac{1}{4}}$	Squared, $\hat{F}_{t,3}(x) = x^2$

In Figure 22, the upper row shows the Tukey-Anscombe plot for the residuals received when using M_t and the QQ-plot when comparing the residuals with normal quantiles. The lower row shows the histogram of the residuals together with MLE-fitted pdf:s, with and without the two most extreme negative residuals and the empirical cdf, together with the cdf:s of the fitted distributions mentioned.

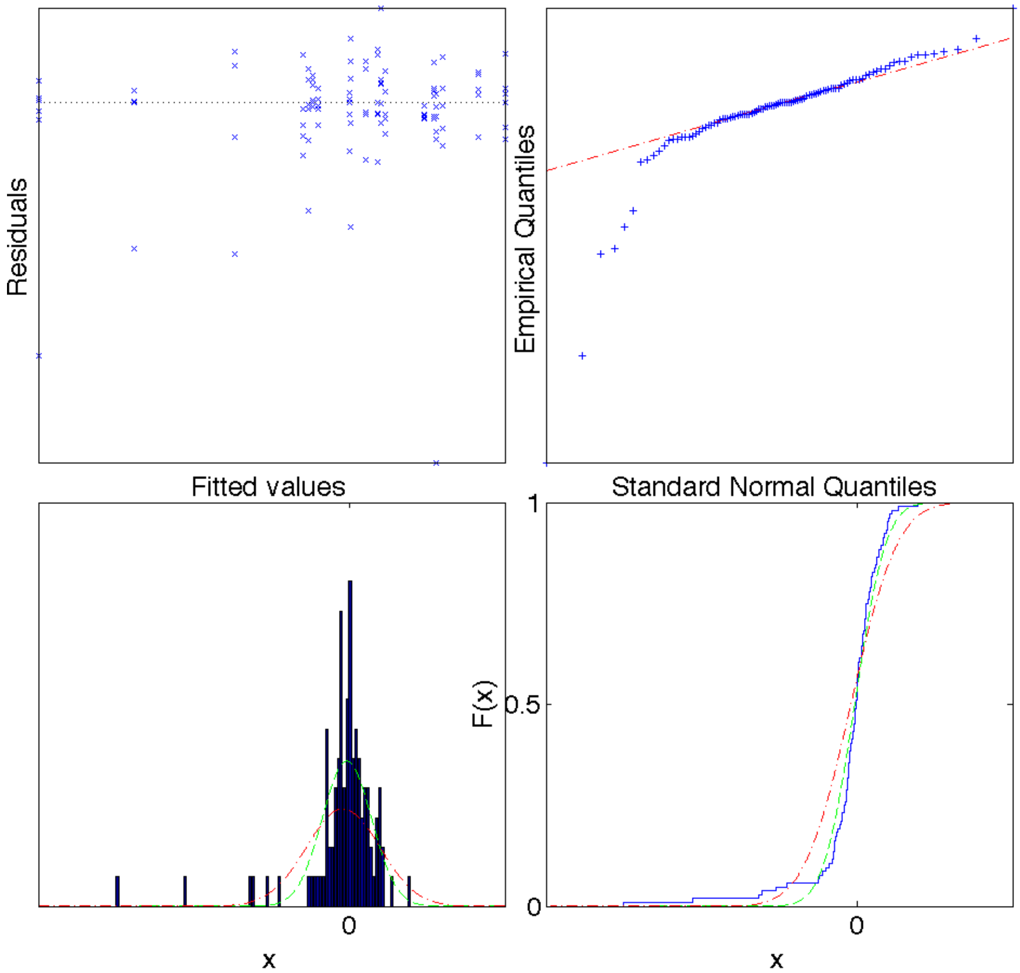

Figure 22: On the upper row, the Tukey-Anscombe (left), QQ-plot against normal quantiles (right) may be seen. On the lower row, the histogram (left) and the empirical cdf (right) for the residuals of M_t are found (both in solid blue). In both lower plots, the red dash-dotted lines show the pdf (left) and cdf (right) of the fitted normal pdfs when including all observations and the green dashed lines the pdf and cdf fitted when removing the two largest negative outliers.

In 23, the R^2 for all 128 candidate models for M_u are marked together with the chosen M_u. Table 5 lists the resulting transforms for M_u.

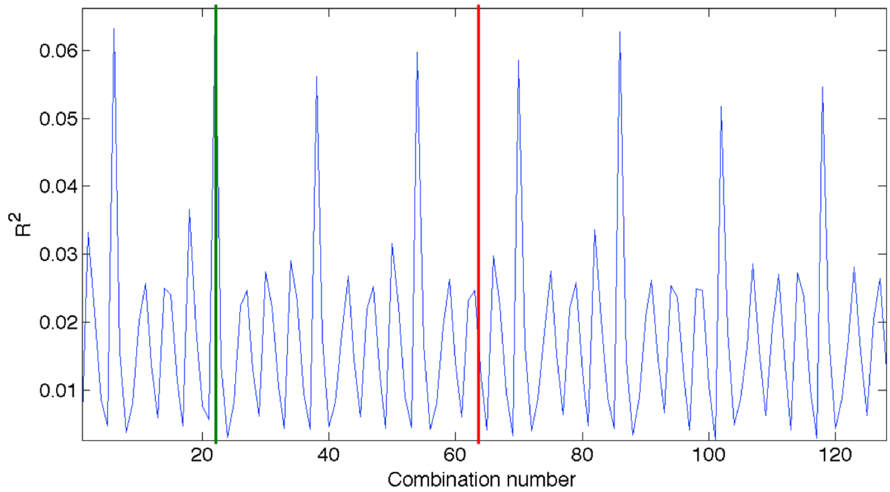

Figure 23: R^2 coefficient for the 128 regression combinations, for unit-linked insurance. The green line shows the chosen model and the red line separates regressions where $\hat{G}_u$ is the logit (left of red line) and profit (right of red line)

Table 5: Resulting transforms the transfer frequencies, equity returns and interest rates for unit-linked insurance.

Variable	Transform
Transfer frequency, f_u	Logit, $\hat{H}_u(x) = log(\frac{x}{1-x})$
Equity returns, r_u	Squared, $\hat{F}_{u,1}(x) = x^2$
10-year interest rate, r_{10}	Squared, $\hat{F}_{u,2}(x) = x^2$
3-month interest rate, $r_{\frac{1}{4}}$	Squared $\hat{F}_{u,3}(x) = x^2$

Figure 24 shows the same residual analysis plots as Figure 22, but for M_u.

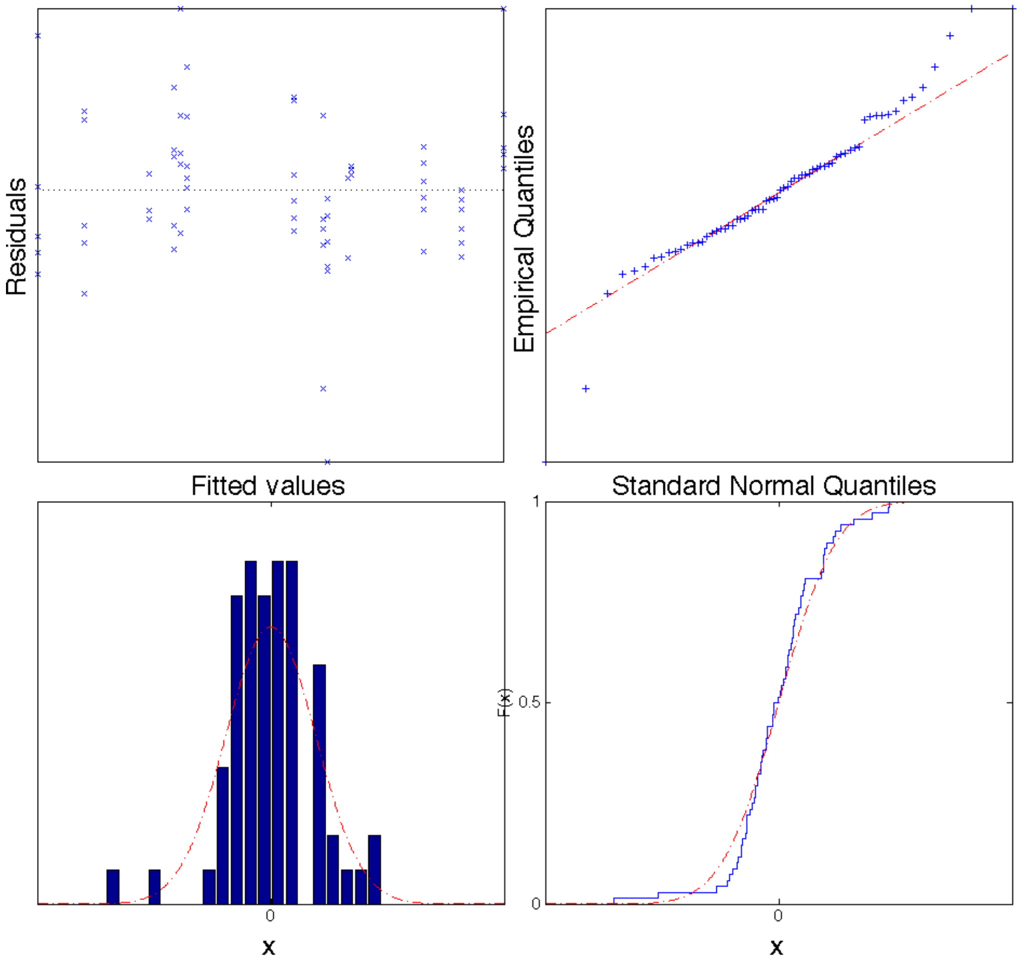

Figure 24: On the upper row, the Tukey-Anscombe (left), QQ-plot against normal quantiles (right) may be seen. On the lower row, the histogram (left) and the empirical cdf (right) for the residuals of M_u are found (both in solid blue). In both lower plots, the red dash-dotted lines show the pdf (left) and cdf (right) fitted to the residuals.

8.1.2 Simulation - Univariate properties

Figure 25 shows the empirical cdf of F_t for several parameter combinations for the distribution of X_t. The solid red line, namely the pair $(\hat{\mu}_t^u, \hat{\sigma}_t^u)$ gives the worst case pair of parameters, so $(\hat{\hat{\mu}}_t, \hat{\hat{\sigma}}_t) = (\hat{\mu}_t^u, \hat{\sigma}_t^u)$, the simulated F_t for this worst case distribution of X_t is from here on denoted as F_t^{WC}. The simulated set for the MLE-estimates of the parameters will be denoted F_t^E. Further analysis of F_t^{WC}:s deviation from F_t^E in terms of their histograms may be observed in Figure 26.

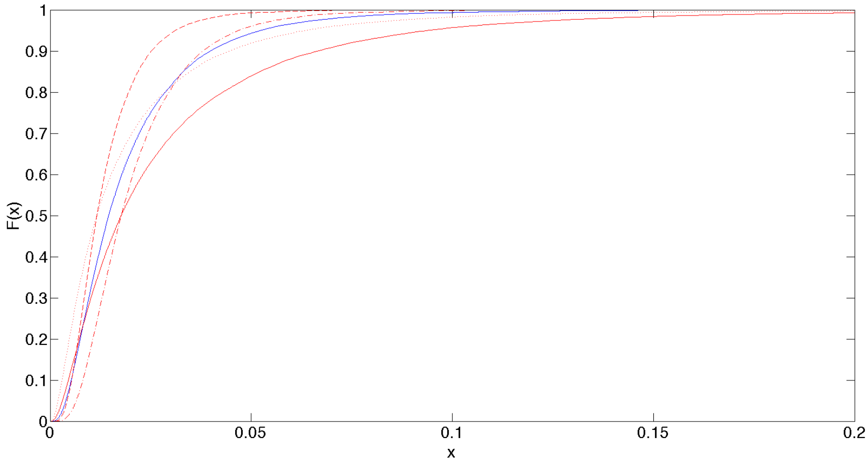

Figure 25: Empirical distribution functions obtained when the parameter combinations $(\hat{\mu}_t, \hat{\sigma}_t)$ (solid blue) , $(\hat{\mu}_t^l, \hat{\sigma}_t^l)$ (dashed red), $(\hat{\mu}_t^l, \hat{\sigma}_t^u)$ (dotted red), $(\hat{\mu}_t^u, \hat{\sigma}_t^l)$ (dot-dashed), and $(\hat{\mu}_t^u, \hat{\sigma}_t^u)$ (solid red) are used in the normal modeling of error term X_t. The pair $(\hat{\mu}_t^u, \hat{\sigma}_t^u)$ gives the fattest right tail for F_t, and are therefore used when obtaining further results.

Figures 27 and 28 show the same as described for F_t, but for F_u. Also in this case, we get the worst case parameter pair $(\hat{\hat{\mu}}_u, \hat{\hat{\sigma}}_u) = (\hat{\mu}_u^u, \hat{\sigma}_u^u)$ and introduce the notations F_u^{WC} and F_u^E.

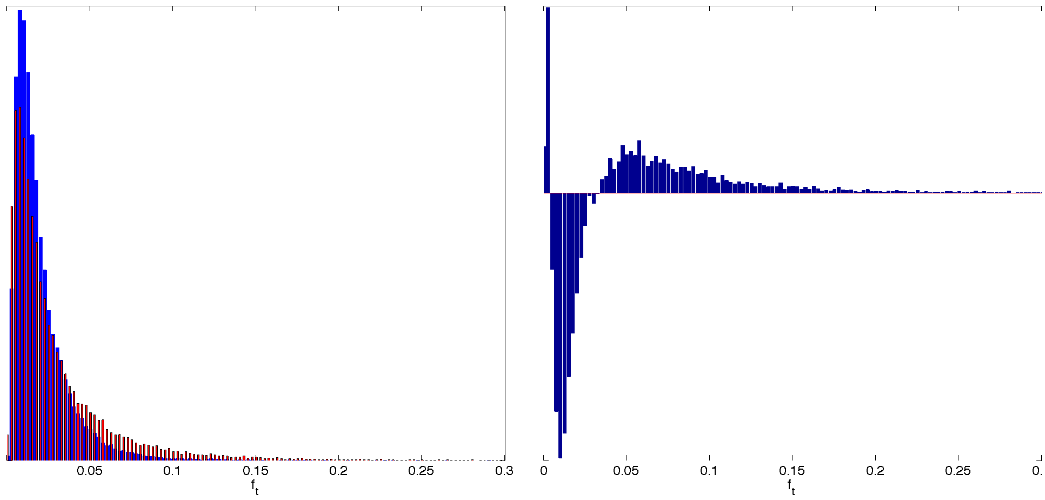

Figure 26: Left plot: Histogram for the simulated transfer frequencies when using the parameters $(\hat{\mu}_t, \hat{\sigma}_t)$ for X_t, F_t^E , (blue bars) and when using $(\hat{\mu}_t^u, \hat{\sigma}_t^u)$ (red bars), F_t^{WC}, which gives the heaviest right tail, as seen Figure 25. Right plot: Histogram for F_t^{WC} minus histogram for F_t^E, aims at assessing how the samples differ in their probability weight distribution, especially in the right tail.

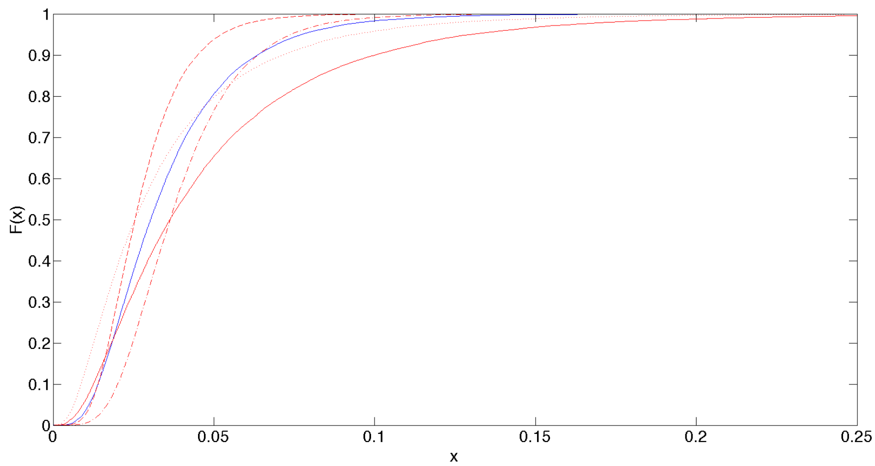

Figure 27: Empirical distribution functions obtained when the parameter combinations $(\hat{\mu}_u, \hat{\sigma}_u)$ (solid blue) , $(\hat{\mu}_t^l, \hat{\sigma}_u^l)$ (dashed red), $(\hat{\mu}_t^l, \hat{\sigma}_u^u)$ (dotted red), $(\hat{\mu}_u^u, \hat{\sigma}_u^l)$ (dot-dashed), and $(\hat{\mu}_u^u, \hat{\sigma}_u^u)$ (solid red) are used in the normal modeling of error term X_u. The pair $(\hat{\mu}_u^u, \hat{\sigma}_u^u)$ gives the fattest right tail for F_u, and are therefore used when obtaining further results.

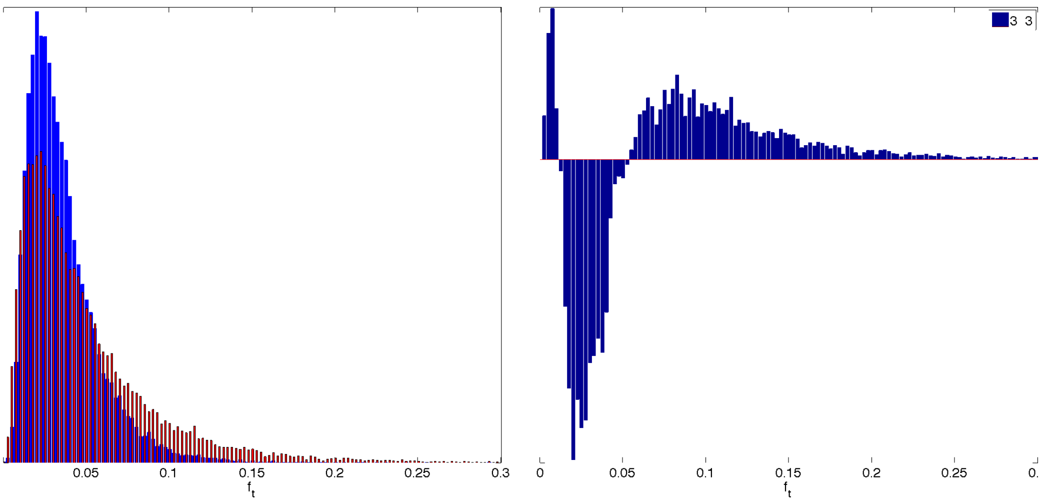

Figure 28: Left plot: Histogram for the simulated transfer frequencies when using the parameters$(\hat{\mu}_u, \hat{\sigma}_u)$ for X_u, F_u^E , (blue bars) and when using $(\hat{\mu}_u^u, \hat{\sigma}_u^u)$ (red bars), F_u^{WC}, which gives the heaviest right tail, as seen Figure 25. Right plot: Histogram for F_u^{WC} minus histogram for F_u^E, aims at assessing how the samples differ in their probability weight distribution, especially in the right tail.

Finally, Table 6 presents the 99.5% quantiles for F_t and F_u, for all the listed parameter combinations, from which one again may see that in both cases $(\hat{\hat{\mu}}, \hat{\hat{\sigma}}) = (\hat{\mu}^u, \hat{\sigma}^u)$.

Table 6: 99.5 % quantile of empirical cdfs.

	$(\hat{\mu}_u, \hat{\sigma}_u)$	$(\hat{\mu}^l, \hat{\sigma}^l)$	$(\hat{\mu}^l, \hat{\sigma}^u)$	$(\hat{\mu}^u, \hat{\sigma}^l)$	$(\hat{\mu}^u, \hat{\sigma}^u)$
F_t	0.1093	0.0562	0.1633	0.0835	**0.2298**
F_u	0.1278	0.0779	0.1832	0.1089	**0.2450**

8.1.3 Simulation - Dependence properties

In Figure 29, one may find scatter plots for the pairs (F_t^{WC}, R_e) and (F_t^{WC}, R_{10}). To assess whether the samples show any indication of stronger correlation in the combined worst cases, the upper tail dependence between F_t^{WC} and $-R_e$ (i.e. when F_t^{WC} is high and R_e is low) and F_t^{WC} and R_{10} (F_t^{WC} and R_{10} both high) may be seen in Figure 30.

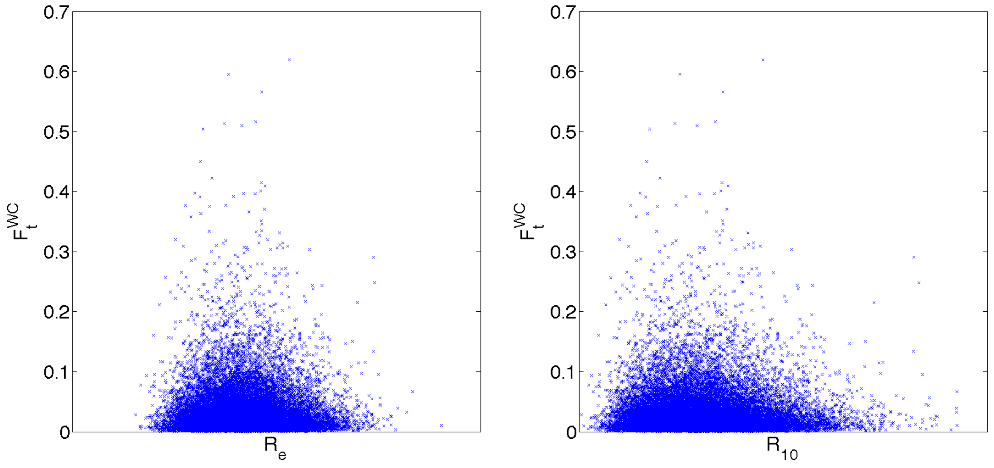

Figure 29: Scatter plots showing the correlation behavior between the F_t^{WC} and the R_e (left) and between F_t^{WC} and R_{10} (right).

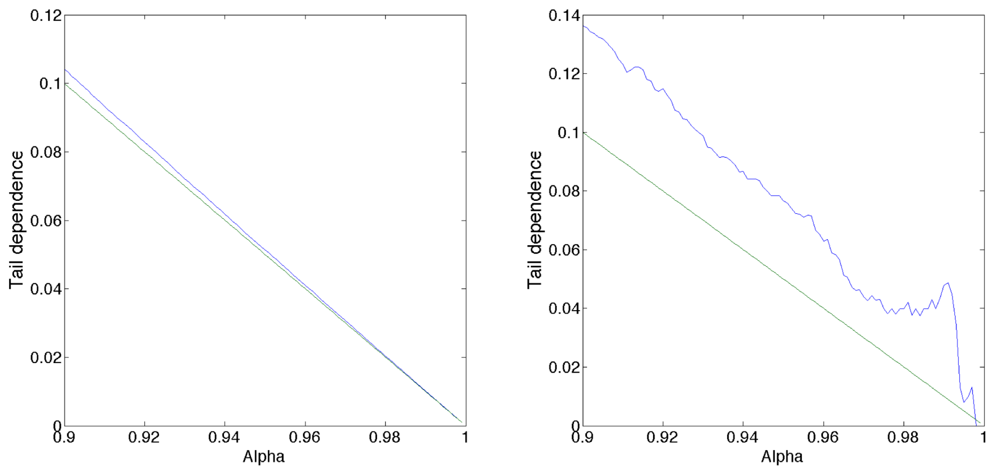

Figure 30: Upper tail dependence between F_t and $-R_e$ (left) and F_t and R_{10}.

Figures 31 and 32 contain the same information as described above, but for F_u^{WC}. Note that the same tail dependencies are relevant.

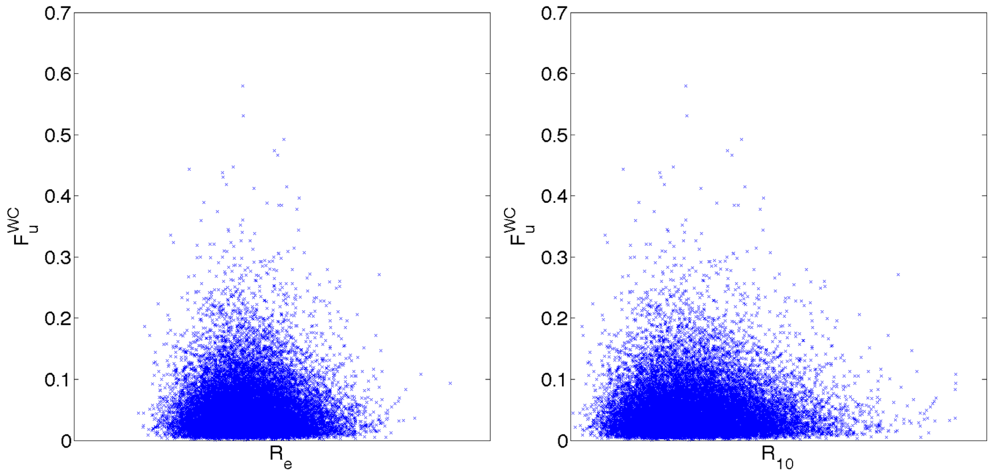

Figure 31: Scatter plots showing the correlation behavior between the F_{WC}^h and the R_E (left) and between F_u^{WC} and R_{10} (right) for the scenarios.

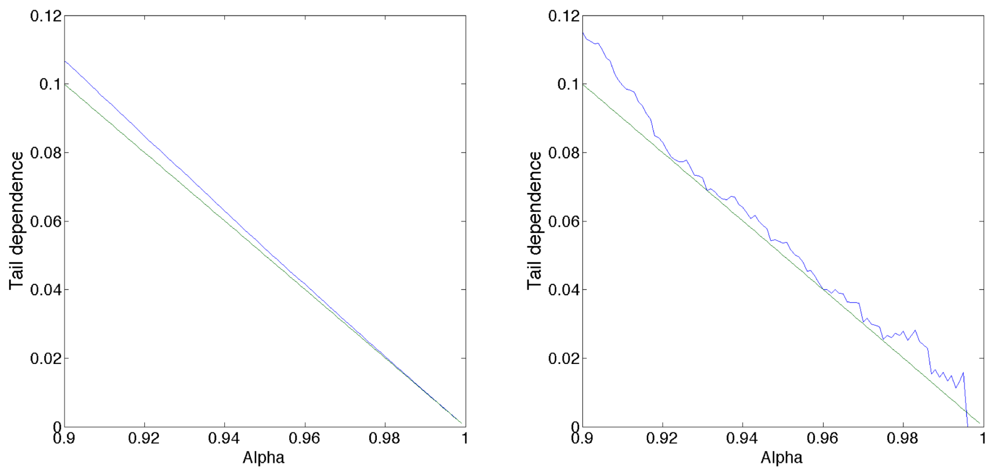

Figure 32: Upper tail dependence between F_u^{WC} and $-R_e$ (left) and F_u^{WC} and R_{10}

8.2 Risk factors C & D

8.2.1 Random variable estimation

As described in the methodology section, the parameters of the beta distribution fitted to S_t', are estimated with MLE estimation. Some analysis of the estimated beta distribution fit to the S_t' may be seen in Figure 33, where the histogram is plotted with the pdf, the empirical cdf with the cdf and two QQ-plots with the residuals of S_t' against two samples drawn from the fitted beta. The same analyisis for the beta distribution fitted to S_u' is found in Figure 34.

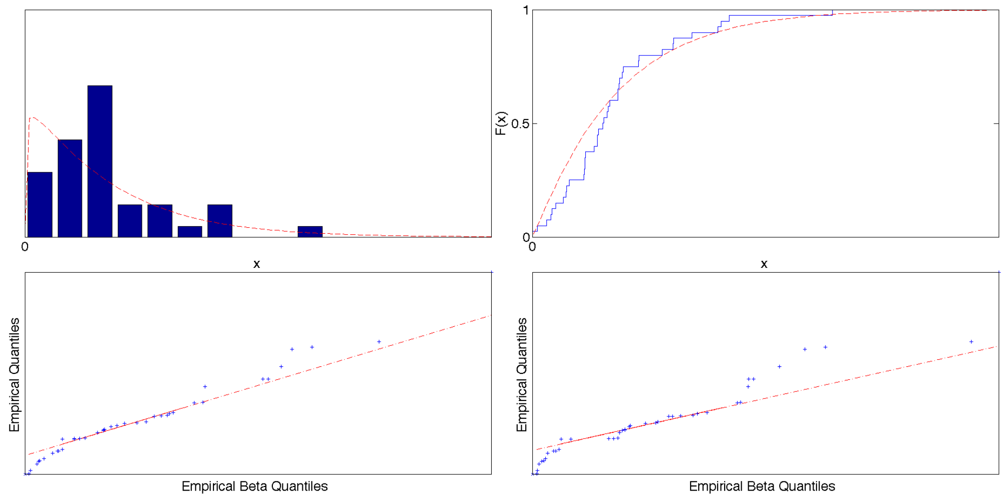

Figure 33: Upper left plot: The histogram for S_t' (blue bars) together with pdf of the MLE-fitted beta distribution (dashed red line). Upper right plot: Empirical cdf of S_t' together with cdf of MLE-fitted the beta distribution (dashed red line). The lower row shows QQ-plots with the quantiles of S_t' on the y-axes and quantiles of a two different samples from the MLE-fitted beta distribution on the x-axes.

8.2.2 Simulation - Univariate properties

The right hand side of Figure 35 shows the empirical distributions received for L_t^{sim} when no weight adjustment or expert judgement has been included into the model, when the MLE-estimates for S_t' and all combinations of their 99.5% confidence interval bounds are used. The left hand side shows the same when including $\gamma = \gamma_1$ and $\alpha = \frac{3}{4}$ in the model. In Table 7, the 0.05% and 99.5% quantiles of the cdfs for the unadjusted and the adjusted versions of L_t^{sim} may be seen. For both versions, the resulting empirical cdfs for all parameter pairs for the distribution of Z_t are also found in the table.

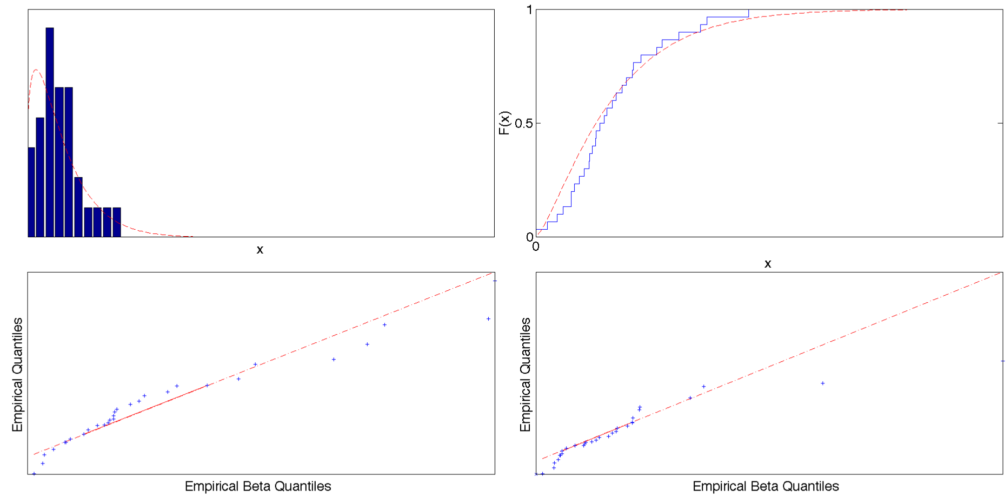

Figure 34: Upper left plot: The histogram for S'_u (blue bars) together with pdf of the MLE-fitted beta distribution (dashed red line). Upper right plot: Empirical cdf of S'_u together with cdf of MLE-fitted the beta distribution (dashed red line). The lower row shows QQ-plots with the quantiles of S'_u on the y-axes and quantiles of a two different samples from the MLE-fitted beta distribution on the x-axes.

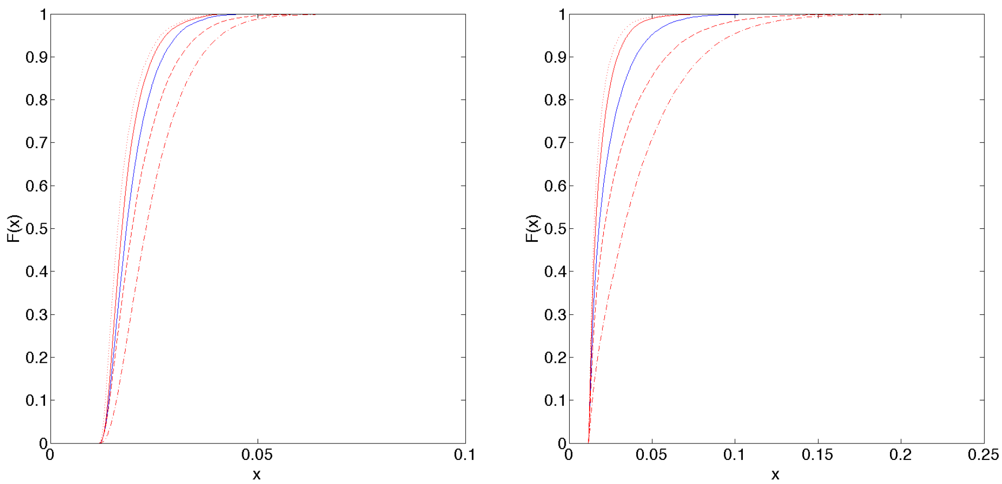

Figure 35: The empirical cdf of L_t^{sim} with adjustments $\alpha = 3/4$ and $\gamma = \gamma_1$ (solid blue, left plot) and the empirical cdf of L_t^{sim} without adjustments (solid blue, right plot). The remainder of the lines indicate the ecdf:s when the combinations $(\hat{\gamma}_t^l, \hat{\kappa}_t^l)$ (red dashed), $(\hat{\gamma}_t^l, \hat{\kappa}_t^u)$ (red dotted), $(\hat{\gamma}_t^u, \hat{\kappa}_t^l)$ (red dash-dotted), and $(\hat{\gamma}_t^u, \hat{\kappa}_t^u)$ (solid red) are used when estimating N_t and N_t^{adj}, for the negative surrender strain contracts.

Table 7: Quantiles for the empirical CDFs of L_t^{sim}, when not adjusted (upper rows) and when adjusted (lower rows).

	Quantile	$(\hat{\gamma}_t^e, \hat{\kappa}_t^e)$	$(\hat{\gamma}_t^l, \hat{\kappa}_t^l)$	$(\hat{\gamma}_t^l, \hat{\kappa}_t^u)$	$(\hat{\gamma}_t^u, \hat{\kappa}_t^l)$	$(\hat{\gamma}_t^u, \hat{\kappa}_t^u)$
Unadjusted	99.5%	0.0804	0.1267	0.0543	0.1538	0.0580
	0.5%	0.0117	0.0117	0.0117	0.0118	0.0117
Adjusted	99.5%	0.0401	0.0488	0.0368	0.0554	0.0376
	0.5%	0.0124	0.0123	0.0121	0.0129	0.0124

The right hand side of Figure 36 shows the empirical distributions received for L_u^{sim} when no weight adjustment or expert judgement has been included into the model, when the MLE-estimates for S_t' and all combinations of their 99.5% confidence interval bounds are used. The left hand side shows the same when including $\gamma = \gamma_1$ and $\alpha = 3/4$ in the model. In Table 8, the 0.05% and 99.5% quantiles of the cdfs for the unadjusted and the adjusted versions of L_u^{sim} may be seen. For both versions, the resulting empirical cdfs for all parameter pairs for the distribution of Z_u are also found in the table.

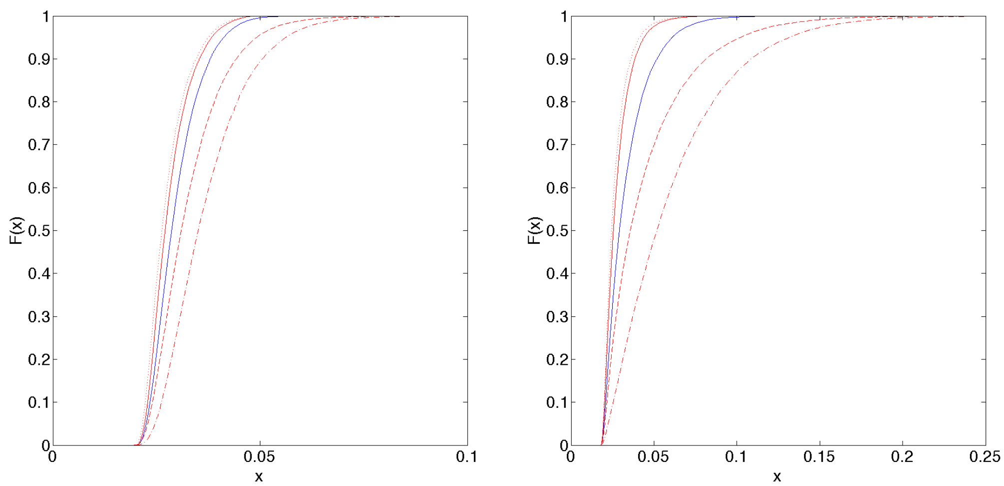

Figure 36: The empirical cdf of L_u^{sim} with adjustments $\alpha = 3/4$ and $\gamma = \gamma_1$ (solid blue, left plot) and the empirical cdf of L_t^{sim} without adjustments (solid blue, right plot). The remainder of the lines indicate the ecdf:s when the combinations $(\hat{\gamma}_u^l, \hat{\kappa}_u^l)$ (red dashed), $(\hat{\gamma}_u^l, \hat{\kappa}_u^u)$ (red dotted), $(\hat{\gamma}_u^u, \hat{\kappa}_u^l)$ (red dash-dotted), and $(\hat{\gamma}_u^u, \hat{\kappa}_u^u)$ (solid red) are used when estimating N_u and N_u^{adj} for contracts with positive surrender strain.

Table 8: 99.5% quantiles for the empirical CDFs of L_u^{sim}, when not adjusted (upper rows) and when adjusted (lower rows).

	Quantile	$(\hat{\gamma}_u^e, \hat{\kappa}_u^e)$	$(\hat{\gamma}_u^l, \hat{\kappa}_u^l)$	$(\hat{\gamma}_u^l, \hat{\kappa}_u^u)$	$(\hat{\gamma}_u^u, \hat{\kappa}_u^l)$	$(\hat{\gamma}_u^u, \hat{\kappa}_u^u)$
Unadjusted	Upper	0.0909	0.1631	0.0611	0.1961	0.0642
	Lower	0.0185	0.0187	0.0184	0.0191	0.0184
Adjusted	Upper	0.0497	0.0655	0.0449	0.0736	0.0456
	Lower	0.021	0.021	0.0205	0.0218	0.0208

8.2.3 Simulation - Dependence properties

For the analysis of the dependence properties, the MLE-estimates for the unadjusted version of the model have been used. Figure 37 shows the scatter plots for the pairs (L_t^{sim}, R_e) and (L_t^{sim}, R_{10}). In Figure 38, the lower tail dependence of (L_t^{sim}, R_e) (Case for co-dependence of low L_t^{sim} and low R_e) and $(L_t^{sim}, -R_{10})$ (Case for co-dependence of low (L_t^{sim}, R_e) and high R_{10}.

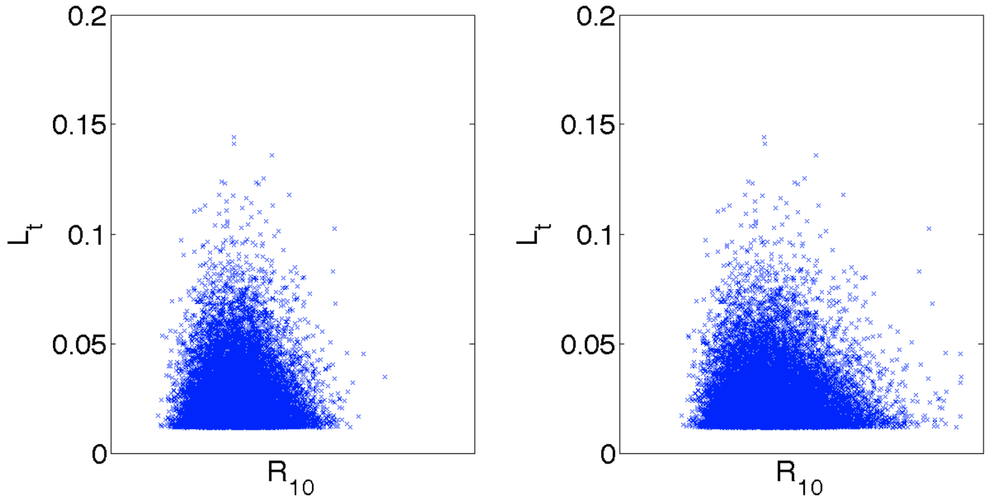

Figure 37: Scatter plots for the pairs (L_t^{sim}, R_e) (left) and (L_t^{sim}, R_{10}) (right).

For the positive surrender strain simulations, Figure 39 contains the scatter plots for (L_u^{sim}, R_e) and (L_u^{sim}, R_{10}). Further, the tail dependencies that are the worst for the Company in this case are; the upper tail-dependence of $(L_u^{sim}, -R_e)$ (high L_u^{sim}, low R_e) and the upper tail dependence of (L_u^{sim}, R_{10}) (High L_u^{sim} and high R_{10}. The assessment of these may therefore be found in Figure 40.

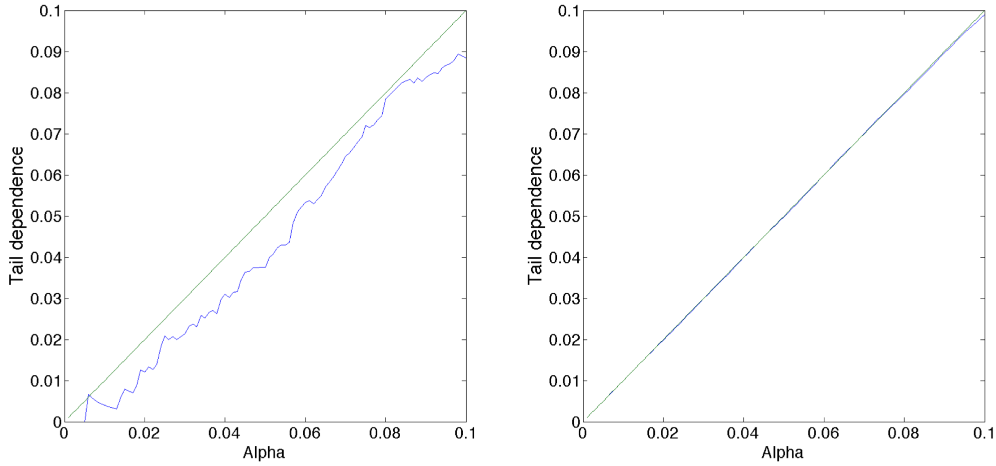

Figure 38: Lower tail dependence of L_t^{sim} and R_e (left) and if L_T^{sim} and $-R_{10}$ (right)

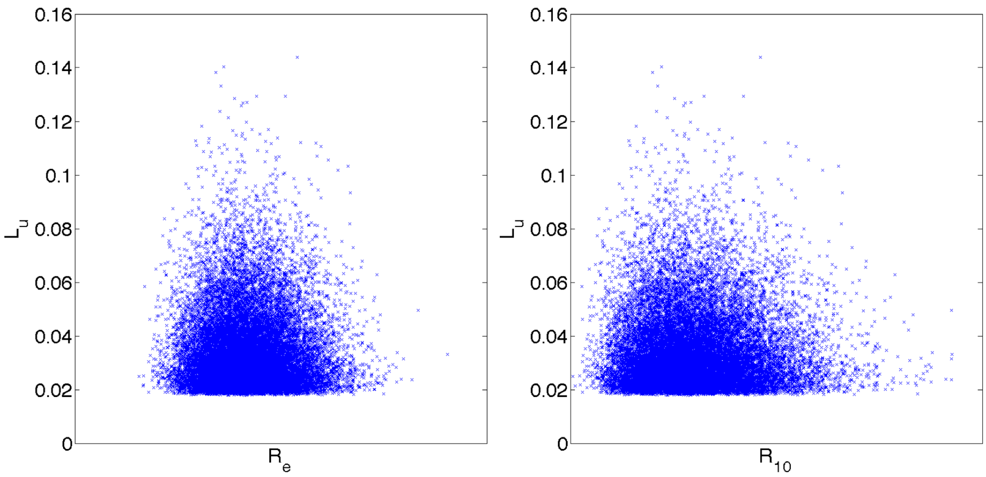

Figure 39: Scatter plots for the pairs (L_u^{sim}, R_e)(left) and (L_u^{sim}, R_{10}) (right)

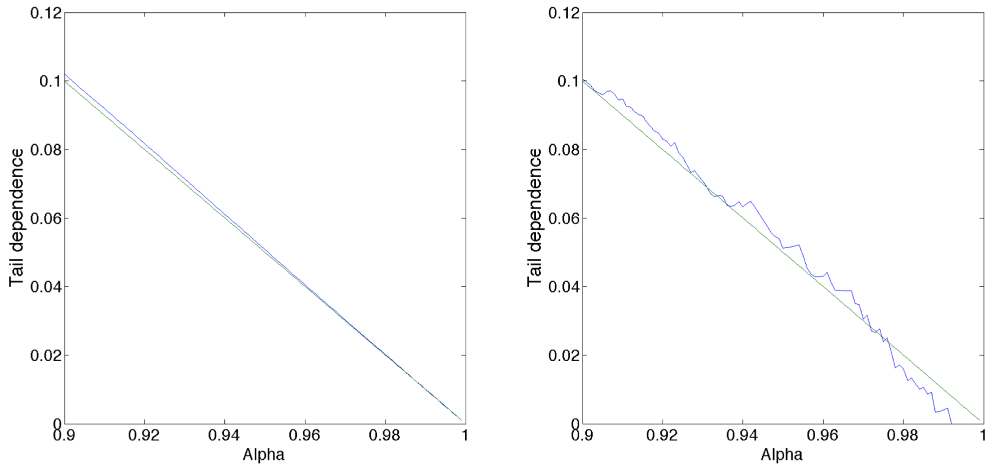

Figure 40: Upper tail dependence of L_u^{sim} and $-R_e$ (left) and upper tail dependence of if L_u^{sim} and R_{10}

8.3 Correlation measures

Finally, Table 9 shows the estimated Pearson's correlation coefficients and Kendall's tau for all the pairs investigated in respective dependence section. It is noteworthy that both correlation measures are negative between all risk factors and R_e and all positive between all risk factors and R_{10}.

Table 9: Estimated person's correlation coefficient (P) and Kendall's tau (K) for the simulated sample pairs of interest.

	Type	R_e	R_{10}
F_t^{WC}	P	-0.0815	0.0908
	K	-0.0662	0.0696
F_u^{WC}	P	-0.1701	0.0297
	K	-0.1254	0.0209
L_t^{sim}	P	-0.0374	0.0367
	K	-0.0345	0.0373
L_u^{sim}	P	-0.0875	0.0066
	K	-0.0723	0.0104

9 Discussion

9.1 Risk factors A& B

The first step in the estimation process of X_t and X_u was to investigate the properties of the residuals obtained for models M_t and M_u. The Tukey-Anscombe plots, situated in the upper left corner of the residual analysis plots seen in Figure 22 and 24, indicate no clear structure in their distribution over the fitted value spectrums, other than varying variances and they being centered around zero. Further, the presence of heteroscedasticity is accounted for through the use of robust regression techniques, averting the threat caused by this.

In the case of the distribution of M_t:s residuals, the QQ-plot indicates that they are indeed normally distributed. However, it is noteworthy that two normal distributions have been fitted; one where all observations have been included, and one where the two smallest extreme values have been omitted. As may be seen, this omitting makes the cdf fit the empirical cdf much better than with all values. We have that $\hat{H}_t(y) = \frac{e^y}{1+e^y}$, meaning that $\hat{H}_t(y)$ is strictly increasing in y. Because of this, negative values of X_t transform into very small transfer frequencies. Since small transfer frequencies are to prefer to large ones, especially in the mass-lapse context, the harm caused by omitting large negative outliers in for the residuals should be limited. Based on this, the decision to use the MLE-parameters for the somewhat reduced sample is made. For M_u however, the QQ-plot, histogram with fitted pdf and the empirical cdf with fitted cdf in Figure 24 all indicate that the residuals come from a normal distribution close to the one fitted with the entire sample, strengthening the notion of X_u being modeled as normal with the MLE-parameters.

When compared to the 99.5%-quantile of 0.4 for the mass-lapse events, as specified in the standard formula, the same quantile in all versions of F_t is substantially less. The chosen combination for further modeling has for conservative reasons been the one yielding the highest value for this residual, as has been described before. The table indicates that this choice shifts the quantile up with about 0.12 for both traditional life and unit-linked insurance.

Table 9 clarifies the contents of figures 29 and 31, namely that both the linear- and rank-correlation measures indicate a negative relationship to the equity returns and a positive relationship to the ten-year interest rates, for both risk factors A and B. In both cases, the sign of the correlation indicates that the two risk factors do not work as a hedge for each other. The relationship may be questioned in the case of traditional life insurance, it does not seem plausible that the policyholders are more likely to leave the Company in times where they benefit from the Company bearing the investment risk, namely when e.g. the equity returns decrease. For unit linked insurance however, the relationship makes sense, since policyholders should be increasingly likely to flee in times of equity recession, as they themselves bear the investment risk. Additionally. we see that the magnitude of correlation is bigger for unit linked than for life. The left plots in Figures 30 and 32, show that in extreme scenarios, namely when the frequency rate is above some high quantile and the equity returns are below some low quantile, the dependence ceases to exist in both cases.

As for the risk factors' relationship to the ten year interest rate, the model suggests a positive relationship in both cases, indicating, again, that the the lapse risk factors do not work as a hedge for the interest rate risk. However, the sign makes sense in both business arms. In traditional life, an increase in the risk-free rate lessens the value of a fix guaranteed growth rate. For unit-linked insurance, an increase in the risk-free rate makes the risky investments in mutual funds and equity less attractive. The co-movement in the worst case scenario, namely when both the transfer rate and interest rates are above some high quantile, is close to zero for unit linked. But for traditional life, the data set shows a weak, barely material, positive upper tail dependence.

9.2 Risk factors C & D

By inspection of Figure 33, the beta-distribution fitted seems to fit the empirical cdf reasonable well. The same goes for the the quantiles of two samples from the fitted beta in the QQ-plots. It is important however to keep in mind that samples that don't fit the empirical quantiles as good may also occur upon further investigation the adequacy of the fit. In the estimation of Z_u however, the sample seems to fit the beta distribution well better, as indicated by Figure 34.

Upon simulating the long term transfer frequency for NSS contracts, using the estimated parameters for Z_t, as well as the combinations of their 99.5% confidence bounds, the empirical cdfs generated form a quite wide band, indicating big uncertainty in the beta-estimations. Further, as also seen in Table 7, the 99.5% quantiles are found at very high levels, e.g. as high as 0.1538. This would indicate a yearly transfer frequency of 15.4%, every year from one year from now and on. In the standard formula, the non-stressed long term assumption would have to be more than 10% for this to be the stress level, a highly unlikely level. The 0.5% quantiles, corresponding to the worst case down-shifts of the transfer frequencies however show reasonable values. For the PSS contracts, Table 8, indicate results of a similar fashion. Quantiles generated when applying the γ_1 and α adjustments are also shown in the tables and on the left sides of figures 35 and 36. The figures clearly show how the bands of the empirical cdfs are tapered by the α-weighting and the table that the 99.5% quantiles are shifted downwards by the γ_1-weighting.

Since the long-term frequencies are built up partly by the mass-transfer frequencies, the results indicate the same signs but somewhat decreased magnitudes for the correlations with equity returns and risk free 10-year interest rates are expected. The analysis of their structure is therefore also covered by the one made for risk factors A & B. Further, no worst-case tail dependencies are observed in the data sets, so the signs of the correlations approach 0 as the variables pair both tend to their worst case extremes.

10 Conclusion

In this thesis, identification of risks arising through contractual options and suggestion of the modeling of factors capturing these risks in life insurance contracts has been conducted. Using various documentation, contractual options of possible materiality for both traditional life and unit-linked insurance have been identified and listed.

The assumption is made that the option to transfer the insurance capital to another insurer solely captures the customer behavior risks that arise through contractual options. With the Solvency II standard formula as base, risk factors are chosen to capture the risks associated with the take-up rate of the contractual option to transfer insurance capital. They are then modeled to fit the Company's existing framework for solvency risk calculation, for both traditional life and unit-linked insurance. One of the factor types assesses the risk associated with the rescaling of the balance sheet upon a sudden large amount of policyholders leaving the company. The other the risk associated with the long term best estimate transfer-rate assumption being deviated from a year into the future.

For the mass transfer risk factors, A & B, Norwegian data has been regressed on three of the Company's risk factors, equity returns and 10-year interest rates, forming a linear model. To complete the model, a random variable has been fitted to the residuals of the model, where mainly qualitative tools have been used in the assessment of the fit. For conservativeness, that combination of the 99.5% confidence parameters which yields the highest 99.5% quantiles risk factors as received when simulated for the Company.

Using the mass-transfer frequency simulated as the unobservable transfer rate of the coming year, a weighted K-year average, one year into the future, may be calculated. The weighted average is complemented with an expert judgment, aiming at taking possible future events into consideration. This combination yields the suggested model for simulation of the long-term assumptions made by the Company, the risk factors C & D. An attempt at approximating the expert judgment mentioned is done by MLE of data of the difference in transfer rates between an economy with full transfer rights and one with limited rights, as the Swedish one.

The univariate behavior of risk factors A & B is deemed reasonable and of conservative nature. However, the sign of the correlation between A and equity risks is not intuitive, and a certain grade of expert judgment may be in order to adjust for this. The same goes for the univariate behavior of risk factors C & D, which without adjustment indicate unreasonably high long term transfer rates for both business arms. The thesis thus suggests through investigation of remedies of the sort presented to weigh the model for long-term transfer rates better in line with the truth.

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