Prediction of French day-ahead electricity prices: comparison between a deterministic and a stochastic approach

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Abstract

This thesis deals with the new flow-based computation method used in the Central Western Europe Area. This is done on the financial side. The main aim is to produce some robust methods for predicting. Two approaches are used: the first one is based on a deterministic and algorithmic method involving the study of the interaction between the fundamentals and the prices. The other one is a more statistical approach based on a time series modeling of the French flow-based prices. Both approaches have advantages and disadvantages which will be discussed in the following. The work is mainly based on global simulated data provided by CASC in their implementation phase of the flow-base in Western Europe.

Keywords: Electricity market, Market coupling, CWE area, Flow Base, modeling

Abstract

Denna avhandling behandlar den nya flödesbaserade beräkningsmetoden som används i Centrala Västeuropa på ekonomisidan. Målet är att producera tillförlitliga metoder för prognostisering. Två tillvägagångssätt kan användas: den första är baserad på en deterministisk och algoritmisk metod som inbegriper studier av interaktionen mellan fundamenta och priserna. Den andra är en mer statistisk metod som bygger på en tidsseriemodellering av de franska flödesbaserade priserna. Båda tillvägagångssätten har fördelar och nackdelar som kommer som diskuteras i det följande. Arbetet är främst baserade på globala simulerade data från CASC i genomförandefasen av flödesbasen i Västeuropa.

Acknowledgments

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Contents

Introduction

This master thesis deals with electricity prices on short-term market in Western Europe. Net operators are on the way to modify the computation method for calculating maximal electrical capacities between countries in Central Western Europe (CWE). This implies a mandatory evolution of the pricing tools developed by the market participants. This thesis exposes adaptation of one of this tool. The method used to compute the prices is deterministic, based on the so-called delta method (sometimes also called resilience). A comparison is realized with a stochastic method to anticipate the prices. The thesis is divided in three parts: presentation of the specifications of the electric market, explanation of the deterministic tool and its evolution, and evaluation of the prediction effectiveness for the stochastic tool. Further analysis of different tools is done in conclusion.

Chapter 1

The purpose and the market

1.1 Aim, motivation and challenge

1.1.1 Global purpose of the thesis

The purpose of my thesis can be stated as follows:

Find a predictive tool in order to get
reliable day-ahead forecast flow-based prices

It may seem quite unclear said like that, but it should become far more explicit after the reading of the first chapter below. Nevertheless, in order to explain the purpose at this stage we can say that: there exists approximately one power exchange for electricity in each country of western Europe. These power exchanges are responsible for the day-ahead prices and the physical electric exchanges between countries. The computation for the flows between countries are done with a method called ATC (for Available Transfer Capacity). This method should be replaced soon in this area for the so-called flow-based method (which is referred to FB in the following). This thesis explores some ways to forecast the day-ahead spot prices under this new exchange computation method.

1.1.2 Motivation

The main motivation is obvious: it is always desirable on a financial market to get previsions which are the closest to reality. As the computation method will change (from ATC to FB) the price previsions has to adapt. This thesis allows also to understand slightly better the mechanisms that link the different markets in CWE. Now the exchange logic between countries is quite well understood between market participants. But the FB methodology will completely reset the system in the sense that as we shall see later the new method is far less legible than the actual one. This paper permits also to estimate the influence of the new computation method on the prices.

1.1.3 Challenges

As stated in the introduction, there are two approaches of the problem. The first one involve a whole analysis of the problem. It is directly related to the physical phenomena. The deterministic approach consists of the adaptation of a so-called delta model which is used by a lot of traders and supposes a precise knowledge of the fundamentals behind the market. The challenge in this part is to well apprehend the fundamental's logic of the electric market, then to understand the delta model algorithm and finally apprehend the flow-based logic in order to implement it in a new rebuilt tool. The second approach is based on time-series modeling. The challenge behind it is much more statistic and mathematical: how to transform correctly the data, how to build a decent model, how to assess its legitimacy. The main goal of this part is to provide a comparison tool, mainly to assess if a better price forecasting can be reached without knowing the fundamentals.

1.2 The electricity market

In this section a short description of the electric market will be done. We will try to underline the strong relationship between the physical and the financial aspects. We will also talk about the differences between this market and a more classical equity financial market. But first of all we will briefly describe the company which welcomed me for my thesis.

1.2.1 "La Compagnie Nationale du Rhône"

La Compagnie Nationale du Rhône (the CNR) was founded in 1933. Its mission has always been to exploit hydro-energy of the most powerful french river: le Rhône. Thanks to 19 hydraulic dams on this river the CNR produces electricity. The CNR has been a public company until 2003. Indeed, in France, EDF (for "Electricité de France") was the only company allowed to produce, convey and sell electricity during all the twentieth century. But in 2000 the European Commission denounced the French monopolistic situation of EDF. In order to be in accord with the European right, France had to sell a part of its energetic industrial estate to private companies. That's what happened with the CNR, half of the company was sold to GDF-Suez. Nevertheless EDF remains by far the strongest actor in the French energetic landscape. For ten years the CNR diversified its activity. Now the company has some wind farms and solar panels. But the major part of the production comes from the water (around 98 %).

Nowadays, the CNR is the leader among the producers of renewable energy in France. Moreover the company has a mission of general interest. In other words the company is responsible for the construction of some public buildings and layouts around the river Rhône.

On the global financial aspect, the CNR is a flourishing company. In 2013 the company had a revenue of 1.3 billion euros and a net result of 211 millions euros. The company employs approximately 1400 persons.

I worked in the trading room. The team is divided in the three classical "sub-teams" of finance sector : back-office, middle-office and front-office. I was integrated to the front-office because the thesis should have direct applications for them. The traders are really few (5) and each of them is able to operate on all of the different trading area (which will be precised in the following).

1.2.2 The French and CWE market, the fundamentals and the ways of trades

As the French market is the central market studied in this work, this presentation will mainly focus on it. Nevertheless some information will be provided about its neighbors. Anyway all electric markets in Europe remain quite similar. Hence a lot of French characteristics apply to other markets.

As it has been said the electric market in France is brand new. It began at the dawn of the twenty first century. The major and omnipotent actor on the French electric market is still EDF. EDF provides electricity to approximately 90% of the consumers in France. We will in the following briefly speak about the production, the consumption and the prices in the French market.

The production

In France a large part of provided electricity comes from nuclear industry. Then, depending on the season, hydro-electricity, wind, solar energy, gas, coal or even oil can be used to produce electricity. One usually ranks the production methods from the cheapest to the most expansive (see Table 1.1)

In France the part of electricity which is produced from wind and sun is extremely small and is often considered as negligible. But it is not the case everywhere. In Germany for example wind represents a far more important part of the production. Basically the means of production used are linked with the demand. But in any case, the kind of electricity which is always produced is the one coming from non-programmable generation. All renewable energies are said non-programmable. Whatever humans do, floods of a strong river will be what they have to be and it is the same for the wind and the sun. Then, depending on the demand of electricity at an instant t, some

Means of production
Hydro-electricity
Solar
Wind
Nuclear
Coal
Gas
Oil

Table 1.1: Ranking of the means of production from the cheapest to the most expensive

electricity coming from programmable generation (nuclear, gas, oil, coal) is produced. That is something we can see on the Figure 1.1.

On this figure we can see that around 4 am in the morning almost all the power plants in France are stopped because the non-programmable production is sufficient to reach the needs.

There is also a clear difference between the curves of production in summer and winter. Of course the needs of the population are not as important in summer as in winter. Then in Summer the energy is mainly produced thanks to the rivers and the nuclear power plants. But in winter the needs exceed the total of the production capacity of the hydro-electric and nuclear power plants. Then producers have to start other types of plants. This can be seen on the Figures 1.2 and 1.3 on the next page: during the winter some gas, coal and even oil is used to produce electricity and to satisfy the demand in energy. The prevision of the "production slice" can be a method to forecast the electricity prices. If the consumption is high enough, some costly programmable means have to be started. Then the electricity prices must be at least equal to the marginal cost of the last mean of production used. This approach (called Stack see [13])can be used in order to compute prices directly, but can also be used as a mean to bypass the non storable constraint for electricity in order to establish a robust stochastic modeling for the forward electricity pricing ([10]).

The consumption

The state regulates the price of the electricity in France. It means that the producers cannot sell electricity to private consumers at the price they want. The result is that the French electric consumption is assumed to be uncorrelated with the market prices. Basically the French consumption is supposed to be only linked to the weather and the temperature. So, during the winter the consumption is more important than during the summer. That can be seen on the Figures 1.4 and 1.5

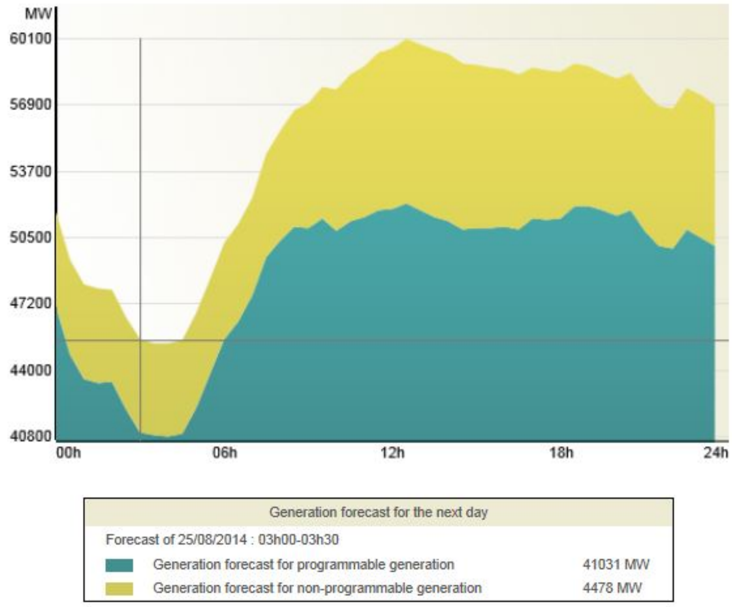

Figure 1.1: Repartition of programmable and non-programmable production for a given summer day

The French market: how trades are done

In France there is an electric power exchange called Epexspot. This stock works differently from a classic financial stock exchange. Indeed prices are calculated each day at 12:45. It means that each actor of the market has to send its sell or buy electricity orders for the day-ahead to Epexspot each day before 12:45. Then the stock computes prices for each hours of next day. Computation is rather simple, it is illustrated in Figure 1.6. The fact is that some companies have to sell their production in any case and at any cost (like the CNR) and others have to buy their needs in any case and at any cost (some steel industry or car industry for example).

We can see on the Figure 1.6 in the lower left corner the orders of the company who have to sell their production and in the upper right corner the companies which have to buy their needs at any cost. In the center we see the orders of all the other actors of the market. When the demand meet the offer, it fixes the price. A curve like that exists for each hour every day of the year. Traders try to forecast these two curves with some modeling, but we will see that in details further. What is described before is the "law of the market" for auctions. But there are two other ways to sell and buy electricity in France. The most intuitive one is the forward market. Of course each company can sell and buy some forwards contract. It concerns

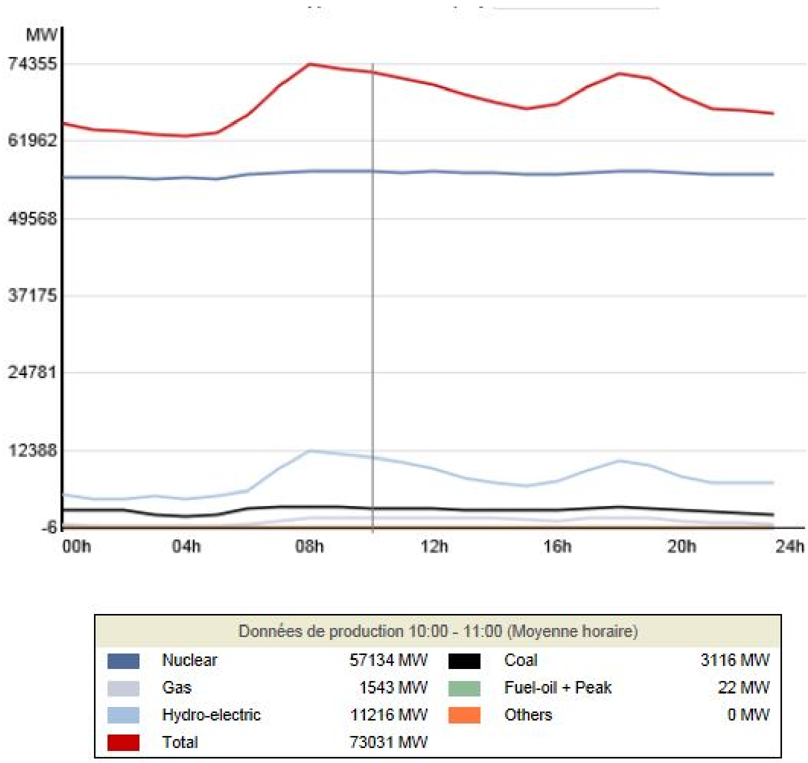

Figure 1.2: Repartition of the production in a classic winter day 2013/02/13

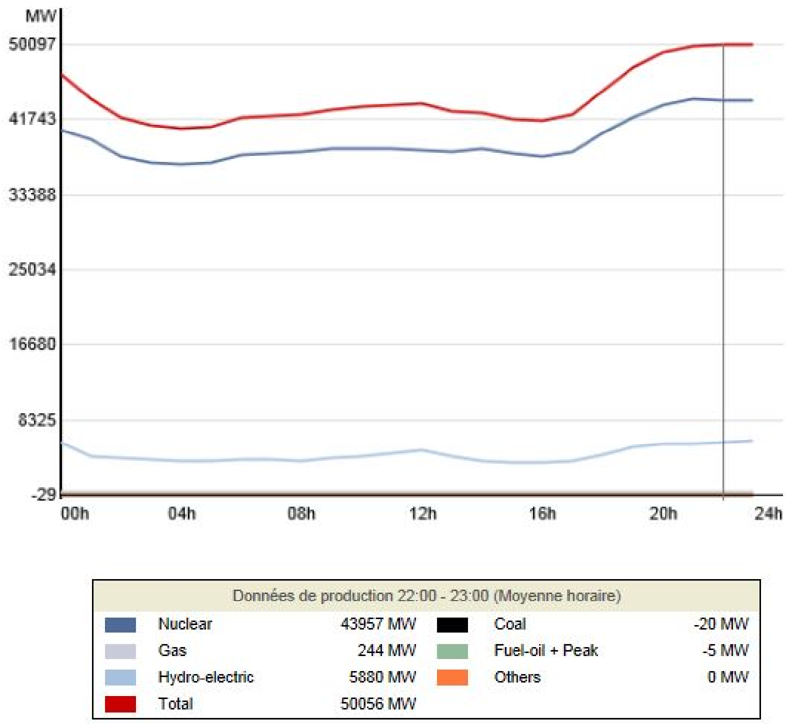

Figure 1.3: Repartition of the production in a classic summer day 08/24/2013

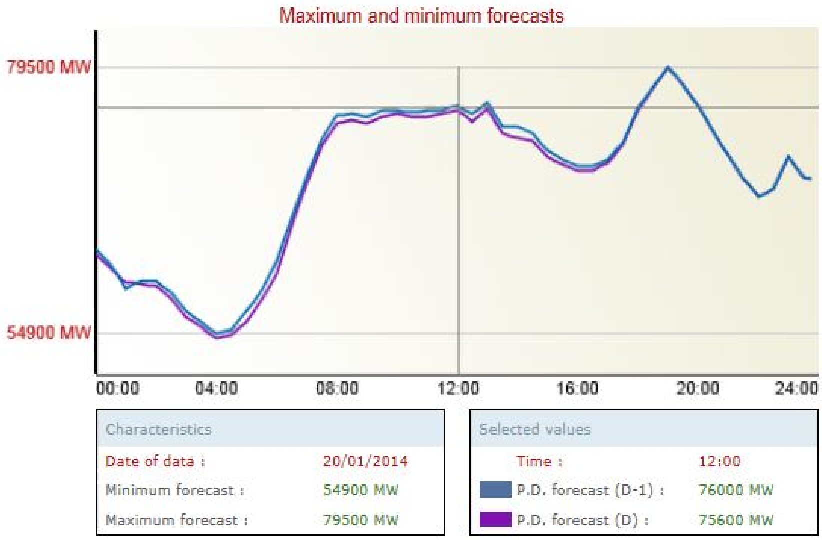

Figure 1.4: French consumption in winter 2014/01/20

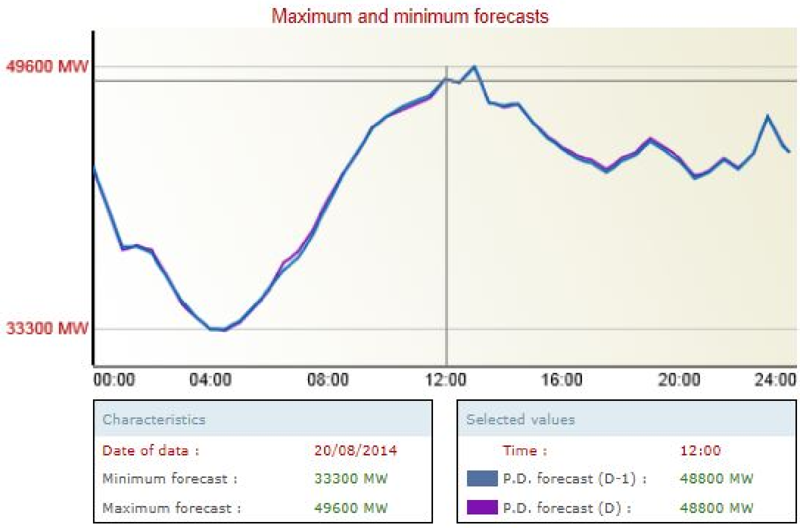

Figure 1.5: French consumption in summer the2013/08/20

Figure 1.6: Aggregated curve of buy and sell orders for 2014/08/26 hour 1

middle and long term trading. It is performed with brokers, so it remains "over the counter". There exists forward contracts for the next days, the next week, the next month, the next quarter, the next year and even the next 3 years. But the last ones are really rare because of the difficulty to forecast the market and the huge amounts of money at stake. If for example a forward contract of 100 MW is sold for the next year for 30 € per MW, it represents

$$100 \times 30 \times 24 \times 365 = 262800000€$$

And 100 MW remains a small amount of electricity. The CNR produces each hour approximately 2000 MW and an optimized computation has shown that around 80% of the future production has to be sold in forward contract by the end of the year. In other words on the 31st of December 2013 the CNR had already sold 80% of what it had and still produce in 2014. The selling of forward contracts is one of the hardest task because it is mainly empirical. The models are not really reliable. The last way to sell and buy electricity is through the intraday market. This market is managed for each country by the different power exchanges. It allows actors of the market to sell and buy some quantity of electricity for the same day but at a future time. It represents a rather small volume of energy but still remain important. This market is the consequence of the fact that the electricity cannot be stored which basically means that all the production has to be consumed. This rule is checked by the Transport System Operator (TSO). For example, the French TSO is called RTE (for "Réseau de transport d'électricité"). Each day, 48 times a day (each 30 minutes) RTE computes the difference between the global production and the global consumption in France and then imposes fees for the ones who do not respect the rule. Let's take an example to be clear. Let's say for example that the CNR has foreseen to produce 2138 MW of electricity between 12:00 and 12:30 the August 25th 2014. This forecasting has been done the 24th of august 2014 and then sent to RTE. With all the data from all the actors of the market RTE can then compute the flows in each hub of the country for the next day. But in reality the real electricity produced is never exactly equal to the forecasted one. Let's say that due to some unexpected rain during the night the effective production of the CNR the 25th of august 2014 between 12:00 and 12:30 is higher than expected and equal to 2158 MW. Then there are two cases: the effective gap between the production and the consumption on the national level can be positive or negative. If it is positive then there is too much electricity on the grid. But we are over-producing. So RTE will buy us our remaining electricity at a really low price (if the CNR was under-producing but the grid is in need of electricity the company should buy the needed electricity at a really high price). On the contrary if the gap is negative then the surplus is "in the good way" (the grid needs electricity and the CNR has more to offer) so RTE buy the electricity at the price computed by the power exchange. The intraday market is here to give the opportunity to market actors to re balance their situation. But the difficulty comes from the fact that the global state of the grid is unknown until RTE publishes it and then it is too late. In the previous example, if the CNR's intraday trader has anticipated that the real production would be too high he could have re sold it on the intraday market to a counterpart.

The European market and exchanges between France and its neighbors

In Europe there exists a lot of different power exchanges. Basically each country have one. But in some area there is one stock for some countries. In Scandinavia for instance, the Nordpool is the power exchange for Sweden, Norway, Denmark, Finland, Estonia, Latvia and Lithuania. In Central Western Europe (CWE) which regroups France, Germany, Belgium and the Netherlands the power exchange integration is on the way. But now each country has still its own power exchange. When one trades in France there is possibility to buy and sell electricity to the neighbors. If a trader wants to exchange with Spain, Italy or Great-Britain he has to do it over the counter. Moreover some auction exists to have the right to use the links between France and Italy and France and Switzerland. For the exchange in the CWE, such auctions are not available, all is integrated thanks to a system called market coupling (see below).

We will discuss about market coupling and flow base which are strongly linked. Market coupling is a major step in the integrating process of electricity stocks in Western Europe. As we have mentioned earlier, before market coupling the only way to buy or sell electricity to your neighbor was an OTC (over the counter) transaction between two parties. Moreover there existed and there still exists (for the border with Switzerland and Italy) an auction system for the capacity. Let's take an example. If you build your model and find that the price tomorrow will be lower in Italy you can make an auction in order to buy capacity from Italy to France. If you foresee that the difference between French and Italian prices will be around 10 € you can post an ask auction until 10 €İf for example you get some capacity for a price of 6 per MW you will make a profit of 4 € per MW. Some years ago this system was generalized in Europe. So if you had some knowledge about the Spanish, French and Italian market you could realized some really interesting arbitrages. This was due to the fact that a lot of Spanish actors reasoned on a national scale but not a continental one for example. But now a new system called market coupling is on the way, and it is more clear and transparent.

1.3 Trading strategies

As an introduction to market coupling and flow base, here is a short summary of the different trading strategies allowed in the energy area. This section is a small summary of the work of S. Fiorenzani, S. Ravelli and E. Edolli [13].

1.3.1 Directional trading

Directional trading is the easiest and most intuitive kind of trading. It can be summarized by the tautology: 'Buy low, sell high'. In other words directional tradings consists in forecasting the trend of spot prices in order to play with spot prices themselves or with linear derivatives. Concerning the electricity market, as it has been explained before there is no real spot price. The main financial instruments traded are futures and forward due to the lack of storability of electricity. These products are usually very liquid. All actors who are not pure traders (meaning that they have to buy or sell something) are committed with directional trading. That is the case for the CNR: the company has to sell all the electricity it produces hence it really matters to anticipate the long-term trend in order to sell the production at the highest prices possible. Mainly two approaches are usually used to determine the price trend: the fundamental approach and the statistical approach. The fundamental approach will be evoked in the second part and is based on the study of the physical foundation of the market. The statistical approach implies a lot of different probabilistic models. Time series and establishment of stochastic models are the most described in literature. As regards stochastic modeling, one of the main difficulties is the perception of peak prices. Such models like the geometric Brownian motion are not satisfactory because they do not exhibit peaks or seasonality. The introduction of Lévy processes is one of the answer brought by the literature to this problem (see [1]). The time series approach will be treated in the third chapter. One refers sometimes to a last approach: technical trading. This approach is a mix of the two precedent approach combined with a more or less powerful algorithmic tool. A lot of articles in the literature deal with machine learning and more precisely with neural network approach. These approaches requires a deep algorithmic knowledge to be implemented (see [8]).

1.3.2 Spread trading

Spread trading consists in trading on more than one commodity. Usually one trades on the differences between two commodities. It is the case in the example described in section 1.2.2.4 about interconnection between France and Italy. Such a spread trading is called location spread and is operated due to the fact that France and Italy do not have the same electricity prices but spread between the two countries can be modeled thanks to knowledge of respective means of production, of respective foreseen consumption. There exists a lot of other spread trades such as dark spread, crack spread, spark spread. The dark spread consists in trading on the difference between the cost of coal and the cost of electricity a coal power plant can produce. Crack spread is the same thing with oil and spark with gas. Usually spread trading can be really valuable when the two commodities are strongly linked thanks to economical reason for example. Spread trading is not specific to commodities, it can be interesting for example to build a portfolio with BP and Total shares as they share a lot of economical links. If two commodities share the same stochastic trend then the spread trading on this two commodities is most of the time really profitable.

1.3.3 Other derivatives

Options and other derivatives are not different in the energy sector than in a classical pure financial market. In Europe really few options are traded. Nevertheless, the most dynamic market concerning energy option is the Nordpool (Scandinavia) where option can be traded both OTC and on the power exchange. In continental Europe some options are sometimes traded but mainly OTC. In any case the energy option market is far from liquid. Traders usually use it on really specific occasions depending on their needs and on available offers. Knowing this, it is obvious that energy option is not one of the main theme in the literature on power market (even if it is slightly different in America where the NYMEX CME exchange is much more dynamic).

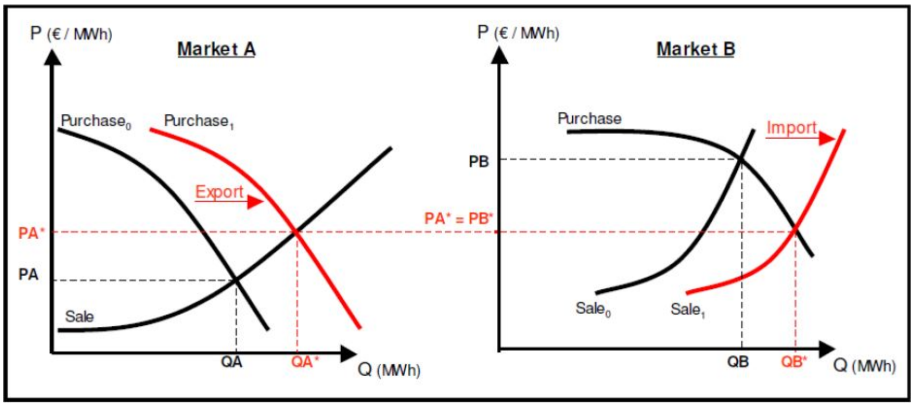

Figure 1.7: Market coupling principle in case of no congestion

1.4 Market coupling

1.4.1 Principle

The basic principle is rather simple: if the price of electricity is lower in a country than in one of its neighbor then the cheaper country will export some of its electricity. But that's only the theory. Practically it is slightly more complicated than this. Let's consider two countries A and B. If the spot price (which is another word for day-ahead price) for electricity is lower in A than in B then buyers in B will want to buy electricity in the country A (because it is cheaper) and sellers in country A will want to sell electricity in country B (because it is more expansive). It is described on the Figure 2.1 where we can see the evolution of demand and offer depending on the exchange between country A and B. The consequence is that both prices will converge.

But as it is briefly said in the legend of the Figure 2.1 there exists some physical constraints to these exchanges between countries. Indeed the physical transfer capacities between the different countries in Europe are not unlimited. There is the possibility that the prices stop their convergence because the maximum allowed capacity between countries A and B is reached before the convergence. This case is described in the Figure 2.2.

Now, the maximum capacities are computed by the grid operators. They are considering the worst case and then deduce a physical capacity which cannot be broken. This computation methodology is known as ATC (for Available Transfer Capacity). We will come back to this problem later on because it is precisely one of the main topic regarding the flow base dilemma.

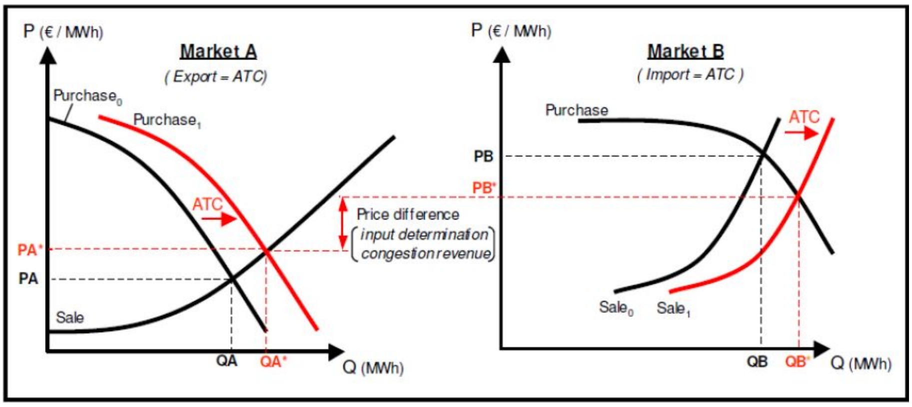

Figure 1.8: Market coupling principle in case of congestion

1.4.2 Computation algorithm

As we have described the principle, we will now describe the market coupling operating system in details, based on the document [11]. We will have a more "mathematical" approach by giving a reasoned description of the algorithm used by the power exchanges in order to compute the spot prices in the CWE area. This algorithm is called COSMOS. COSMOS is an algorithm which was developed by BELPEX (the Belgian electricity spot exchange) but is know co-owned by the power exchanges in CWE. This algorithm solves an optimization problem. Indeed the purpose of the power exchanges is to maximize the social welfare under some market constraints. The social welfare is defined as the sum of gross consumers' surplus minus the production costs. It will be defined more properly soon. The market constraints concerns the orders the power exchanges receive. There exists two kinds of orders: the hourly orders and the block orders. The hourly orders are the basic ones. They concern a certain amount of energy over an hour. As an example let's consider a producer who posts a hourly order at 11am for the next day: the order says that he sells 100 MW of electricity between 8 and 9am (which is also called hour 9) for 40€ per MW. The stock will receive this order at 11am and at 12:45 it will compute the spot prices for each hour of the next day by aggregating the buy and sale orders. If for the hour 9 the price computed is less then 40€ per MW the order of our producer will be rejected. If the computed price is over 40€ then the order of our producer will be accepted.

The block orders are a sum of hourly orders. Let's come back to our previous example. Now the producer has changed his strategy and he wants to sell 33MW for each hours between 8 and 11 pm the day-ahead for 40€ per MW. That is a block order. A block order is an order which concerns strictly more than one hour. The block order offer will not be executed if the average of the rounded market clearing prices (again another word for day-ahead or spot prices) over the relevant hours (between 8 and 11) and weighted by the corresponding volume limits (here all the volumes are the same so there will not be any weight) is lower than the price limit of this order (here 40 €). Recently the power exchanges have chosen to offer two new kind of block orders called smart blocks. The first kind are the linked block orders. These block are a pair or a triplet of blocks. The first one is called the parent block order. The second one is the child block order (and if there is, the third one is the grandchild block order). If the parent block is in the money then the child block can be executed or not depending if it is in or out-the-money. If the parent block is out-the-money and the child block is also out then nothing happens. But if the parent block is out-the-money but the child is in and globally the family is in then all the family is executed. The second kind of blocks are the Exclusive Block Orders. These blocks are a group of blocks for the day-ahead. Only the block of the group which maximizes the total welfare is accepted.

Before we get into the mathematical formulation it may be interesting to underline the fact that an hourly order can be partially executed. That is due to the fact that market participants can give a price interval instead of a precise price for their orders. If a producer give to the stock an hourly offer order of 100 MW for the hour 9 with a low price of 35€ per MW and a high price of 40€ per MW then: the order will be totally accepted if the Market Clearing Price (MCP) is superior to 40 € per MW, it will be totally rejected if the stock price is lower than 35 € per MW. If the MCP is equal to 37.5 € per MW then only half of the order will be accepted, that is to say the producer will only sell 50 MW instead of 100.

The following mathematical description is based on the COSMOS notice made by Epexspot (cf [11]) Now the sets, data and variables of our problem are defined :

SETS:
m = Bidding area
h = Hours
o = Hourly orders
p = Profile block orders
l = Unidirectional transfer line

DATA:
q_o = Quantity of hourly order o, positive for sell orders and negative for buy orders
p_o^0 = Price at which an hourly order start to be accepted
p_o^1 = Price at which an hourly order is fully accepted
$q_{b,h}$ = Quantity of profile block b on period h, negative for buy orders
Bidding area (b) = Area in which block order b originates
Hours (b) = set of hours on which profile block order b spans

Bidding area (o) = Area in which hourly order o originates
Hours (o) = set of hours on which hourly order o spans
from (l) = bidding area from which line l is originating
to (l) = bidding area to which line l is leading
$\text{capacity}_{l,h}$ = capacity on the line l at period h

VARIABLES:
$\text{ACCEPT}_o \in [0;1]$: Acceptance of the hourly order
$\text{ACCEPT}_b \in \{0;1\}$: Acceptance of the block order
$0 \leq \text{FLOW}_{l,h}$: Flow on the line l at hour h
$\text{MCP}_{m,h}$: Market clearing price in area m and hour h
$0 \leq ATCPRICE_{l,h}$: Congestion price of the capacity l at hour h

We will now formulate the different market constraints with the variables we have defined before.

- An hourly order o may be accepted only if it is at or in the money:

$$\text{ACCEPT}_o \succ 0 \Rightarrow q_0.(\text{MCP}_{Biddingarea(o),hour(o)} - p_o^0) \geq 0$$

- An hourly order o must be refused if it is out of the money

$$q_0.(p_o^0 - \text{MCP}_{Biddingarea(o),hour(o)}) \succ 0 \Rightarrow \text{ACCEPT}_o = 0$$

-An hourly order o may be partially rejected only if it is at the money:

$$0 \prec \text{ACCEPT}_o \prec 1 \Rightarrow \text{MCP}_{biddingarea(o),hour(o)} = p_o^0 + (p_o^1 - p_o^0).\text{ACCEPT}_o$$

-An hourly order o must be fully accepted only if it is in the money:

$$q_0.(p_o^1 - \text{MCP}_{Biddingarea(o),hour(o)}) \prec 0 \Rightarrow \text{ACCEPT}_o = 1$$

-An accepted block b must be in the money:

$$\text{ACCEPT}_b = 1 \Rightarrow \sum_{h \in hours(b)} q_{b,h}.(\text{MCP}_{biddingarea(b),h} - p_b) \geq 0$$

Some network constraints also have to be respected:

-There must be a balance at hour h in the area, the sell and import volumes must equal the purchase and export volume:

$$\sum_{\substack{o,biddingarea(o)=m \\ hour(o)=h}} \text{ACCEPT}_o.q_0 + \sum_{\substack{b,biddingarea(b)=m \\ h \in hours(b)}} \text{ACCEPT}_b.q_{b,h} = \sum_{l,from(l)=m} \text{FLOW}_{l,h} - \sum_{l,to(l)=m} \text{FLOW}_{l,h}$$

-Necessarily the flow at hour h on line l can't exceed the capacity:

$$\text{FLOW}_{l,h} \leq \text{Capacity}_{l,h}$$

-If the congestion price is non-equal to zero it means that the flow must be equal to the maximum capacity:

$$\text{ATC\_PRICE}_{l,h} \succ 0 \Rightarrow \text{FLOW}_{l,h} = \text{Capacity}_{l,h}$$

-The congestion price must be equal to the difference between the price corresponding to the two countries which shares a border:

$$\text{ATC\_PRICE}_{l,h} = \text{MCP}_{to(l),h} - \text{MCP}_{from(l),h}$$

Finally the function that has to be maximized under the previous constraints is the so-called social welfare which can be defined more precisely than before as the cumulative amount the buyers are ready to pay and the cumulative amount that sellers want to be paid for. In a more rational way the social welfare is:

$$\sum_o q_o.\mathrm{ACCEPT}_o(\frac{p_o^0 + p_0^1}{2} + \frac{p_o^0 - p_0^1}{2}(1 - \mathrm{ACCEPT}_o)) - \sum_b p_b.\mathrm{ACCEPT}_b \sum_{h(b)} q_{b,h}$$

Here we have a quite decent description of the global problematic for the power exchanges in CWE considering the day-ahead market. This algorithm is interesting in order to understand the global logic of the market but it is only used by the power exchanges to compute the prices. Market participants cannot use it because it is a really heavy and costly algorithm, but mainly because they do not know the orders for the day-ahead until the stock publishes them and by this time it is too late because the prices are already fixed. Market participants do not use this algorithm to forecast the day-ahead prices. They use another prediction method called delta model. It will be described in the second part because it is one of the central point to bring a solution to the flow-base problem.

1.5 Flow base

Here the flow-based computation method will be approached mainly on the financial side. The physical knowledge of the phenomenon will be reduced at the essentials. For a more detailed and complete physical approach see [2] and [12].

1.5.1 Global approach

The flow base is a new kind of computation method for the maximum capacity (the "$\mathrm{Capacity}_{l,h}$" in the previous algorithm) between the countries in CWE area. The idea to implement the flow base came after a really simple assessment: the grid and the interconnections between countries are not used at the maximum of their capacities. Indeed today each grid operator in each country computes for its borders a maximum capacity for each hour with each of its neighbors. For their calculation they are just considering the most constrained case for an hour h whatever the actual grid state is at this hour h. In a nutshell the grid operators do not want to take any risk and they are "over-constraining" the grid. But in reality the Available Transfer Capacity (ATC which is the maximum capacity for a border which can be used for day-ahead predictions) between two countries at an hour h depends on all the other electricity exchanges which are realized in the CWE at the

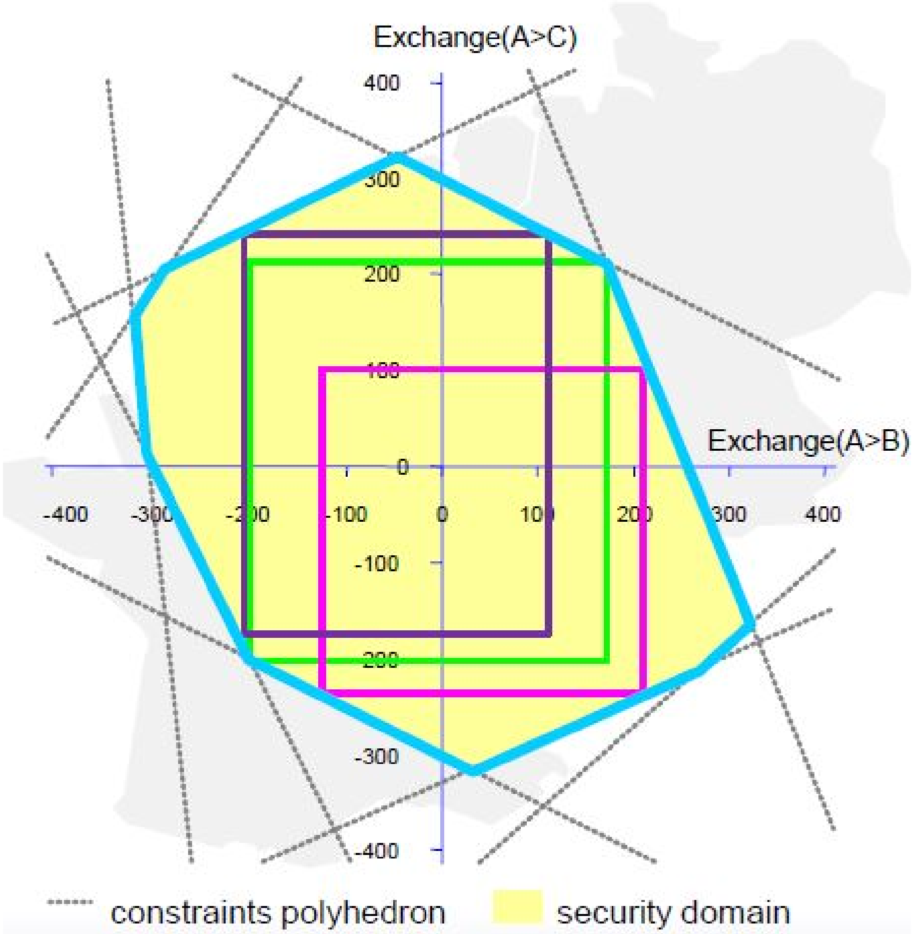

Figure 1.9: Illustration of differences between ATC's and flow-based' security domains

same time. The concept is rather simple. Let's take a simple example with only three countries A, B and C. Let us consider that there are only two borders: one between A and B and one between A and C.

On the Figure 2.3 the X-axis represents the potentials exchanges of electricity between A and B and the Y-axis represents the exchanges between A and C. The blue domain represents the flow-based domain. Purple, green and pink domains represents some possibilities for the ATC domain. The ATC domain is chosen by the grid operator, for some reason (let us say that the operator thinks it is more likely that there will be exchange from A to C and from B to A) he may choose the purple and not the two others. The flow-based domain is bigger which means that more quantities should be exchanged between countries with the flow-based methodology and then the social welfare of CWE should increase.

The flow-based project has been launched in 2007. After years of studies and simulation it shall become effective at the end of March 2015. The flow-base methodology has a lot of impacts on the energetic markets and even at a smaller scale on the economy. Therefore the flow-based implementation process has been (and is still) a tough and long project. It has to deal with the electricity market participants, the operator grid, CASC (which is the central auction office for cross-border transmission capacity for Central Western Europe, the borders of Italy, Northern Switzerland and parts of Scandinavia) which is the initiator of the project and finally the authorities.

1.5.2 Data and tools provided by CASC

As we have said before the responsible for the implementation of the flow base methodology is a pan-European organism called CASC. It publishes a lot of documentation on the project and its evolution. But they provide also some technical data which can be used by the market participants. The flow base introduces a dependance between the maximum transfer capacity (called also Net Transfer Capacity or NTC) and the state of the market in the CWE area. How can this be modeled ? Quite simply actually: with a matrix called Power Transfer Distribution Function. CASC is working jointly with all the Transmission System Operators (TSOs which are the grid operators) of CWE and even beyond. The co-working architecture is shown on the Figure 1.10. The figure is a little bit jumble but it shows the strong interconnection between the different part of the project.

All these people are merging their information and their knowledge. In order to make the system work but also in order to give to the market actors some data which will help them in their predictions. We have often mentioned the physical limits of the flows between countries earlier. We have called these limits NTC or ATC for Net Transfer Capacity and Available Transfer Capacity. There is not a proper definition for these terms but there are usually corresponding to the same maximal capacity for the day-ahead market. Sometimes ATC is equal to NTC minus the long-term nominations. For the flow base, the maximum capacities between countries are variable. They are depending on the entire state of the grid at an instant t. Assuming some flows between the countries CASC furnishes some data to check if these flows are acceptable. Data are divided in two categories: the PTDF matrix and the RAM vectors. PTDF stands for Power Transfer Distribution Function and RAM stands for Remaining Available Margin. Basically the PTDF matrices deliver some information on the influence of the flows in the CWE zone and the RAM is the equivalent of the NTC in the flow based approach.

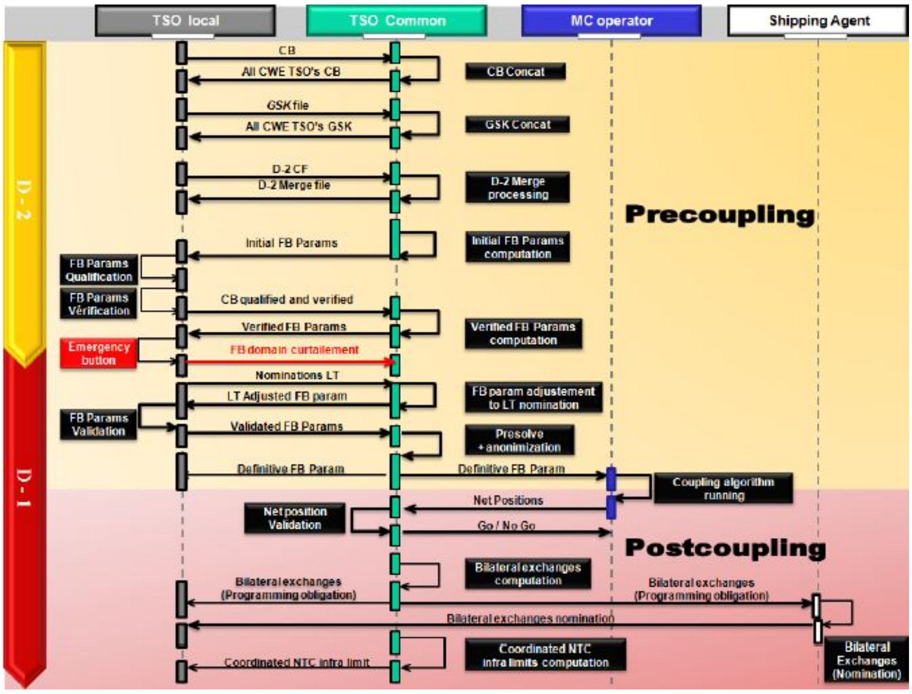

Figure 1.10: Global vision of the flow based process

The Power Transfer Distribution Matrix

A strong connection between the TSOs gives the opportunity to identify the most sensitive areas of the grid. For each hour of each day the TSOs run their algorithms and determine some part of the net which are potentially limiting for the entire CWE zone. These parts can be a cable or a hub. Then, after the identification, the grid operators run some simulations to evaluate the impact of potential cross-boarder flows in the area on these limiting factors. These experimentation are summarized in a matrix made of 4 columns and n raw where n is equal to the number of limiting factors of the grid. An example of such a matrix is given with table 1.2.

The reasoning is not made with bilateral flows but with export flows for each countries. The FR export is for example naturally equal to the sum of the bilateral flows from France to Belgium and from France to Germany. The ID indicates identity of the limiting factors. It does matter to precise that the identification of the physical limiting connectors is only known by the TSOs. It is done in order to not allow the market players to force a congestion on a certain place thanks to their knowledge of the grid. Nevertheless this point is quite sensitive and still opens to discussion between participants and regulators. As regards numbers, they represent a percentage of influence of a certain flow on a specific limiting connector.

ID	FR export	DE export	BE export	NL export
456	0	0	1	0
258	0,32	0,05	0,02	0,235
245	-1	0	0,2	1
269	0,05	0,012	0,58	0,0012
1002	1	0	0	0
758	0,014	0,062	0,26	0,25
362	0,09	0,08	0,236	0,07

Table 1.2: Example of an hypothetical PTDF matrix with realistic values

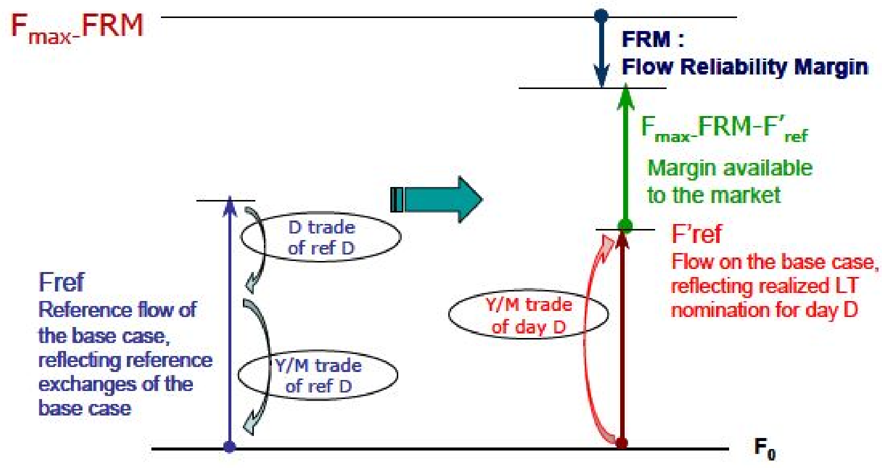

Figure 1.11: Scheme of the RAM computation principle

The Remaining Available Margin

The RAM is a maximal flow which is computed for each limiting factor. It corresponds to the maximum flow allowed on the so-called cable or hub. It is expressed in MW for convenience. Indeed it has the same dimension than the flows. It is computed after some tests and simulations like the PTDF matrices. Some long-term flows and other security factors (Which are summarized in a security number called Flow Reliability Margin) have to be taken in account to compute the RAM for each factor. This is shown on the Figure 1.11. The FRM is computed for each hub or cable which can potentially have a limiting influence on global exchanges. It is computed with a statistical analysis of differences between predicted values of the flows on the limiting factor and realized flows in the simulations. The global idea of the process can be seen on Figure 1.12. If a limiting factor implies more

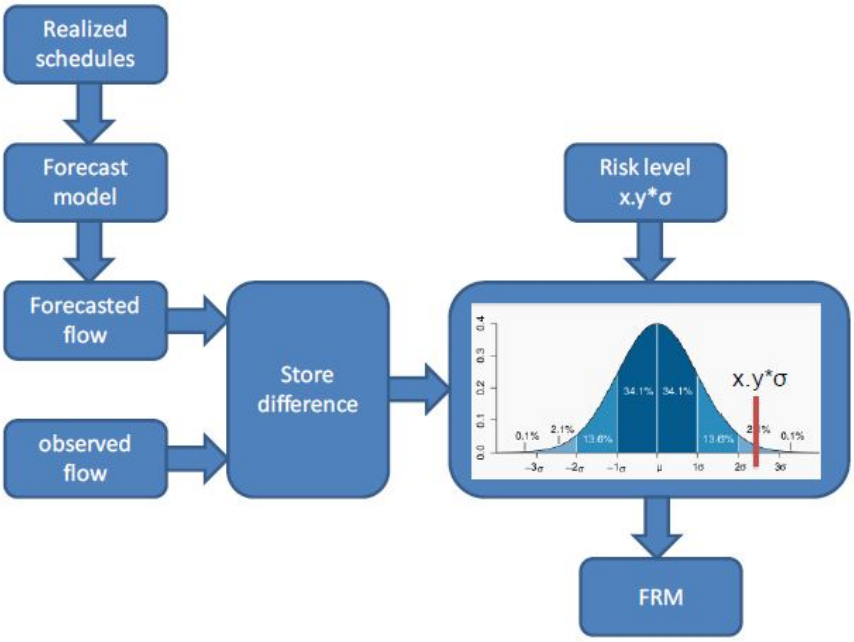

Figure 1.12: Calculation of the FRM, global process description

than one country then the computation of the FRM needs the cooperation of grid operators.

Computation

The computation for the Net Export Flows (NEX) allowed with the flow-based methodology is then quite convenient. It can be expressed with a simple inequality. Let say we have for a given hour:

$$\begin{cases} \text{A PTDF matrix } P \in \Re^{n \times 4} \\ \text{A Net Export Flow vector } N \in \Re^{4 \times 1} \\ \text{A RAM vector } R \in \Re^{n \times 1} \end{cases}$$

Then the condition in order to be in the flow base acceptance domain is rather obvious and simple:

$$\text{N} \in \text{Flow Base domain} \iff P \times N \leq R$$

Summary

The electricity market exhibits strong physical and financial interactions between European countries. The main financial products sold on the markets are forward contracts. There are three ways of selling and buying electricity: long-term (year, month, week), day-ahead and intraday. The prices are highly correlated to a lot of physical factors such as consumption, weather and production events.

Chapter 2

The delta model algorithm: a deterministic approach

2.1 State of the art

The trading team among which I worked uses several tools to make their predictions. For the day-ahead prediction they mainly use one tool which is a delta model.

2.1.1 Data

In this chapter a lot of data is used to make the forecast which can be seen at the end. The fundamentals such as historical values of the consumption, production, interconnections data come from a data provider. The prevision of the fundamentals done for each day, the basic come from this data provider prevision but it is often modified by the CNR's traders regarding their experience of the market. As regards the delta model (see below for definition) and the block orders, data come from the power exchanges. Indeed the power exchange provide a certain amount of past data to all market participants. As regards data specific to flow-based simulation, they are provided by the Capacity Allocating Service Company (CASC) which is a European company in charge of the flow-base installation.

2.1.2 The delta model algorithm

Principle

The principle of a delta model is based on study and implementation of what is called the fundamentals in power markets. Fundamentals are referring to physical data such as light, wind, temperature but also consumption and production. As it can be easily conceived electricity prices are strongly correlated with the weather. If the temperature will be lower tomorrow than today then, ceteris paribus, the consumption of electricity will be higher and the micro-economic reasoning stated above implies that the electricity prices will increase. Then the maximum of weather, consumption and production variation predictions have to be gathered in order to evaluate the day-ahead prices. At the CNR data used are the one provided by a mass media corporation. But there comes the trader main role. He has to be critical with the data he got. Indeed predictions are never perfect and qualitative analysis is an important part of a trader's job. Traders are always connected to the news because as every other markets the power markets are sensible to the macro economical process. But what is specific to power markets is all information regarding the production cessation for some control or whatever. It is often the case for example for the nuclear plants. Even if these anticipations are usually more useful for the mid and long term forecasts they can be in some cases interesting for the short-term.

The delta model is based on the following idea: for a given hour the day-ahead price is equal to the intersection of the ask and bid curve where one of the curve is horizontally translated with an offset equal to the delta of the fundamentals.

The fundamentals are precisely of four types: consumption, production, net export to the neighbors and accepted blocks. For the delta of consumption and production the data from data provider is the one which is basically used (with some slight differences regarding the predictable means of productions which are often adjusted by the trader's analysis). These fundamentals are expressed in delta meaning the difference between tomorrow and today. Finally they are summarized in a global delta which will represent the horizontal offset.

Above that, the delta model algorithm models the flow between two countries, here between France and Germany. If the prices in France computed thanks to the technique above are lower than the prices in Germany computed with the same technique the flow from France to Germany is increased and the total delta is modified then for both countries. Hence a convergence of the prices is achieved. After the convergence or the reach of the physical limit a block order acceptation algorithm is run. It deals with the different kind of block orders evoked before. This process is repeated as many times as necessary. The flow between France and Germany is not unlimited and the limits for the flow of day-ahead are provided by a an European organism called ENTSOE.

The delta model table and price computation

Let us first look at the delta model table. On the table 2.1 the prices are shown for France on October 8^{th} 2014. For more clarity the basic delta is equal to 1000 MW but in the one used in the delta model algorithm at CNR, the basic delta (increment in the left column) is equal to 100.

	1	2	3 ... 23	24
5000	574,044	145,55	...	130,382
4000	150,023	113,736	...	96,619
3000	92,65	79,5	...	69,835
2000	65,953	59,122	...	56,04
1000	49,079	42,863	...	47,928
0	41,253	35	...	38,473
-1000	33,343	32,319	...	34,465
-2000	27,039	25,189	...	30,739
-3000	21,712	18,886	...	26,777
-4000	15,832	14,696	...	23,096
-5000	11,744	11,211	...	19,474

Table 2.1: Extract of the French delta model table for October 8^{th} 2014

On this table we can see that if the delta is positive then the price increases. It means that the convention adopted in the company is that the delta has the same sign as the consumption. If the consumption increases, ceteris paribus, the prices increases too. The price corresponding to a delta equal to zero is equal to the price computed by the power exchange for October 8^{th} with the aggregation of ask and bid orders. If we now imagine that the French global delta is decomposed in the specific deltas shown on the table 2.2. (thanks to the prevision furnished by mass media information and the trader's analysis) then we get a total delta.

Δ between 10/8 and 10/9		Hour 1	Hour 2	Hour3-23	Hour 24
Consumption		3000	2500	...	500
Global production	Coal production	-100	-200	...	-100
	Nuclear production	0	-400	...	-600
	Gas production	200	-100	...	500
	Hydrolic power	2100	3000	...	3200
	Wind	1500	1000	...	1000
	Solar power	0	0	...	0
Interconnections	Fr=>Be	-2000	-2100	...	-2200
	Fe=>Ge	-2000	-2000	...	-2500
	Fr=>Ch	1000	900	...	900
	Fr=>It	500	100	...	400
	Fr=>Esp	300	200	...	400
	Fr=>UK	0	100	...	200
Blocks orders		-500	0	...	300
Δ Total		4000	3000	...	2000

Table 2.2: Example of a hypothetical Delta between 10/08 and 10/09

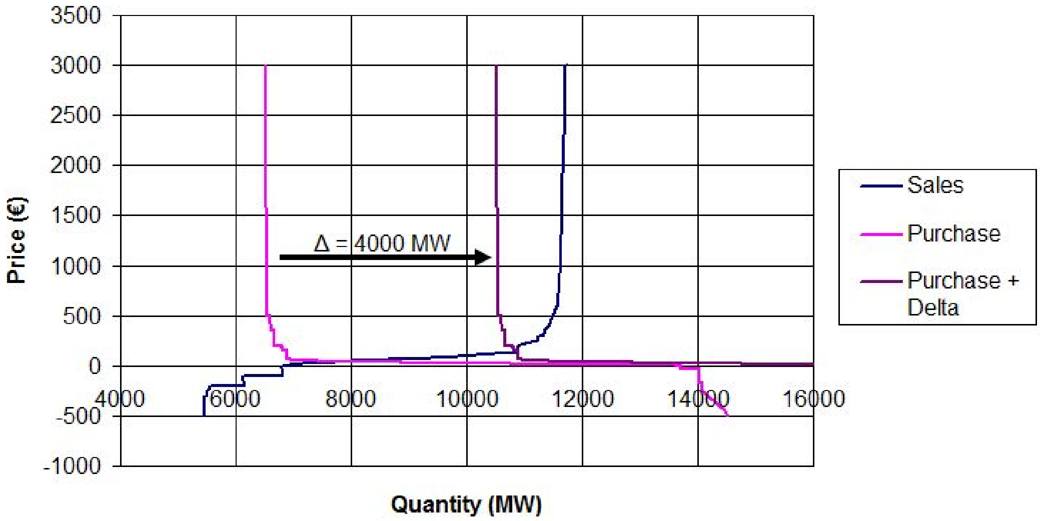

Figure 2.1: New price computation 10/09/2014 H1

With this delta total we can then compute the new forecast price for the day after (here the 9th of October). This is shown on the Figure 2.1 and 2.2.

On these two figures we can graphically see the process which leads to computation of a new price for the day-ahead. Here we can see that for the first cross we get a price of approximately 40 € per MW, this cross corresponds to the price computed by the power exchange for the October 8th. The second cross leads to a price of 140 € per MW. This price correspond to the intersection of the cross of the old sales curve and the new demand curve. The new demand curve represents the prevision made by the traders. We can see with this example that the hypothetical anticipation of the fundamentals for the 9 of October leads to a strong increase in prices.

Flowing between countries

As said above the algorithm contains is partly made of the exchanges between France and Germany. After price computation (thanks to the deltas described above) is done for both countries, a price is assigned to each country. Then if prices are equal nothing is done before the block acceptation computation (see later on). But if prices are not equal then a flow (leading to a new delta and then two new prices) is created through the following steps:

- 1) Computations of prices for France and Germany
- 2) If a difference exists, and the limit capacity is not reached, 100 MW

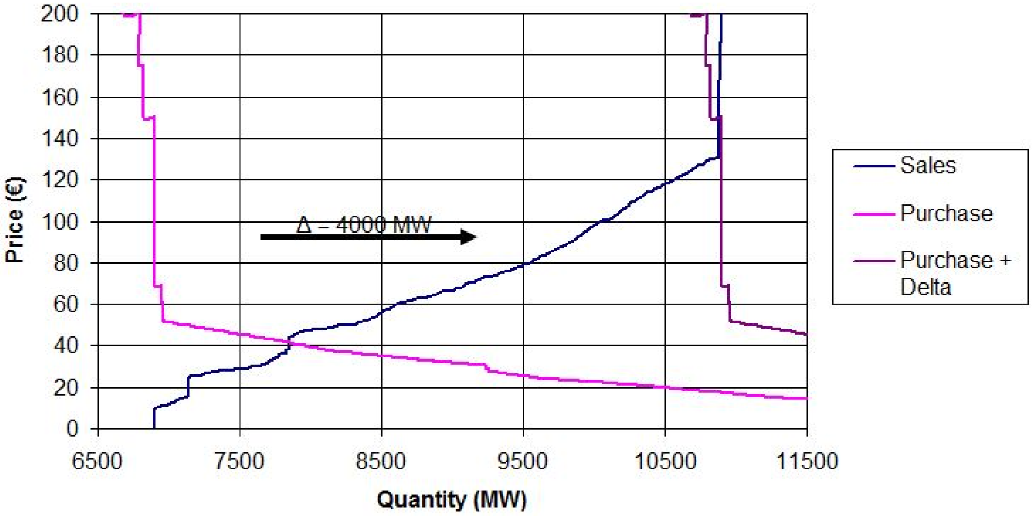

Figure 2.2: New price computation 10/09/2014 H1 zoom

are flowed from the cheapest to the most expensive country.

- 3) New computation of prices
- 4) If the difference has still the same sign, 100 MW are flowed
- 4bis) If the maximal capacity is reached then the loop stops.
- 4ter) If the price difference has not the same sign as the precedent step, the loop stops and the price in both countries is equal to the mean of the two new computed prices.
- 5) etc...

Block order acceptation

Block orders are not behaving like classical orders. They need a special treatment. So at the end of each flowing phase all the block orders are checked. If a block order is in-the-money then it is added to the delta. It changes the global delta and also prices. That's why the algorithm needs more than one global loop. The comprehension of the block order algorithm is not mandatory for the following. What is important to know is that it has to be run at the end of the flowing and implies a new loop of the flowing algorithm since the total delta has changed.

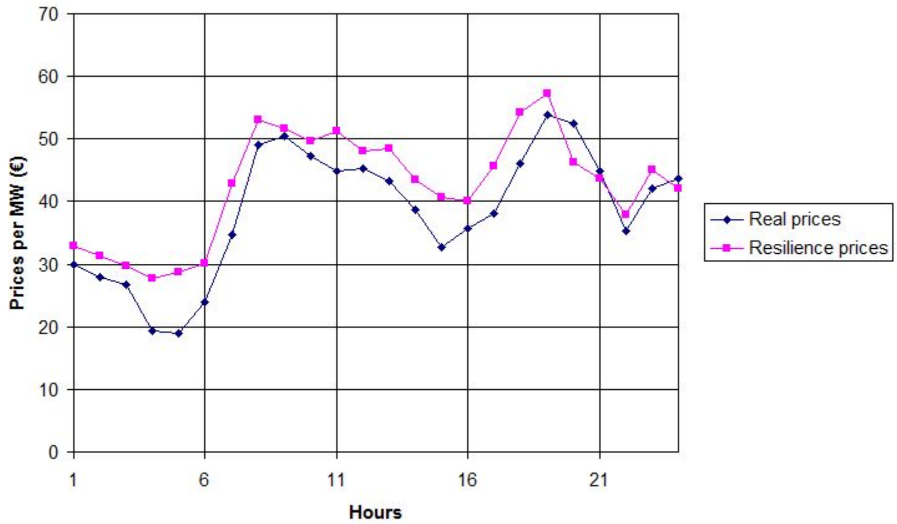

Figure 2.3: French prices for day 2

2.1.3 Some results

With this algorithm the results are quite interesting. The main interest of such a tool is that it can be easily modified. The user can play on numerous fundamental factors and see a direct influence on the forecast prices. Two very important prices are the base price and the peak price. The base price corresponds to the average of the 24 prices for the day-ahead. The peak price is equal to the average of the prices between 9 a.m. and 9 p.m. (approximately the hours of the day where the consumption and the economical activity are strong). These prices are quite interesting because they give a good idea of the trend but mainly because they are the typical financial products which are sold on the OTC market. A sophisticated forecast of prices implies a high probability of earning money by taking a short or long position.

On the table 2.3 we see the differences between real and foreseen peak and base prices. These prices are computed for some week days of November and December 2015 which are not necessarily successive. There we see some decent predictions with an error of less than 10 % in most of the cases. On the Figure 2.4 the forecast and real prices for each hour of the day 2 are drawn, the Figure 2.4 is similar but for day 4.

We can see on these Figures that in the worst case (if we think about mean values) prices are generally underestimated. In the best case prices are not much closer in absolute values but there is a sort of compensation process which cancels errors.

Day	Base	Peak
1	2.3%	1.6%
2	10.5%	9.1%
3	4.7%	3.3%
4	0.6%	0.2%
5	2.2%	0.1%
6	8.0%	7.3%
7	0.1%	0.7%
8	2.1%	3.7
9	4.4%	5.6%
10	8.1%	6.4%
11	2.9%	1.0%
12	1.0%	0.2%
13	4.4%	8.6%
14	7.2%	6.0%
15	1.4%	0.2%
16	2.4%	2.7%
17	1.8%	1.4%
18	0.0%	2.4%
19	4.4%	1.1%
Mean	**3.6%**	**3.2%**
Standard variation	**3.7%**	**3.3%**

Table 2.3: Difference between real and forecast prices

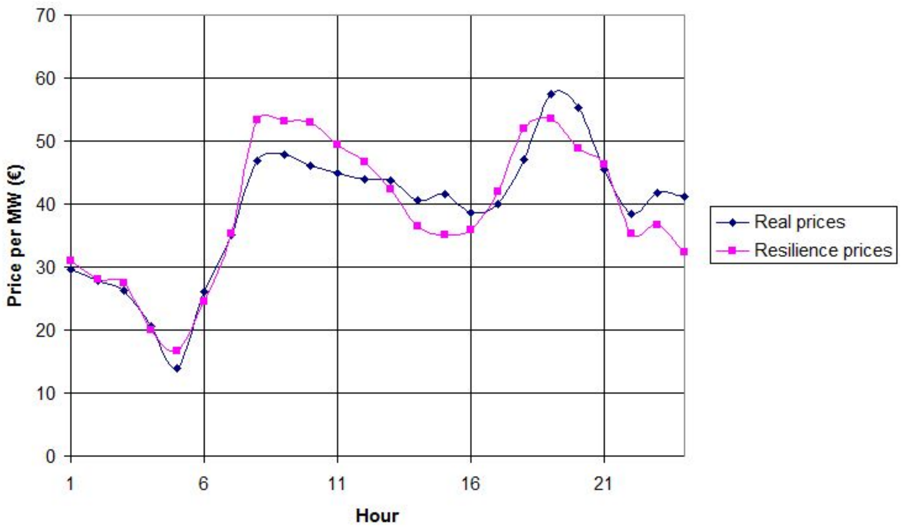

Figure 2.4: French prices for day 4

2.2 New adapted algorithms

2.2.1 Purpose

The purpose of this section can be summarized with the following sentence:

> Adapt the delta model concept to the flow-based methodology in order to get a reliable day-ahead forecast tool.

As we can see the purpose is really clear but the way to achieve it is unclear. That is due to the fact that flow-base is a brand new thing and if a lot of physical publications have been printed (by the grid operators mainly) on this subject, the publications more finance-oriented are basically nonexistent.

One of the first obvious thought is that this delta model tool has to be improved to fit the CWE area. In other words if the old delta model involves only two countries and one connection the new one should necessarily involve 4 countries and 4 connections. Indeed there is no direct link between Belgium and Germany and between France and the Netherlands.

2.3 ATC algorithm with four countries

The first step was to build a tool using the same fixed connection limits between countries but taking into account all four countries.

2.3.1 Scheme of the algorithm

The basic principle is the same as the simpler tool described before. But the problem is slightly more complex due to the increase of variables. Now there are 8 variables: the 4 algebraic flows between countries and the 4 prices. Of course these variables are computed for each hour of the next day. But a strong characteristic of the situation is the independence between the maximal capacities and the flows. Then the idea which grew fast was to act separately. This was the solution implemented. The principle is then to flow between two countries at each time. Two kind of algorithm were used: the "rectangular" one and the "circular" one. They are described on the Figure 2.5 and 2.6. The rectangular one is rather simple: the flows are realized successively until there is a convergence in the French prices (let us say that the price difference between two loops shouldn't exceed 1 € for example). Until some prices are found the Block Order acceptation algorithm is run and then the first flow algorithm is run once again because the total delta has changed due to the blocks. For the circular algorithm, the idea is to flow from Belgium to Netherlands through France and Germany and then flow from Netherlands to Belgium through Germany and France. This is repeated as many time as necessary to reach a convergence in the French prices. Then

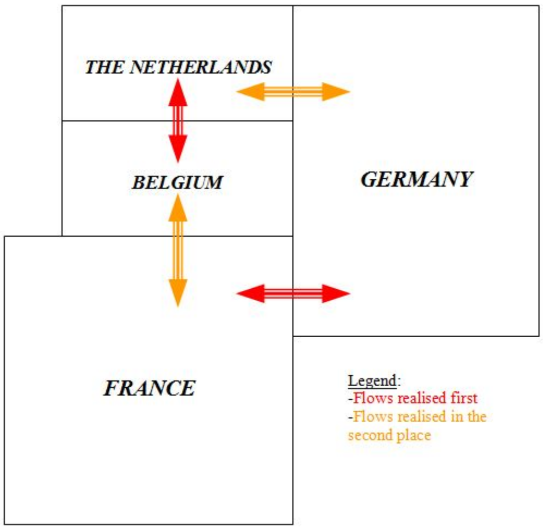

Figure 2.5: Rectangular scheme

the same Block Order acceptation tool is run and then the circular flowing algorithm is run.

2.3.2 Some results

Simulations have been run for the same 19 weekdays of November and December. On tables 2.4 and 2.5 some global numerical results are shown as in the previous section. We can see that the results are quite similar for both cases. This was expected because the independence between maximal capacities and effective flows should lead to the same prices in both cases. The differences come from the fact that the delta model tables have a step of 100 MW. Hence some approximations are done which leads to some small differences.

2.4 Flow-based algorithm

Changes

The global idea of the FB tool is the same as in the previous case but the main difference is inside the flowing part between countries. In this case the maximal transfer capacity is not constant but depends on the effective flows, so a stage has to be added in the flowing algorithm. The principle described at the beginning of this chapter is still the same: a flowing is realized between countries depending on the prices in these countries, 100 MW are flowed and then a new price checking is done and as well as a new maximal capacity. The

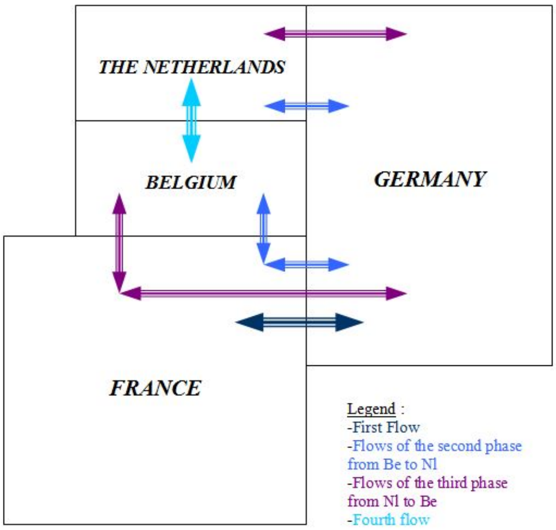

Figure 2.6: Circular scheme

Day	Base	Peak
1	0.2%	1.7%
2	3.7%	2.3%
3	4.5%	1.7%
4	1.2%	0.5%
5	1.9%	0.3%
6	7.7%	6.5%
7	1.6%	0.7%
8	5.5%	7.0
9	3.4%	3.7%
10	8.2%	3.5%
11	3.3%	1.0%
12	4.3%	5.5%
13	4.9%	7.4%
14	5.8%	4.7%
15	4.9%	2.0%
16	2.9%	3.1%
17	6.0%	5.3%
18	3.8%	0.8%
19	0.4%	4.1%
Mean	**3.8%**	**3.2%**
Standard Deviation	**3.9%**	**3.3%**

Table 2.4: Results with rectangular scheme

Day	Base	Peak
1	0.2%	1.4%
2	0.2%	0.9%
3	5.6%	3.0%
4	0.2%	1.9%
5	2.0%	0.0%
6	8.1%	7.0%
7	1.9%	1.4%
8	5.5%	7.1
9	2.8%	2.8%
10	6.0%	4.6%
11	4.2%	2.6%
12	3.6%	5.1%
13	4.6%	7.3%
14	7.0%	6.8%
15	7.0%	4.2%
16	3.3%	4.0%
17	6.0%	5.3%
18	3.9%	2.4%
19	5.0%	10.2%
Mean	**4.1%**	**4.1%**
Standard deviation	**4.3%**	**4.2%**

Table 2.5: Results with circular scheme

definition of the maximal capacity changes. Before the real flowing action, a computation is realized with the PTDF matrix and the RAM vector in order to define the new maximal capacity between the two countries we are interested in at this moment of the algorithm. Let's take a concrete example: in the circular scheme, we are at the end of the third phase and we want to assess the price difference between France and Belgium and then flows electricity if there exists a price difference. At this stage of the algorithm we know all other flows between other countries. The only unknown flow we have to deal with is the one between France and Belgium. Then with this information it is possible to extract a temporary maximal and a temporary minimal capacity for the flow between France and Belgium. This process is sum up in the following:

- 1) Storage of flows between Belgium and Netherlands, between Netherlands and Germany and between Germany and France.
- 2) Computation of the extreme capacities for the Franco-Belgian exchange at this moment with the PTDF and RAM.
- 3) Evaluation of the price difference between the countries.
- 4) If a difference exists, flowing of 100 MW in the appropriate sense.
- 5) New checking of the price difference
- 6) If the temporary maximal capacity is reached then the loop stops.
- 6bis) If the price difference has not the same sign as in the precedent step, the loop stops and the price in both countries is equal to the mean of the two new computed prices.
- 6ter) If none of the precedent cases are realized, 100 MW are added to the flow.
- 7) etc ...

Concerning the paths, the two ways evoked previously are also put into practice in the flow-based case. So for each simulation there exists two results: one is due to the rectangular scheme and the other one is due to the circular scheme.

Results

In the same way as above, some results of this forecasting flow-based day-ahead prices tool are presented.

Comparing the tables 2.6 and 2.7 we see that the rectangular process seems to present generally some more reliable results. But if we look at the

Day	Base	Peak
1	2.6%	5.8%
2	1.5%	1.7%
3	8.2%	4.4%
4	0.1%	0.7%
5	3.1%	0.3%
6	1.0%	2.1%
7	0.8%	0.4%
8	7.8%	8.0%
9	4.8%	6.1%
10	1.6%	1.2%
11	0.7%	2.3%
12	1.6%	3.2%
13	11.5%	11.5%
14	1.0%	1.6%
15	8.2%	1.0%
16	4.4%	2.2%
17	1.5%	3.2%
18	5.7%	1.9%
19	1.2%	5.4%
Mean	**3.5%**	**3.3%**
Standard deviation	**3.2%**	**2.9%**

Table 2.6: FB results with the rectangular scheme

Day	Base	Peak
1	1.3%	4.8%
2	13.7%	14.7%
3	1.6%	0.8%
4	0.7%	1.1%
5	12.2%	12.3%
6	5.3%	4.3%
7	9.8%	2.2%
8	23.0%	22.7%
9	8.2%	14.6%
10	0.8%	0.7%
11	4.6%	3.6%
12	8.9%	0.1%
13	6.2%	7.3%
14	2.4%	0.5%
15	10.9%	2.8%
16	4.0%	1.5%
17	14.5%	4.3%
18	6.6%	11.0%
Mean	**7.4%**	**6.1%**
Standard deviation	**7.7%**	**6.2%**

Table 2.7: FB results, circular

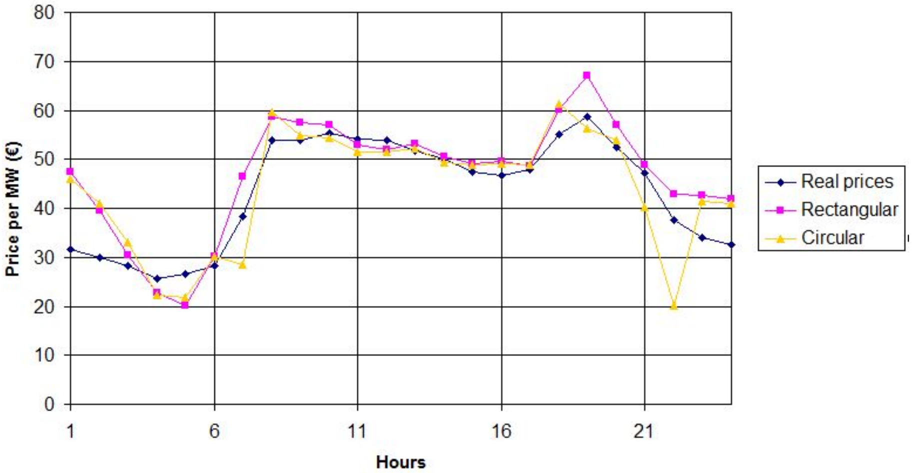

Figure 2.7: FB French prices for day 3

third day we see that the circular scheme gives a better result. That's why the circular scheme is still used, in order to have another look which in some cases might be valuable.

The Figures 2.7 and 2.8 show that the circular scheme seems to display more values which are unexpectedly different from what is expected from rectangular forecasting nor the reality. What is important to underline is that it is normal that the rectangular and circular values are significantly different. The dependance of the maximal capacities with the flows implies that the result should not be the same if we take different paths. Simpler said, two different ways doesn't lead to the same place. Both places should be close but not equal. In the ATC case (in the section before) it was not the case: whatever the way, the result is the same. In the Flow-base calculation, it is not the same thing to first flow between France and Belgium or between France and Germany.

Summary

The delta model algorithm is based on an elementary principle of micro-economy. In this method prices are the crossing point of the modified bid and ask curves. Curves can be modified thanks to external information from the market (such as prediction of wind or consumption). This delta model algorithm can be adapted to the flow based condition with more loops implemented. Results obtained are quite robust. An advantage of the delta method is that it is simple to use with some qualitative knowledge of the market. One of the main disadvantage is that it is almost impossible to predict prices for more than one day. A week-prediction with the delta

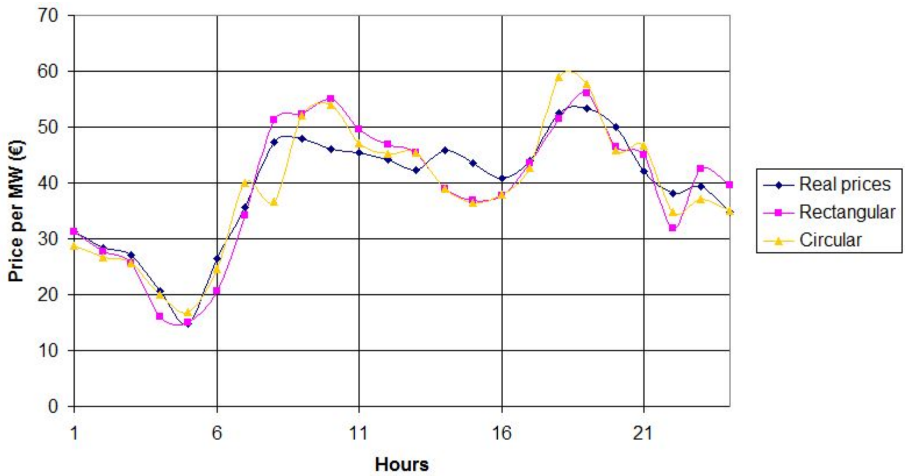

Figure 2.8: FB French prices for day 4

model algorithm would be senseless.

Chapter 3

Time series model: a stochastic approach

In the following the purpose is to try to build a robust predictive model for the French day-ahead electricity prices without using a-priori knowledge of the market. In other words the aim is to build a model based on past values which can be easily found, thanks to statistical tools.

3.1 Introduction and preliminary transformations

The work will be based on the article "Arima Model to Predict Next-Day Electricity Prices" [7]. We will use time series theory to model the evolution of the French electricity spot prices and forecast them.

3.1.1 Why ?

There exists two main probabilistic approaches for the electric market: time-series and classical stochastic model. The first one is usually more adequate for the short-term picture and to the quite strong correlation between prices. The second one is more often used for middle and long term prediction because of its ability to anticipate some non-ordinary scenarios (with modeling of peak-prices thanks to Lévy processes for example). A good introduction of this approach can be found in the Helyette Geman's book [4].

As it can be seen on the Figure 3.1 the serie of French electricity market price presents a high frequency, a non-constant-mean and variance, multiple seasonality effects and a quite high volatility. These characteristics can be taken into account by an ARIMA model.

Finally even if the theory behind the time series approach is not the simplest one, the use of a predictive ARIMA model can be done easily without deep knowledge in mathematics and statistics. This is an advantage since all the actors in electric trading desk are not necessarily mathematicians.

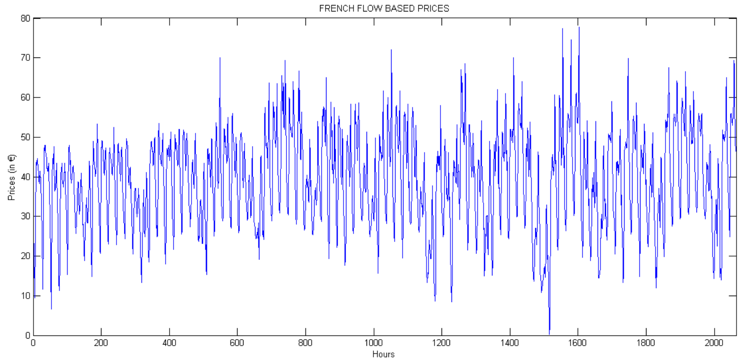

Figure 3.1: The French spot prices in the flow-based computation from 2014/09/01 to 2014/11/25

3.1.2 Data

Data used for this modeling is public. To be precise it is not exactly public but will be as soon as the electric spot launch the flow-based computation. Nevertheless, these data are past simulation prices run by CASC who is in charge of the flow-based implementation. CASC publishes each day 24 flow-based simulated prices for each countries of the CWE area (meaning France, Belgium, Germany and the Netherlands) since the beginning of the year 2013. For the first and second section of this chapter data used will be the French FB prices between the 1^{st} of September 2014 and the 25^{th} of November 2014. They can be seen on the Figure 3.1.

3.1.3 Preliminary transformations

First of all we see that for the 3^{rd} of November (which corresponds to the hour interval [1513,1536]) there are really low prices . The low prices can be explained by a Sunday night, a really low consumption and a combination of really strong wind and flood. In order to get more regular data we remove this day from the time series.

As the period of interest is the beginning of the winter it is normal to see a linear increase of prices mean and variance. We can also clearly see a daily and weekly seasonality on the Figure 3.1 which is perfectly logical knowing the dynamics of the electric market. According to Brockwell and Davis [5] it can be interesting to use a variance-stabilizing transformation such as the Box-Cox transformation. Here we simply and logically take $U_t = log(X_t)$ where X_t is the time series of the prices drawn on Figure 3.1. A computation of the average predictor error on a duration of 55 days give sensibly the same results with or without the logarithm. In order to get more smoothed curves and to slightly reduce the increase of the volatility the logarithm is used in

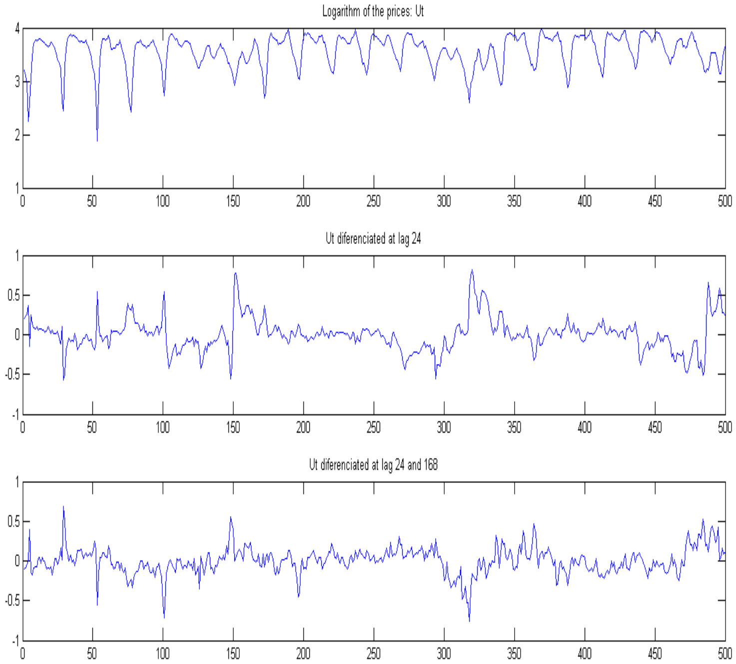

Figure 3.2: Preliminary transformations operated on the French spot prices

the following. Then the serie exhibits a clear daily and weekly seasonality, it seems natural then to differentiate at lag 24 and 168. Finally we want to assess the possibility of a unit root: we want to know if it seems correct to differentiate at lag 1 or not. For that we conduct an augmented Dickey-Fuller test (see appendix B). The results are: $p{-}value = 0.001$ and the test statistic $\widehat{\tau} = -11.9$. We can conclude a really high presumption against the presence of a unit root. On Figure 3.2 the different transformations can be seen. For more clarity only the 500 first terms of the time-series are represented. In a nutshell the third plot of the Figure 3.2 is the representation of the time series $Y_t = (1-B^{24})(1-B^{168})logX_t$ where B defines the back shift operator.

3.2 Modeling

In this section we will try to present the work done and the test run in order to get a satisfying model.

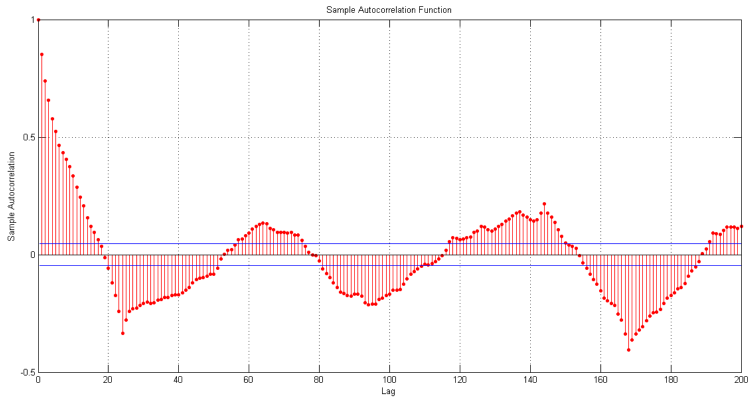

Figure 3.3: Sample auto correlation function of Y_t

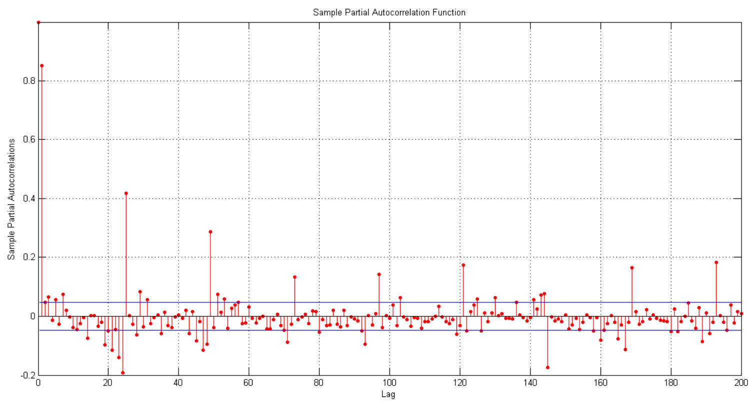

Figure 3.4: Sample partial auto correlation function of Y_t

3.2.1 Auto correlation and partial auto correlation functions

On Figures 3.3 and 3.4 we can see the sample auto correlation and partial auto correlation function of Y_t for the lag 0 until 200.

These Figures clearly exhibit the fact that the serie Y_t cannot be modeled by an AR or MA model but only by an ARMA model. The purpose of this section is to find a model which can be satisfying. One of the main criteria of "satisfaction" is that the model exhibits some tendencies which have been seen on the diverse residual plots and that the final residuals are as close as possible to a white noise serie.

3.2.2 The four models

Four models have been kept. Three have been built by a careful and stepwise analysis of the residuals of the sample minus the fitted model. The ultimate purpose is that the final residuals are as close as possible to a white noise process. The first model is the "unrestricted" one. By unrestricted we mean that it is a model which is long to build with optimization procedure because it exhibits a lot of lag dependencies. The second and third one are two models build step by step regarding the residuals after each new added lag to the model. The second model focuses on the moving average part and then on both parts. It is the opposite for the third model, a close attention have been carried to the auto-regressive part in a first time. The last model is a test model, it is an ARMA(2,2) model. In the following of this section, we will try determine which of the first three models is the best. After it has been done, the selected model will be compared (with prediction error) at the end of the section with the really simple fourth model in order to assess the necessity to build a relatively complex model. The four models are:

$$\text{Model 1: } \boldsymbol{\Phi}_1^1(B)\boldsymbol{\Phi}_{168}^1(B)Y_t = \boldsymbol{\Theta}_1^1(B)\boldsymbol{\Theta}_{168}^1(B)\epsilon_t$$

$$\text{Model 2: } \boldsymbol{\Phi}_1^2(B)Y_t = \boldsymbol{\Theta}_1^2(B)\boldsymbol{\Theta}_{168}^2(B)\epsilon_t$$

$$\text{Model 3: } \boldsymbol{\Phi}_1^3(B)\boldsymbol{\Phi}_{168}^3(B)Y_t = \boldsymbol{\Theta}_1^3(B)\boldsymbol{\Theta}_{168}^3(B)\epsilon_t$$

$$\text{Model 4: } (1 - \phi_1^4 B - \phi_2^4 B^2)Y_t = (1 - \theta_1^4 B - \theta_2^4 B^2)\epsilon_t$$

The full polynomials expression can be found in the appendix B. On the Figure 3.5 the autocorrelation and partial autocorrelation of the residuals of the first three models can be observed.

These graphics seems to show that the residuals are close to a white noise.

3.2.3 Test for independence

In order to assess the independence of the residuals a Ljung-box test is realized (see appendix B). The results are shown on table 3.1. The test is realized for h = 200.

α	0.01	0.05	0.10
$\chi^2_{1-\alpha}(200)$	249.4	234.0	226.0
Model	1	2	3
Test-value	189.7	211.9	270.95

Table 3.1: Ljung box test result

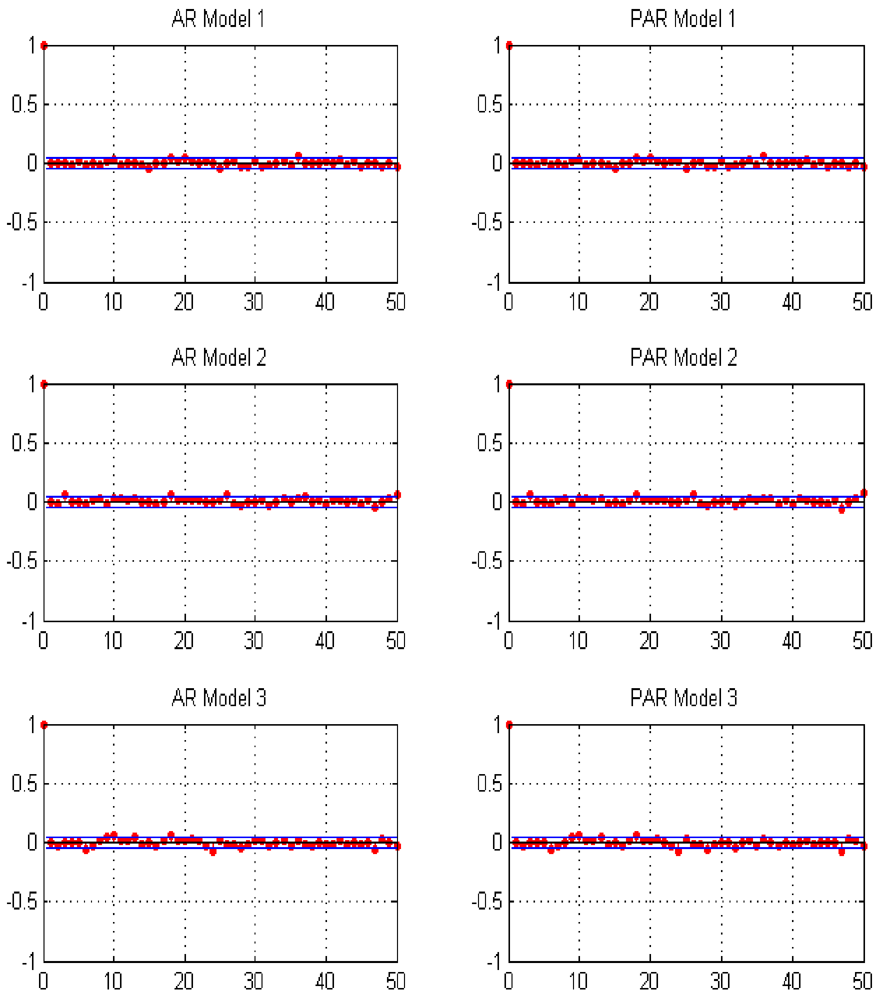

Figure 3.5: Sample normal and partial autocorrelation functions for lag 0 to 50 for residuals of the first 3 models

α	0.01	0.05	0.10
Model 1	1	1	1
Model 2	1	1	1
Model 3	0	0	0

Table 3.2: Acceptation or not of the test

The test seems to clearly say that both model 1 and 2 have white noise residuals but it is not the case for model 3. Hence model 3 is abandoned at this point.

3.2.4 Maximum likelihood, AIC and BIC

On table 3.3 different values for the two remaining models are shown.

	log(L)	AIC	BIC
Model 1	1864	-1135	6021
Model 2	1669	-2519	-254.6

Table 3.3: Results for information criteria

Logically the model 1 (the "unrestricted" model) has the highest AIC and BIC statistics. Then Model 2 seems to be the best fitting model.

3.2.5 A back testing

As we have said above the data data kept was run from the 1^{st} of September until the 25^{th} of November. Some Monte-Carlo simulations are run for some random days of this period. The days are:

- Day 1: 20^{th} of October 2015
- Day 2: 14^{th} of November 2015
- Day 3: 19^{th} of November 2015

The number of simulations is equal to 25. The results can be seen on Figure 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11.

We see that for day 1 and 3 model 2 fits better than model 1. But for the model 2 it seems to be the opposite. Nevertheless, the back-testing seems to show that both models are relevant. Because of its high AIC and BIC criteria the model 2 is retained for the last modeling section. A linked reason for choosing the model 2 is the fact that the computation of the polynomial terms thanks to the likelihood optimization is faster.

3.2.6 A prediction comparison between the model 2 and the model 4

The model 2 has been chosen. We want to compare it with the basic fourth model. To do it we compute the average prediction error for a day between the 1^{st} of October and the 25^{th} of November. Concretely for each day, a recursive prediction of 24 prices is realized using the model and the data of the precedent days. Then the sum of the 24 square of the difference between

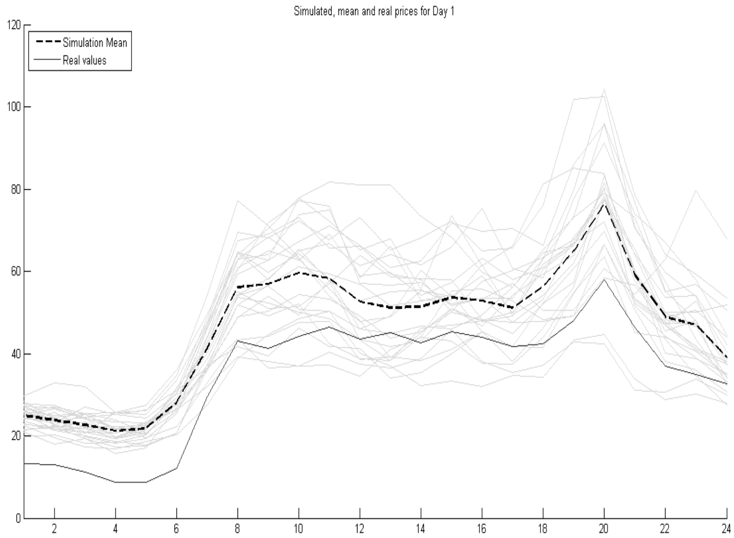

Figure 3.6: Result of the simulation for the 20^{th} of October, Model 1

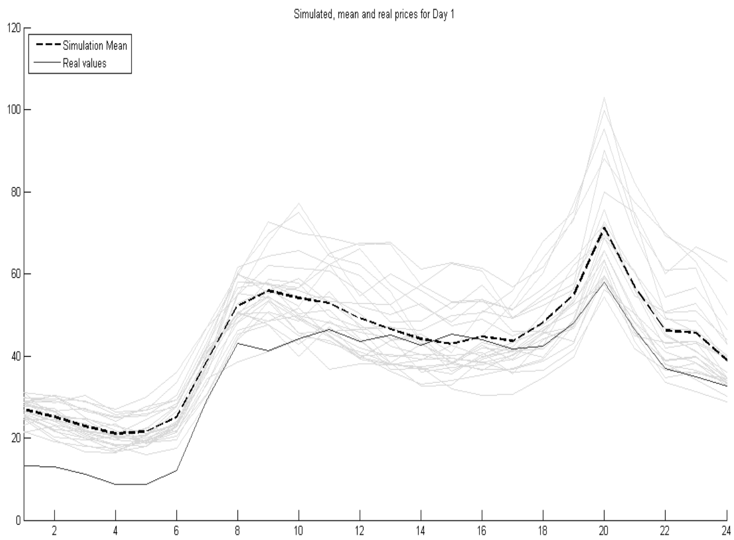

Figure 3.7: Result of the simulation for the 20^{th} of October, Model 2

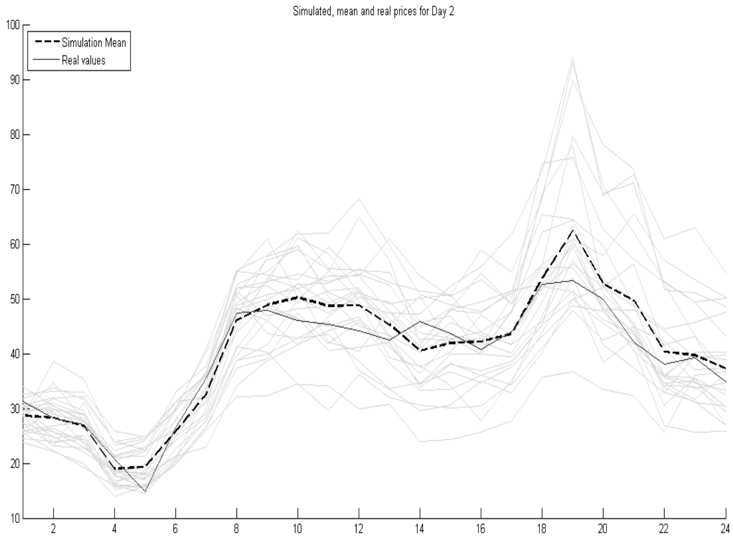

Figure 3.8: Result of the simulation for the 14^{th} of November, Model 1

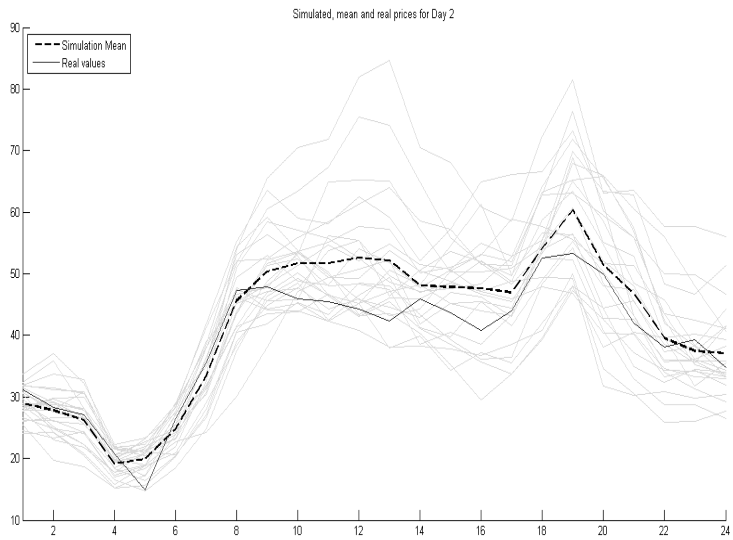

Figure 3.9: Result of the simulation for the 14^{th} of November, Model 2

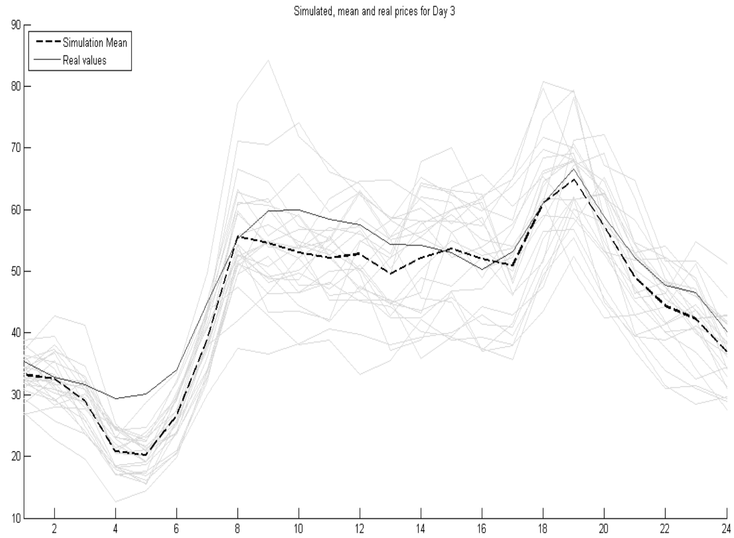

Figure 3.10: Result of the simulation for the 19^{th} of November, Model 1

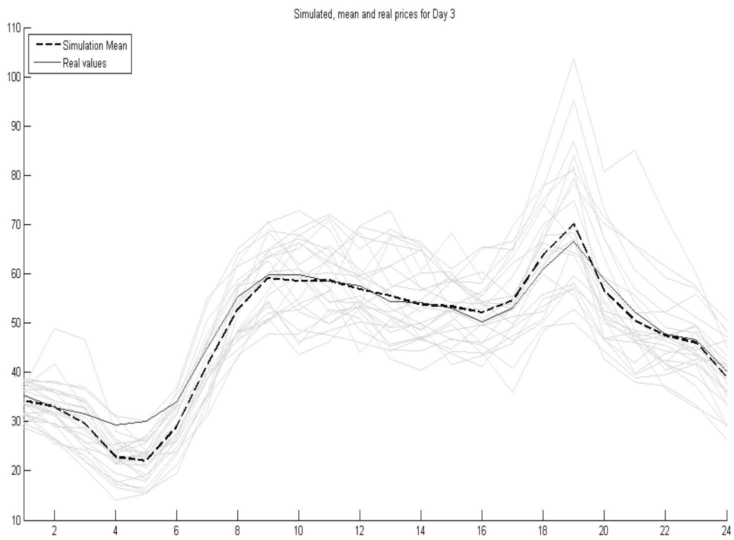

Figure 3.11: Result of the simulation for the 19^{th} of November, Model 2

predicted and real prices are computed. A mean value is computed for both model on the period described above. The results are shown on the Table 3.4.

	Model 2	Model 4
Average prediction error	1181	6790

Table 3.4: Results for prediction error

The results shows that the model 2 seems to present a strong advantage on the model 4 in term of precision of the prediction. It is not useless to try to build a complex and elaborate model. The results between a complex model and a really simple one are sensibly different.

3.3 Forecasting

Here we will use the process described below to forecast the prices for the same days as in chapter 2. This will be done with all data (here flow-based prices) available the day before the evaluated day. An example of this process is given for "Chapter 2 Day 1" (which corresponds in the reality to the 6^{th} of November).

- Parameters of model 2 described in previous section are estimated thanks to a maximum likelihood maximization realized on the flow-based prices from the 1^{st} of September to the 5^{th} of November.
- A Monte-Carlo simulation like the one done previously is done to forecast prices for the 6^{th} of November.
- Spread between the real and forecast based and peak prices are computed and summarized in the table 3.4.

Then we see on table 3.5 the results of the forecast realized for the same days as chapter 2.

The results can be divided in two categories. From the November 6^{th} until November 27^{th} the results are quite comparable with the results obtained in the previous chapter. But they are slightly worse. After December 3^{rd} the results seems to become far less satisfying. This may come from the fact that the model has been built on a series running until the November 25^{th}. It indicates that the steps which have been realized in the two precedent sections must be redone at shorter intervals (less than a week). The model seems to be valid for a quite short term even if it can give quite robust prediction on a weekly scale (see in the conclusion).

For example a quick realization of the process described above in order to forecast the prices for December 4^{th} and December 5^{th} based on data until December 3^{rd} leads to the errors in Table 3.6.

Day	Date	Base	Peak
1	11/06	9.6%	12.3%
2	11/07	23.9%	21.7%
3	11/13	1.5%	6.7%
4	11/14	2.7%	3.4%
5	11/18	2.9%	4.9%
6	11/19	5.6%	6.6%
7	11/20	9.2%	7.5%
8	11/21	4.9%	5.9%
9	11/25	9.2%	16.6%
10	11/26	8.7%	10.5%
11	11/27	3.6%	4.0%
12	12/03	16.7%	10.1%
13	12/04	24.1%	24.5%
14	12/05	26.3%	22.2%
15	12/11	28.4%	28.0%
16	12/12	3.9%	7.2%
17	12/16	22.9%	7.2%
18	12/17	15.5%	26.5%
19	12/18	33.6%	16.4%
Mean		**13.3%**	**13.0%**
Standard deviation		**10.2%**	**8.1%**

Table 3.5: Results of forecasting with time series model

Day	Date	Base	Peak
13	12/04	16.0%	5.0%
14	12/05	3.0%	4.3%

Table 3.6: Results of forecasting adapted data

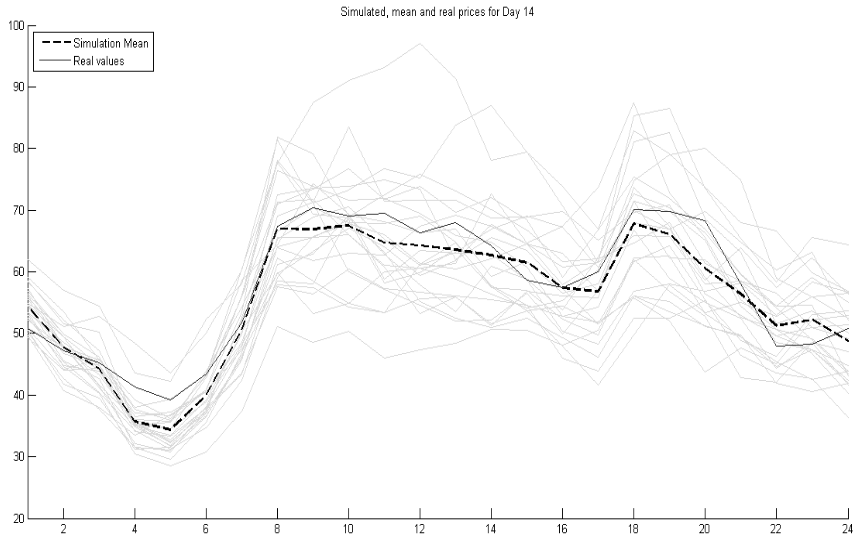

Figure 3.12: Result of the simulation for the 5^{th} of December with updated data

On the Figure 3.1 we can see that the prediction obtained for the 12/05 with the new model better fits the real price values.

Summary

In this section we have fitted an ARIMA model to the prices on a certain period at the beginning of the winter, where the volatility increases. The model is determined regarding dependencies of residuals and some prediction criteria. Results obtained are coherent. The main advantage of such a method is that it can be used for longer term (a week for example see the conclusion below). One of the main disadvantage is that it can be a little bit tricky to find a proper ARIMA model.

Conclusion

Comparison between the two models

Comparing the two models leads to the conclusion that the day-ahead forecast seems to be a bit better with the deterministic tool. This can be seen on Figure 3.13 where the delta model algorithm seems to give in both cases a slightly better price prediction.

It is quite predictable knowing that this tool can be modified much more easily depending on the evolution of the fundamentals. This tool exhibits also a strong correlation between the fundamentals driving the market and the prices. But it is important to underline that the deterministic tool is totally unable of middle-term prevision, such as week for example. It is however not the case for the time series approach. On Figure 3.12 a week prevision can be seen with real prices, this week holds from the 26^{th} of November until the 2^{nd} of December.

Results for this simulation are of course not perfect but they give a good idea of the trend of the prices.

Openings

There is a lot of work to be conducted on this brand-new project which is the financial side of flow-based computation. Many more other approaches can be imagined. Considering the two approaches developed in this work, we will briefly mention some path of improvement.

Concerning the deterministic algorithm, as we have said the "path" found is not the best, another algorithm based on a more brutal approach could be implemented to find the best way. This algorithm could for example conduct a global examination of all of possibilities allowed by the PTDF matrix and then by a complex optimization process find the best way. But this algorithm would be much more time consuming than the one built above and it is not certain that the investment would be rewarded because the price difference between ATC and FB method is not that huge.

As regards the time series approach, the method used here is quite basic and simple to reproduce since it only relies on the past prices. But it could

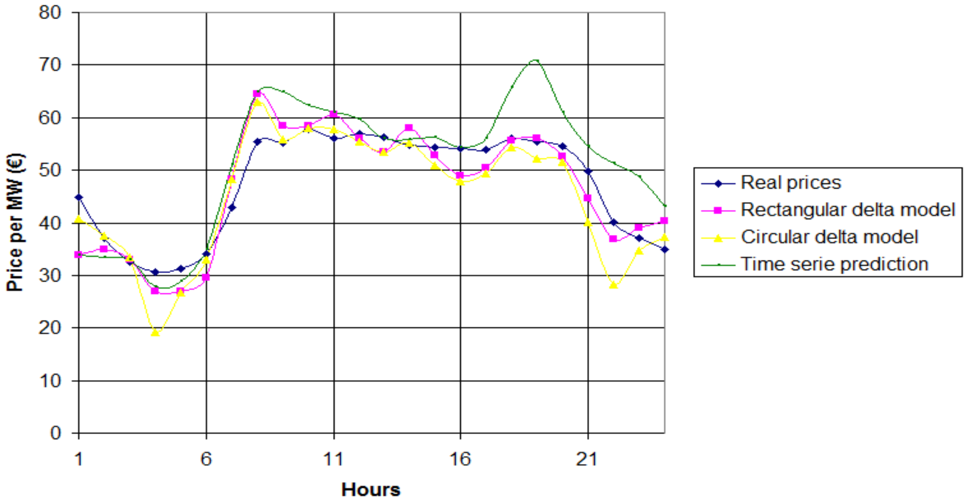

Figure 3.13: Prediction results for the 26th of November with the two methods

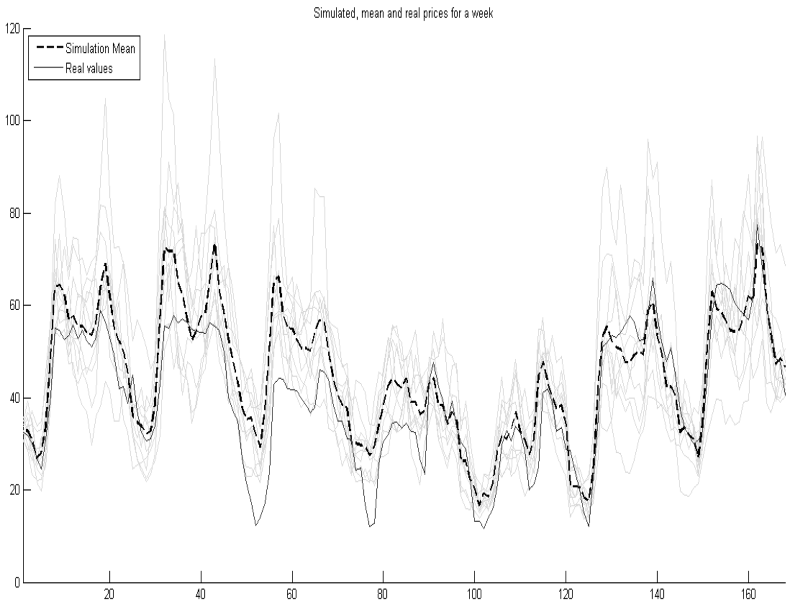

Figure 3.14: Result of the simulation for 7 days between November 26^{th} and December 2^{nd}

be quite interesting to implement exogenous factors (such as consumption and/or load) which have a strong correlation with electricity prices. Some scholars have worked on this improved time series ([6]) and it seems to increase the precision of the results.

A mix of these two approaches could be imagined, for example by using time series modeling in order to predict orders for each hour of the next day and then using these previsions in the algorithm with a better global optimization computation, close to the COSMOS algorithm described in Chapter 1.

Appendix A

Block acceptation algorithm

Here we describe the general principle of the algorithm for acceptation or rejection of block orders.

The main issue with the block order is the interdependence between the fact that they are in-the-money and the price. For example if a sales block order of 100MW and 40 e per MW is in the money then it will be added to the global delta. But the delta will change and it may be possible that the market clearing price changes in a way that the block order in now out of the money. And that is a basic case for a simple block order. This recent implementation of smart block order has made the problem much more complex. The idea is to check more than once the criterion: in or out the money. This is done depending of the nature of the block. There are basically two loops in the algorithm. The first checks all the block orders downward, and the second realizes the same thing but upward. In the first loop if a block order is in the money, then the volume is added to the global delta. If it is not in the money nothing is done. In the second loop all the block orders are checked once more The algorithm is also adapted for the smart blocks. Multiple checking are realized for the different smart blocks in order to avoid if possible to accept out-of-the money blocks, or reject in-the-money blocks.

Appendix B

Time-serie

B.1 Augmented Dickey-Fuller test

The principle of the test is to evaluate the possibility of unit root or not. If we take a simple example with an AR(1) process like that:

$$X_t = \phi_1 X_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathrm{WN}(0, \sigma^2)$$

The Dickey Fuller test will test the two hypothesis:

$$\begin{cases} H_0 : \phi_1 = 1 \\ H_1 : \phi_1 < 1 \end{cases}$$

Now we can rewrite the AR(1) model as:

$$\nabla X_t = X_t - X_{t-1} = \phi_1^* X_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathrm{WN}(0, \sigma^2)$$

where we have $\phi_1^* = \phi_1 - 1$. Now if $\hat{\phi_1}^*$ is the least-square estimator of ϕ_1^* found by regressing X_t on X_{t-1} Dickey and Fuller defined the t-ratio:

$$\hat{\tau} = \frac{\hat{\phi_1}^*}{\hat{SE}(\hat{\phi_1}^*)}$$

If $X_1, X_2, ..., X_n$ are the observations of the AR(1) process the standard error is defined as:

$$\begin{cases} \hat{SE}(\hat{\phi_1}^*) = S(\sum_{t=2}^{n} (X_{t-1} - \overline{X})^2)^{-\frac{1}{2}} \\ S = \frac{1}{n-3} \sum_{t=2}^{n} (\nabla X_t - \hat{\phi_0}^* - \hat{\phi_1}^* X_{t-1})^2 \end{cases}$$

The augmented Dickey-Fuller test reject the null hypothesis if $\hat{\tau} < -2.86$ at the 0.05 level (and if $\hat{\tau} < -3.43$ at the 0.01 level)

B.2 Ljung-Box test

The Ljung-Box test is a refined version of the portmanteau test ([5]). This test aims at quantifying the hypothesis stating that the time serie X_t is a white noise or not. Instead of checcking each of the sample autocorrelation, the idea of the Ljung-Box test is to check if the sum $\tilde{Q}_{LB}$ follows a chi-squared law with h degree of freedom (where h is the number of sample autocorrelations taken in account, which cannot basically exceed one quarter of the total samples n according to Box and Jenkins [9]). The sum is defined as:

$$\tilde{Q}_{LB} = n(n+2)\sum_{j=1}^{h}\frac{\hat{\rho}^2(j)}{n-j}$$

If we assume that X_t is an iid sequence with finite variance (in other words a white noise) then for large n the sample autocorrelation $\hat{\rho}(1), \hat{\rho}(2), ..., \hat{\rho}(h)$ are approximately $N(0, 1/n)$ distributed. Then $\tilde{Q}_{LB}$ is approximately distributed as the sum of squares of independent $N(0, 1/n)$ random variables. Hence:

$$\tilde{Q}_{LB} \sim \chi^2(h)$$

Finally at level α the hypothesis that X_t is a white noise is rejected if $\tilde{Q}_{LB} > \chi^2_{1-\alpha}(h)$.

B.3 Maximum Likelihood, AIC and BIC statistics

The Maximum Likelihood

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If we assume that X_j is a Gaussian time series, that $\hat{X}_j = P_{j-1}X_j$ is the best linear predictor of X_j in term of $X_1, X_2, ..., X_{j-1}$, $\mathbf{X} = (X_1, ..., X_n)'$, $\hat{\mathbf{X}} = (\hat{X}_1, ..., \hat{X}_n)'$ and Γ_n is the covariance matrix $E(\mathbf{X}_n\hat{\mathbf{X}}_n)$; the maximum likelihood is fully defined by:

$$L(\Gamma_n) = (2\pi)^{-n/2}(\det\Gamma_n)^{-1/2}\exp(-\frac{1}{2}\mathbf{X}_n'\Gamma_n^{-1}\mathbf{X}_n)$$

Nevertheless the computation of this expression can be really long and hard even for a powerful machine. In the case where X_t is an ARMA(p,q) process Γ_n can be expressed in term of the polynomials of the model. Even if X_t is not Gaussian which is practically speaking always the case, it still makes sense to look at the likelihood because a large sample distribution of the estimators is the same for a time series iid distributed whether it is Gaussian or not. For an ARMA process defined in a classic way as:

$$\phi(B)X_t = \theta(B)Z_t, \qquad \epsilon_t \sim \mathrm{WN}(0, \sigma^2)$$

using the innovation algorithm, $\check{X}_{n+1}$ can be expressed as (where m = p+ q):

$$\check{X}_{n+1} = \begin{cases} \sum_{j=1}^{n} \theta_{nj}(X_{n+1-j} - \check{X}_{n+1-j}), & 1 \leq n < m \\ \phi_1 X_n + ... + \phi_p X_{n+1-p} + \sum_{j=1}^{q} \theta_{nj}(X_{n+1-j} - \check{X}_{n+1-j}), & n \geq m \end{cases}$$

where θ_{nj} is determined by the innovation algorithm. And if we define:

$$\begin{cases} W_t = \sigma^{-1} X_t, & t = 1, ..., m \\ W_t = \sigma^{-1}\phi(B) X_t, & t = 1 > m \end{cases}$$

then writting $r_n = E(W_{n+1} - \hat{W}_{n+1})^2$ the likelihood for the ARMA process is:

$$L(\phi, \theta, \sigma^2) = \frac{1}{\sqrt{(2\pi\sigma^2)^n r_0 ... r_{n-1}}} \exp(-\frac{1}{2\sigma^2} \sum_{j=1}^{n} \frac{(X_j - \check{X}_j)^2}{r_{j-1}})$$

Estimation of the coefficient of the ARMA process are obtained by maximizing the likelihood (usually one differentiates the logarithm of $L(\phi, \theta, \sigma^2)$).

AIC and BIC criteria

Likelihood approach provides rather good estimators of the model but it doesn't take into account the size of the model. In reality if p and q are huge the model loses some of its robustness because of accumulation of prediction errors done when evaluating the coefficient. AIC and BIC implement a penalty factor within the likelihood function. If $\hat{\beta}$ is the vector of the estimator we have:

$$\mathrm{AIC}(\hat{\beta}) = -2L(\hat{\beta}) + 2(p + q + 1)$$

Some Monte Carlo studies ([5]) have shown that the AIC has a tendency to overestimate p. The BIC is a criterion which corrects this over fitting nature. If n is the number of observations of the time series and if n is large the BIC criteria can be approximated by:

$$\mathrm{BIC}(\hat{\beta}) = -2L(\hat{\beta}) + \log(n)(p + q + 1)$$

The purpose when this estimators are used is to minimize them.

B.4 The polynomials for the 3 models

For model 1 we have:

- $\mathbf{\Phi}_1^1(B) = 1 - \phi_1^1 B - \phi_2^1 B^2 - \phi_3^1 B^3 - \phi_4^1 B^4 - \phi_5^1 B^5 - \phi_6^1 B^6 - \phi_7^1 B^7 - \phi_8^1 B^8 - \phi_9^1 B^9 - \phi_{10}^1 B^{10} - \phi_{11}^1 B^{11} - \phi_{12}^1 B^{12} - \phi_{13}^1 B^{13} - \phi_{23}^1 B^{23} - \phi_{24}^1 B^{24} - \phi_{25}^1 B^{25} - \phi_{26}^1 B^{26} - -\phi_{47}^1 B^{47} - \phi_{48}^1 B^{48} - \phi_{49}^1 B^{49} - \phi_{51}^1 B^{51} - \phi_{71}^1 B^{71} - \phi_{72}^1 B^{72} - \phi_{73}^1 B^{73} - \phi_{95}^1 B^{95} - \phi_{96}^1 B^{96} - \phi_{97}^1 B^{97} - \phi_{119}^1 B^{119} - \phi_{120}^1 B^{120} - \phi_{121}^1 B^{121} - \phi_{144}^1 B^{144} - \phi_{145}^1 B^{145} - \phi_{146}^1 B^{146}$
- $\mathbf{\Phi}_{168}^1(B) = 1 - \phi_{168}^1 B^{168} - \phi_{336}^1 B^{336} - \phi_{504}^1 B^{504}$
- $\mathbf{\Theta}_1^1(B) = 1 - \theta_1^1 B - \theta_2^1 B^2 - \theta_3^1 B^3 - \theta_4^1 B^4 - \theta_5^1 B^5 - \theta_6^1 B^6 - \theta_7^1 B^7 - \theta_8^1 B^8 - \theta_9^1 B^9 - \theta_{10}^1 B^{10} - \theta_{11}^1 B^{11} - \theta_{23}^1 B^{23} - \theta_{24}^1 B^{24} - \theta_{25}^1 B^{25} - \theta_{47}^1 B^{47} - \theta_{48}^1 B^{48} - \theta_{49}^1 B^{49} - \theta_{71}^1 B^{71} - \theta_{72}^1 B^{72} - \theta_{73}^1 B^{73} - \theta_{95}^1 B^{95} - \theta_{96}^1 B^{96} - \theta_{97}^1 B^{97} - \theta_{119}^1 B^{119} - \theta_{120}^1 B^{120} - \theta_{121}^1 B^{121} - \theta_{139}^1 B^{139} - \theta_{140}^1 B^{140} - \theta_{141}^1 B^{141}$
- $\mathbf{\Theta}_{168}^1 = 1 - \theta_{168}^1 B^{168} - \theta_{336}^1 B^{336} - \theta_{504}^1 B^{504}$

For model 2 we have:

- $\mathbf{\Phi}_1^2(B) = 1 - \phi_1^2 B - \phi_2^2 B^2 - \phi_{10}^2 B^{10} - \phi_{11}^2 B^{11} - \phi_{13}^2 B^{13} - \phi_{24}^2 B^{24} - \phi_{25}^2 B^{25} - \phi_{26}^2 B^{26} - \phi_{27}^2 B^{27} - \phi_{30}^2 B^{30} - \phi_{48}^2 B^{48} - \phi_{51}^2 B^{51} - \phi_{72}^2 B^{72} - \phi_{75}^2 B^{75} - \phi_{96}^2 B^{96} - \phi_{120}^2 B^{120} - \phi_{121}^2 B^{121} - \phi_{144}^2 B^{144} - \phi_{145}^2 B^{145} - \phi_{146}^2 B^{146}$
- $\mathbf{\Theta}_1^2(B) = 1 - \theta_1^2 B - \theta_2^2 B^2 - \theta_5^2 B^5 - \theta_8^2 B^8 - \theta_9^2 B^9 - \theta_{10}^2 B^{10} - \theta_{24}^2 B^{24} - \theta_{48}^2 B^{48} - \theta_{96}^2 B^{96}$
- $\mathbf{\Theta}_{168}^2 = 1 - \theta_{168}^2 B^{168}$

For model 3 we have:

- $\mathbf{\Phi}_1^3(B) = 1 - \phi_1^3 B - \phi_3^3 B^3 - \phi_4^3 B^4 - \phi_5^3 B^5 - \phi_6^3 B^6 - \phi_{72}^3 B^{72} - \phi_{144}^3 B^{144} - \phi_{145}^3 B^{145}$
- $\mathbf{\Phi}_{168}^3(B) = 1 - \phi_{168}^3 B^{168} - \phi_{192}^1 B^{192}$
- $\mathbf{\Theta}_1^3(B) = 1 - \theta_6^3 B^6 - \theta_{24}^3 B^{24} - \theta_{96}^3 B^{96}$-$\theta_{120}^3 B^{120} - \theta_{144}^3 B^{144} - \theta_{145}^3 B^{145}$
- $\mathbf{\Theta}_{168}^3 = 1 - \theta_{168}^3 B^{168}$

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