Evaluation of Alternative Weighting Techniques on the Swedish Stock Market

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Abstract

The aim of this thesis is to evaluate how the stock index SIX30RX compares against portfolios based on the same stock selection but with alternative weighting techniques. Eleven alternative weighting techniques are used and divided into three categories; heuristic, optimisation and momentum based ones. These are evaluated from 1990-01-01 until 2014-12-31.

The results show that heuristic based weighting techniques overperform and show similar risk characteristics as the SIX30RX index. Optimisation based weighting techniques show strong overperformance but have different risk characteristics manifested in higher portfolio concentration and tracking error. Momentum based weighting techniques have slightly better performance and risk-adjusted performance while their risk concentration and average annual turnover is higher than all other techniques used.

Minimum variance is the overall best performing weighting technique in terms of return and risk-adjusted return. Additionally, the equal weighted portfolio overperforms and has similar characteristics as the SIX30RX index despite its simple heuristic approach. In conclusion, all studied alternative weighting techniques except the momentum based ones clearly overperform the SIX30RX index.

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List of Abbreviations

ARIMA	Autoregressive integrated moving average.
ERC	Equal risk contribution.
HHI	Herfindahl-Hirschman index.
OMXS30	The Nasdaq OMX Stockholm 30 index. Market capitalisation weighted.
RSI	Relative strength index.
SIX30RX	The SIX30 return index. Market capitalisation weighted with reinvested dividends.

1 Introduction

During the last decade there has been a strong growth of index funds in Sweden. Index funds are promoted through low fees and the fact that they, on average, give a competitive return in comparison to actively managed mutual funds ([19, 25, 23]). Today, several popular index funds are based on the market capitalisation weighting technique.

An index fund tracks a stock index by attempting to follow the same methodology as the one used in the stock index. The stock index is a virtual stock portfolio using a passive strategy that automatically rebalances the portfolio through some pre-determined method. The rebalancing is done by first determining a subset of stocks and then weighting these according to some weighting technique. The stock selection approach varies across different indices although the most common versions is to select stocks based on size in terms of market capitalisation or to select stocks based on trading volume over some previous period, e.g. the last six months ([29]).

Nasdaq offers several different stock market indices with the most prominent in the Swedish market being OMXS30, which is a selection of the 30 most traded stocks on the Swedish Stock Exchange weighted by their market capitalisation. The reason for choosing 30 constituent stocks is that for the Swedish Stock Exchange this is a good trade-off between ensuring liquidity of the shares in the index while simultaneously showing the underlying trend of the stock market as a whole ([31]). OMXS30 does not reinvest dividends.

Another index is SIX30RX that is based on the same methodology but reinvests dividends and is provided by SIX Financial Information. This index is used as a benchmark for professional investors and there are popular index products on the Swedish market replicating the performance of the SIX30RX index ([33]).

Another aspect to consider when constructing an index is how often stock selection and rebalancing should occur. This aspect produces a trade-off between the desire to keep the index weights up to date while not changing the index constituents too frequently. The most common approaches are quarterly (e.g. S&P 500, FTSE 500) or semi-annual rebalancing (e.g. OMXS30, SIX30RX). This study employs semi-annual rebalancing in line with the OMXS30 and SIX30RX indices. With the above preliminaries in place, the focus of this study was on different portfolio weighting techniques while using the same stock selection as the OMXS30 and SIX30RX indices.

The standard weighting technique across all equity indices is to use a market capitalisation based weighting technique, sometimes with a free-float adjustment factor to account for stocks not traded. The idea behind the market capitalisation weighting as a benchmark index is to create a portfolio that replicates owning the whole stock market, or at least fully owning the largest publicly traded companies. The rationale behind the market capitalisation weighting technique, as an investment strategy, is the following:

1. It is an easy strategy to implement,
2. It is automatically rebalanced as stock prices fluctuate,
3. The largest weights are assigned to the companies with the highest market capitalisation and
4. Under the CAPM paradigm it can be interpreted as the market portfolio and thus automatically Sharpe ratio maximised.

Points (1)-(3) above ensure that the portfolio is operationally easy to implement, mainly because it requires little active oversight, have limited rebalancing and automatically limits transaction costs since it trades in highly liquid stocks ([16]).

However, optimality of capitalisation-weighted indices is questioned. The market capitalisation weighting technique overweights stocks whose prices are high relative to their fundamentals and underweight stocks whose prices are low relative to their fundamentals. The size of this underperformance is increasing with the magnitude of price inefficiency and is roughly equal to the variance of the noise in prices ([16]). Furthermore, the market capitalisation weighting technique often produces a portfolio that is overly exposed to a few very large stocks leading to under-diversification of the portfolio. As an example, in July 2014 the stock of Hennes & Mauritz made up 12.4% and in July 2000 the stock of Ericsson B made up 38.9% of the OMXS30 index ([21]). In response to this criticism alternative weighting techniques have been introduced. One example is fundamental weighting, where company fundamentals indicating company size, such as number of employees, sales, book value or dividends are used as an indicator instead of price.

In this study eleven alternative weighting techniques have been grouped into three categories; heuristic, optimisation and momentum based ones. Heuristic based weighting techniques are ad hoc weighting techniques established on simple and, arguably, sensible rules ([10]). Both market capitalisation and equal weighting belong to this category. Optimisation based weighting techniques are based on theoretical foundations forming a maximisation or minimisation problem of some mathematical function. An example of this is the minimum variance technique that is based on the CAPM model and concerned with minimising the portfolio variance. Momentum based techniques use an alternative approach altogether, taking the standpoint that stock prices show signs of momentum and trend which can be exploited. All weighting techniques considered in this study requires little active oversight and all trade in the same stock selection of highly liquid stocks. However, the transaction costs generated in each technique is not known in advance.

Previous research shows that many heuristic and optimisation based alternative weighting techniques overperform against the market capitalisation weighting technique. Fundamental, equal, diversity, minimum variance, maximum diversification and risk efficient all overperform against the market capitalisation weighting technique on the MSCI World and S&P500 stock indices over the time periods 1987-2009 and 1964-2009 ([10]). The inverse volatility and equal risk contribution weighting techniques have also shown interesting results ([11]). Additionally, the recently introduced risk-weighted alpha weighting technique gave promising results on the Chinese HengSeng index over the time period 2002-2012 ([1]). There is lacking academic empirical evidence for the momentum weighting techniques used in this study. However, momentum behaviour of stocks can be observed in many markets ([14]). The concept was introduced in 1993 ([18]) and is still applied by several mutual funds despite critique.

It is interesting to study whether the results also hold for the Swedish market since previous research is unable to draw general conclusions over different stock markets and indices. Therefore, the aim of this thesis was to investigate how the SIX30RX index compares against portfolios based on the same stock selection but with alternative weighting techniques. The SIX30RX index is in this study constructed by using data of historical constituents together with historical stock data.

This study investigated the weighting techniques presented in section 2 and considered the time period from 1990-01-01 to 2014-12-31. To make the results of this study relevant for both stock indices and index funds, transaction costs were considered for each weighting technique. However, transaction costs were not measured in absolute levels but in terms of average annual turnover because of different transaction cost levels for different investors and fund managers. This study did not consider any operational aspects of implementing alternative weighting techniques, e.g. resources required to set up and manage a portfolio. Furthermore, taxes and regulations were not taken into account.

The OMXS30 and SIX30RX indices are price return indices constructed with the objective of replicating the whole stock market based on owning a limited number of shares. The OMXS30 and SIX30RX indices have the same rules but the difference between the two is that the OMXS30 index does not include any dividends.

For a security to be included in the index it must be listed on Nasdaq Stockholm and be of an eligible type. Generally, eligible types of securities are ordinary shares and depositary receipts. Security types generally not included in the index are closed-end funds, convertible debentures, exchange traded funds, limited partnership interests, rights, shares of limited liability companies, warrants and other derivative securities. The index is evaluated semi-annually to allow for continued and correct representation of changing stock markets. The index share selection criteria described below are applied using data through the end of November and May, respectively.

Index share additions and deletions are made effective after the close of last trading day of each December and June. The following rules are applied:

1. If, during the control period, an index share is not among the 45 most traded shares on Nasdaq Stockholm, the index share is replaced by the non-index share with the highest traded volume during the control period.
2. If a share is listed on Nasdaq Stockholm, but is not an index share, and is among the 15 most traded shares on Nasdaq Stockholm during the control period, that share is replacing the index share with the lowest traded volume.

There are several more detailed rules regarding e.g. corporate actions that can determine both inclusion and exclusion from the index as well as how the weights are calculated and assigned. However, since this study used Nasdaq's historical constituents for constructing the indices all these rules are implicitly included in the data ([29]).

The remainder of this thesis is structured as follows. Section 2 explains the studied stock index weighting techniques. In section 3 the data and method used in the study is described and necessary assumptions are stated. In section 4 the results are presented and in section 5 they are discussed. Section 6 concludes the study.

2 Stock index weighting techniques

There is a large number of weighting techniques available in varying combinations but this study was limited to investigating the most common methods found in academic literature (e.g. in [2]). A few other methods, some of which are showing promising results, were also tested. An alteration done in this study compared to others was that the cap on how large weight the index can put into a single stock was dynamic and not fixed. This is further described in section 2.2.1.

For each method applied in this study consider a portfolio with weights $w = (w_1, w_2, ..., w_n)^T$ of n risky assets where $\sum_{i=1}^{n} w_i = 1$. No short-selling was allowed, i.e. $w \geq 0$. Let σ_i^2 be the variance of asset i, σ_{ij} be the covariance between assets i and j, and Σ be the covariance matrix. The vector of asset volatilities is denoted by $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T$.

2.1 Heuristic based weighting techniques

2.1.1 Market capitalisation weighting (SIX30RX index)

The market capitalisation weighting technique is easy to implement, automatically rebalanced as stock prices fluctuate and assigns the largest weights to the largest companies. Additionally, under the CAPM paradigm it can be interpreted as a market portfolio and is thus automatically Sharpe ratio maximised ([37]). As previously mentioned, the SIX30RX index uses this heuristic weighting technique.

The market capitalisation weights are specified as

$$w_i = \frac{p_i * n_i}{\sum_{i=1}^{n} p_i * n_i}, \tag{2.1}$$

where p_i is the price and n_i the number of outstanding shares of stock i at the time of rebalancing.

2.1.2 Fundamental weighting

The rationale behind the fundamental weighting technique is critique of market pricing. It is argued that stocks are mispriced and that traditional market capitalisation weighting underperforms since overvalued stocks get larger weights and vice versa ([4]). The fundamental weighting technique tries to find the true stock value by analysing its fundamentals in order to better reflect each individual company's value in relation to others ([20]). The American RAFI Fundamental Index, which is well-known in both practice and academia, uses the following fundamentals as proxy for the stock value in the index construction: sales, book value, cash flow and dividends ([10]).

The weights are determined by calculating four different index weights, one for each fundamental value, based on a five-year average of historical fundamentals. Thereafter the individual company's weight in relation to the other companies is calculated. Finally, the weights each company would have in the four fundamental indices are combined in equal proportion ([4]).

The fundamental weights are specified as

$$w_i = \left(\frac{Sales_i}{\sum_{i=1}^{n} Sales_i} + \frac{Book\,Value_i}{\sum_{i=1}^{n} Book\,Value_i} + \frac{Cash\,Flow_i}{\sum_{i=1}^{n} Cash\,Flow_i} + \frac{Dividends_i}{\sum_{i=1}^{n} Dividends_i} \right) * \frac{1}{4}, \tag{2.2}$$

where $Sales_i$, $Book\ Value_i$, $Cash\ Flow_i\ and\ Dividends_i$ is the 5-year averages for each metric for stock i.

2.1.3 Equal weighting

The rationale behind the equal weighting technique is to avoid a large concentration of only a few stocks in the portfolio. One simple way to do this is to assign equal weights to each stock, thus lowering the concentration of large companies in favour of smaller companies. The equal weighting technique is a simple heuristic model widely used due to its ease of implementation and uniform diversification ([2]).

The equal weights are specified as

$$w_i = \frac{1}{n}, \tag{2.3}$$

where n is the number of stocks in the index.

2.1.4 Diversity weighting

The rationale behind the diversity weighting technique is that the traditional market capitalisation weighting is not sufficiently diversified since it puts too much weight on companies with high market capitalisation. Equal weighting is a simple way to solve this problem whereas diversity weighting is more advanced where one sets a maximum weight on an individual stock.

Mathematically it can be represented as a modification of the traditional market capitalisation weighting by introducing the constraint that $w_i \leq c$, where $c \, \epsilon \, \left[\frac{1}{n}, 1\right]$ is a cap variable used to avoid putting too much weight in only a few stocks. The weighting procedure follows an iterative process, where in the first step the portion of the weights exceeding c is redistributed to the remaining stocks not exceeding c according to their market capitalisation. The iteration is repeated until the constraint is satisfied for all stocks ([11]). If the cap is set to $\frac{1}{n}$ the diversity portfolio would become the equal portfolio. If the cap is set much higher the diversity portfolio would become the market capitalisation portfolio. Thus, the diversity weighting technique can be seen as a middle ground between the equal and market capitalisation weighting techniques.

2.1.5 Inverse volatility weighting

The rationale behind the inverse volatility weighting technique is that although finance theory proclaims that low volatility stocks should have low returns, this is not always true ([15]). Thus, one assigns large weights to low volatility stocks. However, the overall portfolio volatility is not necessarily lowered since covariances of stocks are not taken into account. The minimum variance weighting technique presented below does incorporate this aspect.

The inverse volatility weights following [11] are specified as

$$w_i = \frac{1/\sigma_i}{\sum_{i=i}^{n} 1/\sigma_i}, \tag{2.4}$$

where σ_i is the volatility of stock i.

2.1.6 Risk-weighted alpha weighting

The rationale behind the risk-weighted alpha weighting technique is to assign a large weight to stocks with high returns and low variance. To achieve this, Jensen's alpha is employed and risk-adjusted through dividing by the stocks volatility ([1]).

The risk-weighted alpha weights are specified as

$$w_i = \frac{RA_i}{\sum_{i=1}^{n} RA_i}, \tag{2.5}$$

where

$$RA_i = \frac{\alpha_i}{\sigma_i} \tag{2.6}$$

is Jensen's risk-adjusted alpha of stock i. Each α_i is obtained through the regression of excess asset returns over the risk-free rate against excess returns of the market over the risk-free rate

$$r_i - r_f = \alpha_i + \beta_i \left(r_m - r_f\right) + \varepsilon_i, \tag{2.7}$$

where r_f is the risk-free rate, r_i the return of stock i, r_m is the market return, β_i the beta of stock i obtained from the regression and ε_i the error terms.

2.2 Optimisation based weighting techniques

2.2.1 Minimum variance weighting

The rationale behind the maximum diversification weighting technique is to maximise the portfolio Sharpe ratio. The definition of a portfolio Sharpe ratio is $\frac{E[r_p]}{\sqrt{w^T \Sigma w}}$, where $E[r_p]$ is the expected portfolio return and the denominator is the expected portfolio risk ([8]). With this weighting technique it is assumed that expected returns are not possible to estimate and that all stocks have the same expected return $E[r_i] = k$, where k is a constant and r_i the return of stock i. However, it is assumed that variance can be estimated with a fair level of confidence ([11]).

Thus

$$\frac{E[r_p]}{\sqrt{w^T \Sigma w}} = \frac{w^T (E[r_1], E[r_2], \ldots, E[r_n])}{\sqrt{w^T \Sigma w}} = \frac{w^T (k, k, \ldots, k)}{\sqrt{w^T \Sigma w}} = \frac{k}{\sqrt{w^T \Sigma w}}, \tag{2.8}$$

where it is used that

$$w^T * (1, 1, \ldots, 1) = \sum_{i=1}^{n} w_i = 1.$$

Under these assumptions, the portfolio that is obtained through maximising the right hand side of equation 2.8 becomes the Sharpe ratio optimised portfolio. The minimum variance weights are specified as

$$w^* = arg \min_w w^T \Sigma w, \tag{2.9}$$

$$such\,that \begin{cases} 1^T w & = 1, \\ w & \geq 0, \\ w & < c, \end{cases}$$

where c is a cap constraint. The minimum variance optimisation problem is a convex quadratic optimisation problem since the covariance matrix is by its definition positive semi-definite. Without constraints this expression has a simple analytical solution ([17]). When introducing constraints, a numerical optimisation is required ([34]).

Previous research use different cap constraints, some set a fixed level and some use a dynamic one. The rationale in this study for how to choose a suitable cap c was to set the constraint so that a portfolio does not put more weight into single stocks than the market capitalisation weighting technique would have put into a single stock at each rebalancing period. The cap was specified as

$$c = \max(w_{market\,capitalisation}), \tag{2.10}$$

where $w_{market\,capitalisation}$ is the vector of weights obtained from the market capitalisation weighting technique at the rebalancing day considered.

2.2.2 Maximum diversification weighting

The rationale behind the maximum diversification weighting technique is the same as in the minimum variance weighting technique, i.e. to maximise the portfolio Sharpe ratio. As in the minimum variance technique, it is assumed that variance can be estimated with a fair level of confidence. However, in this weighting technique it is assumed that expected returns of assets are proportional to their variances, i.e. $E[r_p] = kw^T\sigma$, where k is a constant. The diversification ratio for any portfolio is defined as $D(w) = \frac{w^T\sigma}{\sqrt{w^T\Sigma w}}$ ([8]). Noting that

$$\frac{E[r_p]}{\sqrt{w^T\Sigma w}} = \frac{kw^T\sigma}{\sqrt{w^T\Sigma w}} = k * D(w), \tag{2.11}$$

maximising *D(w)* is equivalent to maximising $\frac{E[r_p]}{\sqrt{w^T\Sigma w}}$, which is the Sharpe ratio of the portfolio. Thus, with the assumptions above the maximum diversification portfolio also maximises the portfolio Sharpe ratio.

The maximum diversification weights are specified as

$$w^* = arg\max_w \frac{w^T\sigma}{\sqrt{w^T\Sigma w}}, \tag{2.12}$$

$$such\,that \begin{cases} 1^T w & = 1, \\ w & \geq 0, \\ w & < c, \end{cases}$$

where c is set in the same manner as described in equation (2.10) ([8]). This is a quadratic programming problem on a convex set for which a solution exists ([9]).

2.2.3 Risk efficient weighting

The rationale behind the risk efficient weighting technique is the same as in the minimum variance and maximum diversification techniques, i.e. to maximise the portfolio Sharpe ratio. However, in the risk efficient weighting technique it is assumed that expected returns are better estimated through downside deviation than through standard deviation. It is argued that downside deviation is a more meaningful definition of risk than standard deviation since it takes into account only deviations below the mean ([7]). Additionally, a growing number of practitioners are using downside risk in portfolio management applications. It is further argued that expected returns are proportional to downside deviation multiplied by the market risk premium ([7]). This assumption was used to calculate expected returns in a risk efficient weighting technique ([3]).

Thus, the expected return of stock i is estimated as

$$E[r_i] = r_f + Downside\,Deviation_i * (r_m - r_f), \tag{2.13}$$

where r_f is the risk-free rate and r_m is the market return ([3]).

The downside deviation of stock i is defined as

$$Downside\,Deviation_i = \left(\frac{1}{T}\sum_{t=1}^{T} min\,(r_{i,t} - \mu_i,\, 0)^2\right)^{1/2}, \tag{2.14}$$

where $r_{i,t}$ is the return of stock i in day t, μ_i is the average return of the i-th stock and T is the last day ([3]).

In order to increase the robustness of the estimated expected stock returns, the stocks are sorted into deciles according to their downside deviation and the median downside deviation of each decile is assigned as the risk measure of each stock in that decile. This procedure is consistent with cross-sectional asset pricing tests in financial literature ([3, 13]).

Let

$$R(n) = (E[r_1], E[r_2], \ldots, E[r_n]) \tag{2.15}$$

be the vector of expected stock returns for n stocks in any portfolio where $E[r_i]$ is defined as in equation 2.13. Note that under the assumptions of this model the portfolio Sharpe ratio can be rewritten as

$$Sharpe\,ratio = \frac{E\left[r_p\right]}{\sqrt{w^T \Sigma w}} = \frac{w^T R(n)}{\sqrt{w^T \Sigma w}}. \tag{2.16}$$

Thus, the maximisation of the portfolio Sharpe ratio and the risk efficient weights are specified as

$$w^* = arg\ \max_{w} \frac{w^T R(n)}{\sqrt{w^T \Sigma w}}, \tag{2.17}$$

$$such\,that \begin{cases} 1^T w & = 1, \\ w_i & \geq 0, \\ w_i & < c, \end{cases}$$

where c is set as described in equation (2.10) ([3]). Similarly as in section 2.2.2, a solution exists for the quadratic programming problem on a convex set.

2.2.4 Equal risk contribution (ERC) weighting

The rationale behind the ERC weighting technique is that a middle ground should exist between the minimum variance portfolio and the equal portfolio. In this technique, as in the minimum variance technique, it is assumed that expected stock returns estimates are too unstable to be useful but that the expected stock return variance can be estimated with a fair level of confidence. However, it is argued that minimum variance portfolios often suffer from too high portfolio concentration ([24]). This could be mitigated using a simple equally weighted portfolio, although this too has its drawbacks. Thus, the ERC weighting technique seeks to construct a portfolio in the middle ground between the minimum variance portfolio and the equally weighted portfolio.

This is achieved through equalising risk contributions from different portfolio components ([24]). This means that the risk contribution of a component, defined as the share of the total portfolio risk attributable to that component, is not larger than of any other component in the portfolio.

Let $\sigma(w) = \sqrt{w^T \Sigma w} = \left(\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j \neq i}^n w_i w_j \sigma_{ij}\right)^{1/2}$ be the total portfolio risk. Then, the marginal risk contribution of asset i is defined as

$$\delta_{w_i} \sigma(w) = \frac{\delta \sigma(w)}{\delta w_i} = \frac{w_i \sigma_i^2 + \sum_{j \neq i} w_j \sigma_{ij}}{\sigma(w)} \tag{2.18}$$

and the weights corresponding to the ERC portfolio are

$$w^* = \left\{ w \in [0,1]^n : \sum w_i = 1,\, w_i \delta_{w_i} \sigma(w) = w_j \delta_{w_j} \sigma(w)\, for\, all\, i,j \right\}. \tag{2.19}$$

Let $(\Sigma w)_i$ denote index i of the vector product Σw. Now note that $\delta_{w_i} \sigma(w)$ is proportional to $(\Sigma w)_i$ since on vector form the n marginal risk contributions can be written as $\frac{\Sigma w}{\sqrt{w^T \Sigma w}}$. The Euler decomposition[1] gives that

$$\sigma(w) = \sum_{i=1}^n \sigma_i(w) = \sum_{i=1}^n w_i \frac{\delta \sigma(w)}{\delta w_i}. \tag{2.20}$$

[1] A function $f : \mathbb{R}^n \setminus \{0\} \to \mathbb{R}$ is positive homogeneous of degree k if

$$f(ax) = a^k f(x)$$

for all $a > 0$. Euler's homogeneous function theorem says that if f is continously differentiable, then f is positive homogeneous of degree k if and only if

$$\mathbf{w} \nabla f(\mathbf{w}) = k f(\mathbf{w}).$$

It is now verified that the total portfolio risk is the weighted sum of the marginal risk contributions, i.e. that

$$w^T \frac{\Sigma w}{\sqrt{w^T \Sigma w}} = \sqrt{w^T \Sigma w} = \sigma(w). \tag{2.21}$$

Now, the problem can be written on the form

$$w^* = \left\{ w \in [0,1]^n : \sum w_i = 1,\, w_i \left(\Sigma w\right)_i = w_j \left(\Sigma w\right)_j \; for\, all\, i, j \right\}. \tag{2.22}$$

This problem does not offer a closed-form solution and thus requires the use of a numerical optimisation. One way of finding the weights corresponding to the ERC portfolio is to consider the optimisation problem

$$w^* = arg\, \min_w \sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j\right)^2, \tag{2.23}$$

$$such\, that \begin{cases} 1^T w & = 1, \\ w & \geq 0. \end{cases}$$

In effect, the ERC portfolio is found when the goal function is equal to zero or, equivalently, when all risk contributions are equal. The ERC portfolio problem can be specified in an alternative way to more explicitly show the relation between the minimum variance, equally weighted and ERC portfolios, as described in [24]. However, the above specification allows for a stable numerical computation with a unique solution and is the one often preferred ([24]).

2.3 Momentum based weighting techniques

The rationale behind the relative strength index (RSI) and autoregressive integrated moving average (ARIMA) weighting techniques is that stock prices can show behaviours of momentum and trend. These methods have been applied by both practitioners and academics in order to capture stock movements, although neither one is seen as being able to capture the full picture. RSI is a contrarian strategy that buys a stock when its price has fallen whereas ARIMA follows the trend, i.e. it buys a stock when its price is rising.

In the RSI method, the index is weighted proportionately to the strength of the buy signal by maximising the weighted buy signal value constrained by a cap c on the weights as described in section 2.2.1. For the ARIMA method the weights are calculated by maximising the weighted forecasted return using the same cap c.

2.3.1 Relative strength index (RSI) weighting

The RSI was introduced over 30 years ago and has been widely used among traders focusing on technical analysis ever since ([35]). The RSI is an oscillator that shows the strength of the asset price by comparing the individual upward or downward movements of the consecutive closing prices. The RSI value ranges between 0 and 100. An asset with RSI value lower than or equal to 30 is seen as oversold, generating a buy signal, and an asset with RSI value higher than or equal to 70 is seen as overbought, generating a sell signal ([35]).

The relative strength index at time t considering d days is specified as

$$RSI_t(d) = \frac{\sum_{i=0}^{d-1} \left(P_{t-i} - P_{t-i-1}\right) 1 \left\{P_{t-i} > P_{t-i-1}\right\}}{\sum_{i=0}^{d-1} \mid P_{t-i} - P_{t-i-1} \mid} * 100, \tag{2.24}$$

where P_i is the stock price at time i and $1\{\cdot\}$ is the indicator function. The weights are obtained by minimising the weighted RSI value

$$w^*(c) = arg \min_w w^T R, \tag{2.25}$$

$$such\,that \begin{cases} 1^T w & = 1, \\ w & \geq 0, \\ w & < c, \end{cases}$$

where w is the portfolio weights, R the vector of RSI values for each stock and c the cap constraint, set as described in equation (2.10).

2.3.2 Autoregressive integrated moving average (ARIMA) weighting

The ARIMA model is a stochastic time-series model used both for analysing and forecasting time series. The first step in applying an ARIMA model is to choose the parameters p, d and q, which are non-negative integers that refer to the order of the autoregressive, integrated and moving average parts of the model. The model is then referred to as ARIMA(p,d,q). In this study an ARIMA(1,1,1) model was used for all stocks in line with [27].

The model is fitted to a historical time series and thereafter forecasts can be generated. The weights are then obtained by maximising the forecasted portfolio return. Additionally, a cap constraint is used in the maximisation problem.

If d is a non-negative integer, then $\{X_t\}$ is an ARIMA(p,d,q) process if $Y_t := (1-B)^d X_t$ is a causal ARMA(p,q) process. This means that $\{X_t\}$ satisfies

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right) (1-L)^d X_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_t, \tag{2.26}$$

where L is the lag operator, ϕ_i is the parameters of the autoregressive part, θ_i is the parameters of the moving average part and ε_t the error terms. Furthermore, $\{Y_t\}$ is weak-sense stationary.[2]

The best one-day predictor of an ARIMA(p,d,q) process $\{X_t\}$ following [6] is obtained through

$$P_n X_{n+h} = P_n Y_{n+h} - \sum_{j=1}^{d} \begin{pmatrix} d \\ j \end{pmatrix} (-1)^j P_n X_{n+h-j}, \tag{2.27}$$

where $h = 1$, n is the time of the last observed value of $\{X_t\}$ and P_n is the linear prediction operator.

The prediction $P_n Y_{n+h}$ of the ARMA(p,q) process $\{Y_t\}$ in terms of $\{1, Y_1, Y_2, \ldots, Y_n\}$ is described in [6]. To calculate the best predictor 2 days forward in time, the same procedure is applied using the now calculated value for $P_n X_{n+1}$. This procedure is then repeated recursively to obtain longer forecasts.

The ARIMA weights are obtained by maximising forecasted returns

$$w^* = arg \max_w w^T \mu_f, \tag{2.28}$$

$$such\,that \begin{cases} 1^T w & = 1, \\ w & \geq 0, \\ w & < c, \end{cases}$$

where w is the vector of portfolio weights, μ_f the vector of forecasted expected returns from the day of rebalancing until the last forecasted day and c the cap constraint set as described in equation (2.10).

[2] Weak-sense stationary processes are processes where the first moment and the auto-covariance do not vary with time.

3 Data and Method

3.1 Time period and index constituents

Index constituents of OMXS30 were available from Nasdaq since the inception of OMXS30 in January 1987 until January 2015. This period was chosen since it covers several full market cycles including the dot-com bubble in the early 21st century as well as the financial crisis in 2008 and it was the most complete data set available. For each rebalancing period of OMXS30, Nasdaq provided the following data for each constituent; share name, market capitalisation and weight. The data was received in PDF form and manually put into Microsoft Excel.

3.2 Stock data

Thomson Reuters Datastream was used in order to get daily stock data for all current and historical constituents of the index during the time period 1987-01-01 to 2015-01-01. Stock data used for this study were adjusted closing prices, dividend amount, ex-dividend date, payment date and fundamental values. Adjusted closing prices are daily closing prices adjusted for additional information such as stock splits and corporate actions. It was difficult to find stock data for the period 1987-01 until 1989-01. Additionally, the stock of Framtidsfabriken was excluded from the dataset since no stock prices were found in Thomson Reuters Datastream or any other data source available for this study.[3]

Dividend data was available for all constituents for the time period. This included dividend amounts, ex-dividend dates and payment dates. The dividend amount was not always in the same currency as the stock prices, which were in Swedish currency (SEK). Dividends were manually transformed into SEK using the exchange rates mentioned below.

Fundamental values including sales, book value and cash flow were found for all constituents for the time period 2005-2015. Between 2001 and 2005 some data points were missing and between 1987-2000 a larger number was missing, no supplementary data source was found.

3.3 Risk-free rates and exchange rate data

Additional data used in this study includes the Swedish risk free interest rate and foreign exchange rates, available for the whole time period of this study, i.e. 1987-2015. The Swedish risk free interest and exchange rates were obtained from the Swedish National Bank. There seems to be no common standard risk-free rate to be used in this context. The chosen risk free rate was the three-month Treasury bill SSV3M, in line with [36]. This risk-free rate was used throughout the study whenever a risk-free rate was needed. Daily exchange rates were used for the following: SEK/EUR, SEK/USD, SEK/CHF and SEK/FIM.

3.4 Portfolio construction

In order to answer the research question, a portfolio for each weighting technique was constructed. The aim of each portfolio was to illustrate how different index weighting techniques would have performed over the chosen time period. Each portfolio was constructed in the same way except when portfolio weights were calculated at inception and at each rebalancing date. Calculations were done in the Matlab environment.

3.4.1 Assumptions

Some assumptions were made when constructing the portfolios. These are in line with previous research ([24, 8]) and presented below.

[3] Framtidsfabriken was part of the index during two rebalancing periods; July 2000 and January 2001 with weights of 0.45% and 0.07% respectively.

- Short selling is not allowed. Since many fund managers are not allowed to take short positions this assumption is realistic.
- Each portfolio is self-financing, i.e. there is no inflow/outflow of funds into the portfolio except at inception. This also means that there is no need to continuously buy/sell stocks in between rebalancing periods. In reality a fund tracking an index will have inflow and outflow of capital at different points in time, but in research this is often omitted. However, there are inflows in terms of dividend from stocks into the portfolio and therefore stocks were in those cases bought between rebalancing periods.
- It is allowed to trade in fractions of assets. This is a standard assumption in most research papers. It makes the portfolio computations easier to handle and means that the absolute value of the portfolio is not taken into account.
- It is always possible to trade any volume of each stock at the adjusted closing price. In reality, a large portfolio will have some market impact if trading large quantities in an illiquid stock. However, since the stocks included in this study are all highly liquid stocks of large companies, the market impact at rebalancing is negligible.
- No transaction costs are considered. Despite the fact that transaction costs are an important factor in the success of a systematic trading strategy, they are assumed negligible in the construction of each portfolio. Instead, as described in section 4.5, a turnover statistic is used to compare the number of transactions between each weighting technique.

3.4.2 Daily performance updating

The portfolio construction algorithm calculated the performance of each stock in the portfolio as well as the total portfolio each trading day. The portfolio performance was updated after rebalancing the portfolio, using yesterdays closing price. Additionally, each trading day was checked for ex-dividend dates. If a stock included in the portfolio had an ex-dividend date on the current trading day, a marker was set to remember if the portfolio should receive a dividend on the payment date regardless whether the stock was in the portfolio at the payment date or not.

3.4.3 Reinvestment of dividends

This study replicated the SIX30RX index that reinvests dividends on the ex-dividend date ([32]). The reinvestment of dividends on the ex-dividend date can be done easily since SIX30RX is an index constructed without any real assets under management and thus real cash flows does not need to be considered. However, mutual funds like Avanza Zero replicating the SIX30RX index invests dividends on the ex-dividend date although the dividend has not yet been received. This is done using futures contracts, which affects the performance of the index, albeit slightly ([22]).

In this study, dividends were reinvested at the dividend payout date reflecting the actual cash flow of the portfolio. They were reinvested in the index in relation to their weight, as in the SIX30RX methodology. The implication of this is that the SIX30RX index constructed will have a tracking error in relation to the real SIX30RX index. The alternative would be to model the buying and selling of futures contracts to make the constructed portfolios as realistic as possible. However, this was not done in this study since different fund managers can handle dividends differently. It is not common to model this aspect.

3.4.4 Rebalancing

Rebalancing was performed at the same dates as the SIX30RX and OMXS30 indices were historically rebalanced. The rebalancing occurs every six months and includes an update of the index constituents according to historical constituents and calculation of new portfolio weights.

3.4.5 Weighting method considerations

Unless stated otherwise, the time period considered for calculating the risk and return statistics in each weighting technique was 1 year prior to the rebalancing day. If there was less than 1 year of stock data available prior to the rebalancing day, the full period available was used.

The optimisation problems presented in section 2 include both linear and quadratic optimisation problems with linear constraints. All optimisation problems have a unique solution and were solved in Matlab using the *fmincon* method. The *fmincon* method was chosen since it applies to all linear and quadratic optimisation problems with general smooth constraints ([26]).

It is important to remember that most portfolios have a cap constraint on how large one weight can be, which will have a significant effect on the results. Additional weighting technique specific considerations are described below.

Market capitalisation weighting

Market capitalisation data for each stock at each rebalancing date was supplied by Nasdaq. Thus, information on the number of stocks for each company is not needed.

Fundamental weighting

In the fundamental weighting technique described above, the 5-year average of each fundamental value was used. However, in some cases the fundamentals value was not accessible for the whole 5 year period prior to the rebalancing date. In such cases, the average was calculated of all accessible values prior to the rebalancing date. There was insufficient data to construct the portfolio over the whole 25-year period.

Diversity weighting

The weight c was chosen to 10%. The lowest possible value would be $\frac{1}{n}$, where n is the number of stocks in the index, i.e. roughly 3.33%. Historically, the largest share one stock has ever had in the index on a rebalancing day was Ericsson B at 38.9% in July 2000.

Risk-weighted alpha weighting

The market return was calculated as the return of the constructed market capitalisation portfolio. In order to have one year of historical market return data at inception of the portfolio an additional market capitalisation portfolio was constructed starting from 1989-01-02.

Risk efficient weighting

The market return was calculated in the same way as in the risk-weighted alpha weighting technique described above.

RSI weighting

Traders and academics often use a time period of 14 days when calculating RSI ([30]). Thus, 14 days were applied in this study.

ARIMA weighting

In the ARIMA weighting technique, the model was fitted to a historical time series from one year prior to the rebalancing day until the rebalancing day. The chosen forecasting period is until the next rebalancing day, roughly corresponding to 130 days.

3.5 Portfolio evaluation

3.5.1 Time periods

Each portfolio was constructed over the time periods; 1990-01-01 until 2014-12-31, 2005-01-03 until 2014-12-31 and 2012-01-02 until 2014-12-31. The starting date 1990-01-01 was chosen since most weighting techniques considered in this study require one year of historical data and 1989 was the earliest year with a near complete data set. The ten-year period starting in 2005-01 was chosen in order to enable a proper evaluation of the fundamental weighting technique. Fundamental data was available for many stocks from 2000 but for some stocks only available from 2003. Thus, the ten-year period achieves a good trade-off between length of time and quality of the required five-year average fundamental data. Finally, a three-year period was chosen to validate results in the short-term.

A rolling window analysis was also performed. Four windows were evaluated, each with a length of ten years and an overlap of five years. This complements the other time periods in the sense that the end date is varied. The rolling window time periods were 1990-01-01 until 1999-12-31, 1995-01-02 until 2004-12-31, 2000-01-03 until 2009-12-31 and 2005-01-03 until 2014-12-31.

3.5.2 Measures

The following groups of measures were used in order to evaluate the constructed portfolios; performance statistics, Jensen's alpha, correlation, concentration and turnover. Generally, there are many different measures for evaluating performance of funds and portfolios. The chosen measures are in line with previous research within the area of alternative indices ([10, 3, 16, 24]). Not all measures have been included since many of them describe similar characteristics.

Measures in performance statistics are annual return, annual standard deviation, Sharpe ratio, tracking error, information ratio and maximum drawdown. Jensen's alpha describes risk-adjusted return in relation to the market capitalisation portfolio and complements the performance statistics. Correlations were measured between all constructed portfolios. Two concentration measures were included to assess the concentration risk in each portfolio. Finally, a turnover measure was included as an indication of transaction costs. The above mentioned measures should allow an in-depth discussion of the differences between studied weighting techniques.

4 Results

In this section the results are presented. Each group of measurements is analysed and the portfolios with highest and lowest values are mentioned. The returns over the 25-year period for all portfolios are illustrated in appendix A.2.

4.1 Performance statistics

In table 1 the risk and return statistics of the 25-year period are presented. Annual return shows how well a portfolio has performed. Minimum variance (19.1%), risk efficient (18.9%) and risk-weighted alpha (18.7%) had highest annual return over the 25-year period. The market capitalisation (10.8%), diversity (11.7%) and equal (12.7%) portfolios had lowest annual returns. Annual standard deviation measures how risky the portfolio is, i.e. how volatile its returns are. The portfolios with highest annual standard deviation were ARIMA (26.5%), RSI (26.4%) and the risk weighted alpha portfolio (26.1%). The portfolios with lowest annual standard deviations were maximum diversification (21.5%), diversity (21.9%) and equal (22.0%). The Sharpe ratio measures risk-adjusted return and a high value is desired. In terms of Sharpe ratio the top three portfolios were minimum variance (0.75), risk efficient (0.74) and maximum diversification (0.71). Lowest Sharpe ratio was found in market capitalisation (0.42), ARIMA (0.48), RSI (0.50) and equal (0.50).

Table 1: Risk and return statistics over the 25-year period. Daily data was used from 1990-01-01 until 2014-12-31. The highest value in each column is in bold.

Weighting technique	Annual return (%)	Annual SD (%)	Sharpe ratio	Tracking error (%)	Information ratio	Maximum drawdown (%)
Market capitalisation	10.8	22.8	0.42	-	-	**70.2**
Fundamental†	-	-	-	-	-	-
Equal	12.7	22.0	0.50	7.2	0.21	54.0
Diversity	11.7	21.9	0.46	4.2	0.14	56.2
Inverse volatility	16.5	22.0	0.65	9.7	0.50	48.5
Risk-weighted alpha	18.7	26.1	0.66	14.9	0.51	72.9
Minimum variance	**19.1**	22.3	**0.75**	13.3	**0.53**	47.6
Maximum diversification	17.7	21.5	0.71	12.5	0.46	50.9
Risk efficient	18.9	22.2	0.74	13.1	**0.53**	47.5
ERC	16.4	22.3	0.64	11.9	0.41	66.2
RSI	13.8	26.4	0.50	**15.8**	0.22	66.2
ARIMA	13.5	**26.5**	0.48	14.1	0.23	68.7

†The fundamental weighting technique was excluded due to lack of data.

Annual return is the annualised geometric average return. Annual standard deviation is the annualised standard deviation. Sharpe ratio is defined as the annualised average excess return over the risk-free rate divided by the annual standard deviation of excess returns ([28]). Tracking error is defined as $TE = \omega = (\mathrm{Var}(r_p - r_b))^{1/2} = \left(\frac{1}{n}\sum_{i=1}^{n}(r_{p,\,i} - r_{b,\,i})^2 - \left(\frac{1}{n}\sum_{i=1}^{n} r_{p,\,i} - r_{b,\,i}\right)^2\right)^{1/2}$, where $r_{p,\,i}$ is the portfolio return in day i and $r_{b,\,i}$ is the return of the benchmark portfolio in day i. The benchmark portfolio is the market capitalisation portfolio. Information ratio is defined as $IR = \frac{\alpha}{\omega}$ where ω is the tracking error as above and α is the expected value of the active return measured as $\alpha = E[r_p - r_b] = \frac{1}{n}\sum_{i=1}^{n} r_{p,\,i} - r_{b,\,i}$. Maximum drawdown is the largest peak-to-through decline of the portfolio value defined as $\mathrm{MDD}(T) = \max_{\tau\in(0,T)}\left[\max_{t\in(0,\tau)} X(t) - X(\tau)\right]$, where T is the time at the end of the period and $X(t)$ is the stock price at time t.

Table 2: Risk and return statistics over the ten-year period. Daily data was used from 2005-01-03 until 2014-12-31. The highest value in each column is in bold.

Weighting technique	Annual return (%)	Annual SD (%)	Sharpe ratio	Tracking error (%)	Information ratio	Maximum drawdown (%)
Market capitalisation	10.3	22.9	0.49	-	-	53.8
Fundamental	11.4	22.3	0.54	5.2	0.16	51.2
Equal	13.1	23.8	0.58	4.8	0.57	54.0
Diversity	11.0	23.0	0.51	0.9	0.69	53.5
Inverse volatility	**15.7**	23.2	**0.69**	5.2	**0.93**	48.5
Risk-weighted alpha	14.9	**28.8**	0.58	**14.1**	0.39	**72.9**
Minimum variance	14.3	21.6	0.67	9.2	0.35	47.6
Maximum diversification	12.8	21.5	0.61	10.0	0.19	50.9
Risk efficient	14.3	21.6	0.67	9.3	0.35	47.5
ERC	13.4	24.8	0.58	10.8	0.30	66.2
RSI	15.4	26.8	0.62	12.3	0.45	53.4
ARIMA	12.7	25.8	0.55	10.3	0.28	65.0

Performance statistics are defined as in table 1.

The tracking error measures how closely a constructed portfolio follows a benchmark portfolio, which in this case is the market capitalisation portfolio. A low value indicates similar returns in the evaluated portfolio and the market capitalisation portfolio. The three portfolios with lowest tracking error were diversity (4.2%), equal (7.2%) and inverse volatility (9.7%). The portfolios with highest tracking error were RSI (15.8%), risk-weighted alpha (14.9%) and ARIMA (14.1%).

The information ratio is a risk-adjusted return measure that measures the active return of the portfolio divided by the amount of risk the portfolio takes relative to a benchmark, which in this case is the market capitalisation portfolio. A high value indicates that the portfolio beats the benchmark. The portfolios with highest information ratio were minimum variance (0.53), risk efficient (0.53) and risk-weighted alpha (0.51). The portfolios with lowest information ratio were diversity (0.14), equal (0.21) and RSI (0.22). Maximum drawdown measures the largest peak-to-through decline and can be seen as a measure on how much one could possibly lose by investing in the portfolio. The portfolios with lowest maximum drawdown were risk efficient (47.5%), minimum variance (47.6%) and inverse volatility (48.5%). The portfolios with highest maximum drawdown were risk-weighted alpha (72.9%), market capitalisation (70.2%) and ARIMA (68.7%).

The market capitalisation portfolio underperformed all constructed portfolios in terms of annual return, Sharpe ratio and information ratio. Furthermore, it had the second highest maximum drawdown.

In table 2 the risk and return statistics of the ten-year period are presented, including all portfolios. The ten-year period showed similar results as the 25-year period. The market capitalisation portfolio had lower annual return and Sharpe ratio than the other portfolios. The tracking error was lowest for diversity, as over the 25-year period. The fundamental portfolio, which was not included in the 25-year period, performed above market capitalisation and diversity with regards to annual return, Sharpe ratio and information ratio but worse than most other portfolios. The fundamental portfolio also exhibited a low tracking error, information ratio and maximum drawdown.

In table 3 the risk and return statistics of the three-year period are presented. This time period showed different results compared to the other time periods. The market capitalisation portfolio (14.9%) had lower annual return than other portfolios except maximum diversification (13.4%) and equal (14.9%). The Sharpe ratio showed that market capitalisation underperformed all other portfolios except maximum diversification, RSI and equal. The tracking error showed similar results. However, the diversity portfolio had a tracking error of 0.4% indicating that it performed very similar to the market capitalisation portfolio. The maximum drawdown showed similar results as well, where risk-weighted alpha had the highest value. However, market capitalisation had one of the lowest maximum drawdowns in contrast to the 25-year period where it had one of the highest values.

In table 4 annual returns from the rolling window analysis are presented. The portfolios with highest annual return in each time period were respectively risk-weighted alpha (34%), minimum variance (23.2%), minimum variance (12.4%) and inverse volatility (15.7%). The portfolios with lowest annual return in each time period were respectively equal (19.5%), RSI (10.7%), ARIMA (-0.4%) and market capitalisation (10.3%).

In table 5 Sharpe ratios from the rolling window analysis are presented. The portfolios with highest Sharpe ratio were respectively maximum diversification (1.15), minimum variance (0.87), minimum variance (0.51) and inverse volatility (0.69). The portfolios with lowest Sharpe ratio in the first two periods were respectively equal (0.71) and RSI (0.39). In the third period, the portfolios with the lowest Sharpe ratio were ARIMA (0.07) and market capitalisation (0.07). In the fourth period, market capitalisation (0.49) had the lowest Sharpe ratio.

Table 3: Risk and return statistics over the three-year period. Daily data was used from 2012-01-02 until 2014-12-31. The highest value in each column is in bold.

Weighting technique	Annual return (%)	Annual SD (%)	Sharpe ratio	Tracking error (%)	Information ratio	Maximum drawdown (%)
Market capitalisation	14.9	14.5	1.11	-	-	11.4
Fundamental	20.7	15.3	1.43	5.5	**1.05**	10.4
Equal	14.9	14.9	1.08	3.2	0.02	11.6
Diversity	15.0	14.5	1.12	0.4	0.46	11.4
Inverse volatility	16.6	14.5	1.23	3.9	0.43	11.1
Risk-weighted alpha	**29.1**	**21.0**	**1.45**	**14.9**	0.96	**14.7**
Minimum variance	19.5	15.5	1.34	8.0	0.58	12.6
Maximum diversification	13.4	15.3	0.96	8.1	-0.17	11.9
Risk efficient	19.5	15.5	1.33	8.1	0.57	12.6
ERC	14.9	15.4	1.30	7.1	0.55	12.0
RSI	15.7	17.4	1.00	9.4	0.13	14.5
ARIMA	18.6	15.8	1.26	6.7	0.57	14.4

Performance statistics are defined as in table 1.

Table 4: Return statistics over rolling window periods. Daily data was used from 1990-01-01 until 2014-12-31. The time periods were 1990-01-01 until 1999-12-31, 1995-01-02 until 2004-12-31, 2000-01-03 until 2009-12-31 and 2005-01-03 until 2014-12-31. The highest value in each column is in bold.

	Annual return (%)			
Weighting technique	1990-1999	1995-2004	2000-2009	2005-2014
Market capitalisation	21.5	12.7	0.3	10.3
Fundamental†	-	-	-	11.4
Equal	19.5	14.0	5.8	13.1
Diversity	19.9	14.2	3.6	11.0
Inverse volatility	22.6	18.5	11.8	**15.7**
Risk-weighted alpha	**34.0**	18.9	2.1	14.9
Minimum variance	29.7	**23.2**	**12.4**	14.3
Maximum diversification	31.7	21.2	7.3	12.8
Risk efficient	30.0	22.5	11.6	14.3
ERC	19.6	14.1	5.8	13.1
RSI	22.4	10.7	5.4	15.4
ARIMA	26.6	16.9	-0.4	12.7

†The fundamental weighting technique was excluded from some periods due to lack of data.
Annual return is defined as in table 1.

Table 5: Sharpe ratio over rolling window periods. Daily data was used from 1990-01-01 until 2014-12-31. The time periods were 1990-01-01 until 1999-12-31, 1995-01-02 until 2004-12-31, 2000-01-03 until 2009-12-31 and 2005-01-03 until 2014-12-31. The highest value in each column is in bold.

	Sharpe ratio (%)			
Weighting technique	1990-1999	1995-2004	2000-2009	2005-2014
Market capitalisation	0.78	0.49	0.07	0.49
Fundamental†	-	-	-	0.54
Equal	0.71	0.58	0.27	0.58
Diversity	0.73	0.57	0.18	0.51
Inverse volatility	0.83	0.74	0.48	**0.69**
Risk-weighted alpha	1.10	0.71	0.15	0.58
Minimum variance	1.03	**0.87**	**0.51**	0.67
Maximum diversification	**1.15**	0.83	0.32	0.61
Risk efficient	1.05	0.85	0.48	0.67
ERC	0.72	0.58	0.27	0.58
RSI	0.73	0.39	0.26	0.62
ARIMA	0.88	0.57	0.07	0.55

†The fundamental weighting technique was excluded from some periods due to lack of data.
Sharpe ratio is defined as in table 1.

4.2 Jensen's alpha

In table 6 Jensen's alpha values are presented for the 25-year period. The alpha value indicates whether a portfolio is under- or overperforming in terms of its risk-adjusted return in relation to the market capitalisation portfolio. Not all alphas obtained showed significance and the non-significant alphas should therefore only be seen as indicators. The results indicated that all portfolios overperform in terms of their risk-adjusted return in relation to the market capitalisation index. Half of the constructed portfolios showed an alpha greater than 6%, each with a significance level higher than 99%. The highest alpha was obtained for the minimum variance portfolio with an alpha of 8.95%. The portfolio with the lowest obtained alpha was the fundamental portfolio although its alpha was not significant.

Table 6: Jensen's alpha over the 25-year period. The table shows the coefficient estimates from the regressions of daily returns for each portfolio against the market capitalisation portfolio from 1990-01-01 until 2014-12-31. The highest value in the first two columns is in bold.

Weighting technique	Alpha (%)	Beta (%)‡	Alpha t-statistic
Market capitalisation	-	0.0	-
Fundamental†	1.39	95.2	0.86
Equal	2.31	91.9	1.66**
Diversity	1.11	94.6	1.40*
Inverse volatility	6.02	87.7	3.23***
Risk-weighted alpha	8.21	94.5	2.76***
Minimum variance	**8.95**	80.8	3.55***
Maximum diversification	7.70	79.4	3.32***
Risk efficient	8.75	80.9	3.53***
ERC	6.31	84.3	2.79***
RSI	4.20	93.0	1.34*
ARIMA	3.42	**98.6**	1.21*

†The fundamental weighting technique is measured during a ten-year period, from 2005-01-03 until 2014-12-31. ‡All beta values are highly statistically significant at the 99.99% level.
*** significant at the 99% level, ** at the 90% level, * at the 75% level.

The single factor CAPM-based model was used to calculate Jensen's alpha. The model is defined as $r_i - r_f = \alpha_i + \beta_i * (r_m - r_f) + \varepsilon_i$, where r_i is the portfolio return, r_m is the market return, r_f is the risk-free rate, α_i is the intercept and β_i describes the volatility of the asset with respect to that of the market and ε_i is the error term ([19]). If an asset return is higher than the risk-adjusted return, its alpha is positive. The t-statistic is the estimated parameter divided by the estimated standard error of the estimator, e.g. for β calculated as $t_{\hat{\beta}} = \frac{\hat{\beta}}{S.E.(\hat{\beta})}$.

4.3 Correlation

In table 7 the correlation coefficients for the portfolios are presented, showing how well portfolios correlate to one another. In general, all portfolios correlated highly to one another, with correlations ranging between 91.4% (fundamental and RSI) to 99.9% (risk efficient and minimum variance). The portfolios that correlated most with the market capitalisation portfolio were diversity (98.9%), ARIMA (97.7%) and RSI (97.7%).

Table 7: Portfolio correlations over the 25-year period. Daily data was used from 1990-01-01 until 2014-12-31. The highest value in the first two columns is in bold.

Weighting technique	Annual return (%)	Annual SD (%)	Correlations (%)											
Market capitalisation	10.8	22.8	100	97.1	97.0	98.9	93.9	93.1	94.0	95.9	94.1	94.4	97.7	97.7
Fundamental†	11.4	22.3		100	94.8	96.8	94.0	92.8	96.9	91.7	96.9	94.1	94.0	96.0
Equal	12.7	22.0			100	99.3	99.3	95.5	98.9	99.6	99.0	98.3	94.1	98.2
Diversity	11.7	21.9				100	97.6	95.4	97.7	98.7	97.8	97.6	96.2	98.7
Inverse volatility	16.5	22.0					100	94.7	99.6	99.4	99.6	98.4	90.4	96.4
Risk-weighted alpha	18.7	26.1						100	95.5	94.5	95.6	97.3	92.5	97.8
Minimum variance	**19.1**	22.3							100	99.2	99.9	98.8	91.0	96.7
Maximum diversification	17.7	21.5								100	99.2	98.4	92.1	97.2
Risk efficient	18.9	22.2									100	98.8	91.2	96.8
ERC	16.4	22.3										100	91.5	97.7
RSI	13.8	26.4											92.6	96.7
ARIMA	13.5	**26.5**												100

†The correlations for the fundamental weighting technique are calculated during a ten-year period, from 2005-01-03 until 2014-12-31.

Annual return and annual standard deviation were calculated as described in table 1. The correlation coefficients are the Pearson's correlation coefficients calculated as the covariance of the two portfolios divided by the product of their standard deviations.

4.4 Concentration

The portfolio concentration was evaluated using the average Gini and Herfindahl-Hirschman Index (HHI) value, providing information about concentration risk in each portfolio.

In table 8 the average Gini and HHI values are presented. The portfolios with highest average Gini values were the RSI and ARIMA portfolios, each with a value of 0.78. They also had highest average HHI values. The inverse volatility portfolio had the lowest average Gini value, except for the equally weighted portfolio. The equal portfolio had both a low average Gini and HHI value since it is a perfectly balanced portfolio after each rebalancing day.

Table 8: Portfolio concentration over the 25-year period. Daily data was used from 1990-01-01 until 2014-12-31. The highest value in each column is in bold.

Weighting technique	Average Gini	Average HHI
Market capitalisation	0.47	0.08
Fundamental†	0.45	0.06
Equal	0.07	0.04
Diversity	0.41	0.06
Inverse volatility	0.40	0.06
Risk-weighted alpha	0.74	0.15
Minimum variance	0.75	0.15
Maximum diversification	0.75	0.14
Risk efficient	0.75	0.15
ERC	0.49	0.09
RSI	**0.78**	**0.17**
ARIMA	**0.78**	0.16

†The fundamental weighting technique is measured during a ten-year period, from 2005-01-03 until 2014-12-31.

Average Gini is the average of the daily Gini values and average HHI is the average of the daily Herfindahl-Hirschman Index (HHI) values, both calculated as described in appendix A.1.

4.5 Turnover

Transaction costs were not measured in absolute values but rather as portfolio turnover since different investors and fund managers apply different transaction costs. The turnover of a portfolio illustrates how much of the portfolio, in relation to the average portfolio value, is redistributed at each rebalancing date. Turnover is closely correlated with total transaction cost of the portfolio and used as a proxy for this ([12]). The average annual turnover should be seen as support for a discussion about transaction costs. It is important to keep in mind that although a portfolio might perform well in comparison to the market capitalisation portfolio it might not overperform when adjusted for transaction costs. The market capitalisation portfolio had low turnover due to its inherent rebalancing behaviour and is a benchmark for a portfolio with low average annual turnover and transaction costs.

In table 9 average annual turnover of each portfolio is presented. The portfolios with highest average annual turnover were RSI (1.58), ARIMA (1.16) and maximum diversification (1.11). These three portfolios had much higher average annual turnover than the market capitalisation portfolio (0.11), which had the lowest value. The fundamental (0.14) and diversity weighted (0.14) portfolios had only slightly higher average annual turnover than the market capitalisation portfolio (0.11).

Table 9: Portfolio turnover over the 25-year period. Daily data was used from 1990-01-01 until 2014-12-31. The highest value is in bold.

Weighting technique	Average annual turnover
Market capitalisation	0.11
Fundamental†	0.14
Equal	0.23
Diversity	0.14
Inverse Volatility	0.42
Risk-weighted alpha	0.98
Minimum Variance	0.84
Maximum Diversification	1.11
Risk Efficient	0.86
ERC	0.70
RSI	**1.58**
ARIMA	1.16

†The fundamental weighting technique is measured during a ten-year period, from 2005-01-03 until 2014-12-31.

The turnover is defined as the one-way portfolio turnover where only purchases and not sales are considered. The turnover between each rebalancing date is defined as $\frac{Total\,Purchases}{Average\,portfolio\,value} = \frac{\sum_{i=b}^{b+130} purchases_i}{\frac{1}{130-1}\sum_{i=b}^{b+130} value_i}$, where b is any rebalancing day except the first, $value_i$ is the portfolio value at day i and $purchases_i$ is the total value of purchases at day i. Since there are two rebalancing days each year there are 130 trading days between each rebalancing day. This turnover value was thereafter annualised by a factor of 260/130 and thus the annual turnovers were obtained. The average annual turnover was calculated as the average of these annual turnovers.

5 Discussion

In the discussion, results from each alternative weighting portfolio were benchmarked against the market capitalisation portfolio, i.e. the constructed SIX30RX index, and the outcome for each alternative weighting technique was compared and commented.

Fundamental weighting

The rationale behind the fundamental weighting technique is critique of market pricing. It is argued that stocks are mispriced and that traditional market capitalisation weighting underperforms since overvalued stocks get larger weights and vice versa. Thus, if this assumption is true and the fundamental weighting technique would capture the true value of the stock better than the stock price itself, it should be able to overperform the market capitalisation technique.

The results clearly showed that the fundamental portfolio overperformed the market capitalisation portfolio both in terms of return and risk-adjusted performance over both time periods considered. The alpha obtained indicates significant overperformance. Furthermore, the portfolio had a low tracking error and slightly lower maximum drawdown in comparison to the market capitalisation portfolio. Additionally, the correlation with the market capitalisation portfolio was moderate in comparison to the other constructed portfolios. Finally, the fundamental portfolio had slightly lower average Gini and HHI values while keeping a low but slightly higher average annual turnover than the market capitalisation portfolio.

In conclusion, the fundamental weighting technique succeeded in its objective since it overperformed the market capitalisation portfolio in terms of return and risk-adjusted performance while keeping the other attractive attributes of the market capitalisation portfolio.

Equal weighting

The rationale behind the equal weighting technique is to avoid a large concentration of the portfolio in only a few stocks.

The equal portfolio overperformed against the benchmark in regards to annual return and Sharpe ratio over the 25-year and ten-year period. However, it underperforms against the benchmark in the first rolling window period. The alpha obtained showed a slight but significant overperformance. The portfolio had over all time periods a relatively low tracking error and a positive information ratio indicating slight overperformance against the benchmark. Its correlation with the benchmark was average in comparison to the other portfolios. The portfolio concentration was the lowest of all portfolios both in terms of average Gini and HHI values. Its average annual turnover was about twice the benchmark.

In summary, the equal portfolio was a highly diversified portfolio able to overperform the benchmark while keeping similar risk characteristics but with a slightly higher average annual turnover.

Diversity weighting

The rationale behind the diversity weighting technique is that the traditional market capitalisation weighting is not sufficiently diversified since it puts too much weight on large companies.

The diversity portfolio showed slight overperformance in relation to the market capitalisation portfolio in terms of return and risk-adjusted performance over all time periods, except the first rolling window period, while keeping a low tracking error. Furthermore, the alpha obtained showed overperformance but without significance. The maximum drawdown was lower over the 25-year period but similar over other periods compared to the market capitalisation portfolio. Additionally, correlation with the market capitalisation portfolio was high, average Gini and HHI values were slightly lower and average annual turnover was slightly higher.

Thus, the diversity weighting technique achieved its objective since it had lower concentration and overperforms the benchmark. The results showed that the diversity portfolio lies between the equal and market capitalisation weighted portfolios.

Inverse volatility weighting

The rationale behind the inverse volatility weighting technique is that although finance theory proclaims that low volatility stocks should have low returns, this is not always true ([15]). Thus, one assigns large weights to low volatility stocks.

The inverse volatility portfolio has consistently overperformed the benchmark in terms of annual return and Sharpe ratio while annual standard deviations are on comparable levels. Furthermore, the alpha obtained showed significant overperformance. The tracking error was relatively low and the information ratio was rather high over all time periods. Maximum drawdown was on similar levels as the benchmark but lower over the 25-year period. The correlation between the benchmark and inverse volatility was relatively low. The Gini and HHI averages were lower than those of the benchmark, indicating a more diversified portfolio. The average annual turnover was four times higher for the inverse volatility portfolio than for the benchmark.

The inverse volatility portfolio performed well against the benchmark and its characteristics are quite similar except for the average annual turnover. In summary, the portfolio succeeded in its objective to choose low volatility stocks and at the same time perform well in regards to the benchmark. The annual standard deviation of the portfolio was similar to the benchmark despite focusing on low-volatility stocks. One explanation for this is that the covariances are not taken into account and the portfolio variance is thus not necessarily lowered.

Risk-weighted alpha weighting

The rationale behind the risk-weighted alpha weighting technique is to assign a high weight to those stocks with high returns and low variance.

The risk-weighted alpha portfolio showed significant overperformance in comparison to the benchmark in terms of returns and risk-adjusted performance over all time periods despite having one of the highest annual standard deviations. Furthermore, the alpha obtained showed significant overperformance. The portfolio's information ratio varied across the time periods, while the tracking error and maximum drawdown were consistently high. Furthermore, the risk-weighted alpha portfolio had a low correlation with the benchmark. Its average Gini and HHI values as well as average annual turnover were high, indicating that the portfolio was concentrated in few stocks and that these are changing over time.

In summary, the risk-weighted alpha portfolio achieved its objective of finding those stocks with an attractive risk and return trade-off at the expense of higher overall risk-taking and considerably higher average annual turnover. An explanation for the high portfolio variance is that covariances are not taken into account when calculating weights.

Minimum variance, maximum diversification and risk efficient weightings

The rationale behind the minimum variance, maximum diversification and risk efficient weighting techniques is to maximise the portfolio Sharpe ratio. The three weighting techniques have different approaches for this.

In terms of returns and risk-adjusted performance all three portfolios have performed well compared to the benchmark during all time periods with one exception. Over the three-year period the maximum diversification portfolio did not overperform against the benchmark. The annual standard deviations were lower for the three portfolios than for the benchmark except over the three-year period. Additionally, the alphas obtained showed high and significant overperformance. The portfolios had relatively high tracking errors and information ratios. The maximum drawdown was lower over the 25-year period but otherwise similar to the benchmark. Correlations with the benchmark were quite low but correlations between these three portfolios are high, indicating similar performance. The three portfolios have high Gini and HHI averages in relation to the benchmark, indicating that they were are not as diversified as the benchmark and that their risk characteristics were different from the benchmark. The portfolios had high annual average turnover values; maximum diversification was tenfold higher than the benchmark, minimum variance and risk efficient were eightfold higher than the benchmark.

These three portfolios all succeeded in their objective to maximise Sharpe ratio although minimum variance and risk efficient overperformed against the maximum diversification portfolio in terms of annual return and Sharpe ratio. The portfolios had good performance and low standard deviation at the expense of high average annual turnover and high portfolio concentration.

ERC weighting

The rationale behind the ERC weighting technique is that a middle ground should exist between the minimum variance portfolio and the equal portfolio.

The ERC portfolio overperformed in terms of return and risk-adjusted performance relative the benchmark over all time periods except the first rolling window period. The alpha obtained showed high and significant overperformance. Furthermore, tracking error and information ratio were modest. Maximum drawdown was low over the 25-year period and high over the other two periods relative to the benchmark. Additionally, correlation with the benchmark was quite low. Finally, the ERC portfolio had moderate average Gini and HHI values resembling the benchmark but with average annual turnover seven times higher than the benchmark.

The ERC portfolio was in most measures in between the results of the equal and the minimum variance portfolios. Exceptions are tracking error, information ratio and maximum drawdown. Notably, the ERC portfolio had a higher maximum drawdown than the equal and minimum variance portfolios over the 25- and ten-year period.

In conclusion, the ERC portfolio achieved its objective of finding a middle ground between the equal and minimum variance portfolio. This since it had performance characteristics resembling the minimum variance portfolio but with a portfolio concentration in between the minimum variance and equal portfolios. This results in a concentration similar to the benchmark.

Momentum weightings

The rationale behind the RSI and ARIMA weighting techniques is that stock prices can show behaviours of momentum and trend, and they aim to capture and exploit this behaviour.

In terms of returns and risk-adjusted performance RSI and ARIMA overperformed the benchmark over the 25-, ten- and three-year periods with one exception. The RSI portfolio did not overperform the benchmark in terms of risk-adjusted return during the three-year period. The RSI portfolio showed overperformance in two out of four rolling window periods and the ARIMA portfolio showed overperformance in three out of four periods. The obtained alphas for RSI and ARIMA showed overperformance without significance. Furthermore, the portfolios had among the highest tracking errors over the 25- and ten-year periods but slightly lower over the three-year period. The RSI information ratio was considerably lower during the three-year period than in the longer time periods. The portfolios showed a relatively high maximum drawdown over all time periods but were not highly correlated to each other. Furthermore, they had high average Gini and HHI values and the highest average annual turnover.

In summary, the momentum weighting techniques indicate better performance and risk-adjusted performance than the benchmark while their risk concentration and average annual turnover was higher than all other techniques considered.

Summary

The results obtained in this study are in line with previous research in the sense that the heuristic and optimisation based weighting techniques overperformed against the market capitalisation weighting technique ([10, 16]). The momentum based weighting techniques showed tendencies of overperforming the market capitalisation portfolio while having high risk concentration and average annual turnover.

Minimum variance was the best overall performing weighting technique in this study. It had the highest annual return and Sharpe ratio over the 25- and three-year period and second highest during the ten-year period. It had some of the lowest maximum drawdown values over all periods, high Gini and HHI averages and relatively high average annual turnover. Thus, the minimum variance portfolio was the most profitable portfolio to have been invested in but also the portfolio that had one of the highest portfolio concentrations. Additionally, the minimum variance weighting technique was found to have generated the best performing portfolio over the time period 1969-2011 on the U.S. stock market ([11]). However, despite performing well the minimum variance portfolio is not always the clearly best performing portfolio ([10, 24]).

The worst performing weighting technique was the market capitalisation portfolio. It had the lowest return and Sharpe ratio over the 25- and ten-year periods and performed moderately well over the three-year period. It had one of the highest maximum drawdown values over the 25-year period and moderate maximum drawdown values during the other periods. Additionally, it had relatively low average Gini and HHI values and a relatively low average annual turnover. All together, the market capitalisation portfolio is the least profitable portfolio to have been invested in when not accounting for transaction costs.

The weighting technique most similar to the market capitalisation portfolio was, not surprisingly, the diversity portfolio. More surprisingly, the equal weighted portfolio overperformed and had similar characteristics as the market capitalisation portfolio despite its simple heuristic approach.

6 Conclusion

In this study, different alternative weighting techniques have been analysed and compared to the SIX30RX index. The conclusions are:

- All studied alternative weighting techniques except the momentum weighting techniques clearly overperformed against the SIX30RX index.
- Minimum variance was the overall best performing weighting technique in terms of return and risk-adjusted return.
- The heuristic based weighting techniques overperformed and showed similar risk characteristics as the SIX30RX index. However, the risk-weighted alpha portfolio showed similar characteristics as the optimisation based portfolios in terms of performance and risk. The equal weighted portfolio overperformed and had similar characteristics as the SIX30RX index despite its simple heuristic approach.
- The optimisation based weighting techniques showed strong overperformance relative the SIX30RX index but had different risk characteristics manifested in higher portfolio concentration. However, the ERC portfolio had a portfolio concentration resembling the SIX30RX index.
- The momentum based portfolios RSI and ARIMA had slightly better performance and risk-adjusted performance while their risk concentration and average annual turnover was higher than all other weighting techniques considered.

Suggestions for further research

Although this study considers the SIX30RX index on the Swedish stock market, previous research on the subject has mainly been studied on the U.S. stock market. Thus a natural area for further research is to investigate whether the results are still relevant if considering other geographical markets and other indices with a different stock selection. It would also be interesting to study what impact the weighting cap variable and the length of the rebalancing period has for the characteristics of the alternative weighting techniques. Furthermore, since there is a continuous development of new alternative weighting techniques there will be a continuous need for evaluating these against both each other and the market capitalisation weighting technique. Finally, combining the studied weighting techniques might yield interesting results; e.g. combining the market capitalisation and fundamental weighting techniques.

7 References

[1] Agarwal, N. Helping Identify the Most Efficient Chinese Companies Using the Risk-Weighted Alpha Index Method. *Strategic Change*, 22:447–460, 2013.

[2] Amenc, N., Goltz, F., and Lodh, A. Choose Your Betas: Benchmarking Alternative Equity Index Strategies. *The Journal of Portfolio Management*, 39(1):89–111, 2012.

[3] Amenc, N., Goltz, F., Martellini, L., and Retkowsky, P. Efficient Indexation: An Alternative to Cap-Weighted Indices. *EDHEC-Risk Institute*, 2010.

[4] Arnott, R. D., Hsu, J., and Moore, P. Fundamental Indexation. *Financial Analysts Journal*, 61(2):83–99, 2005.

[5] Ávila, F., Flores, E., López-Gallo, F., and Márquez, J. Concentration indicators: assessing the gap between aggregate and detailed data. *IFC Bulletin*, pages 1–18, 2013.

[6] Brockwell, P. J. and Davis, R. A. *Introduction to Time Series and Forecasting.* Springer, New York, 2nd edition, 2002.

[7] Chen, D., Chen, C., and Chen, J. Downside risk measures and equity returns in the NYSE. *Applied Economics*, 41(8):1055–1070, 2009.

[8] Choueifaty, Y. and Coignard, Y. Toward Maximum Diversification. *The Journal of Portfolio Management*, 35(1):40–51, 2008.

[9] Choueifaty, Y., Froidure, T., and Reynier, J. Properties of the Most Diversified Portfolio. *Journal of Investment Strategies*, 2(2):49–70, 2013.

[10] Chow, T. M., Hsu, J., Kalesnik, V., and Little, B. A survey of alternative equity index strategies. *Financial Analysts Journal*, 67(5):37–57, 2011.

[11] Clare, A., Motson, N., and Thomas, S. An evaluation of alternative equity indices Part 1: Heuristic and optimised weighting schemes. *Cass Consulting*, (March):1–42, 2013.

[12] Dow, C. Portfolio turnover and common stock holding periods. *Dow Publishing Company*, pages 1–20, 2007.

[13] Fama, E. F. and French, K. R. The Cross-Section of Expected Stock Returns. *The Journal of Finance*, 47(2):427–465, 1992.

[14] Fama, E. F. and French, K. R. Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 105(3):457–472, 2012.

[15] Haugen, R. A. and Heins, A. J. Risk and the rate of return on financial assets: Some old wine in new bottles. *The Journal of Financial and Quantitative Analysis*, 10(5):775–784, 1975.

[16] Hsu, J. C. Cap-Weighted Portfolios are Sub-Optimal Portfolios. *Journal of Investment Management*, 4(3):1–10, 2006.

[17] Hult, H., Lindskog, F., Hammarlid, O., and Rehn, C. J. Quadratic Investment Principles. In *Risk and Portfolio Analysis: Principles and Methods*, chapter 4, pages 85–126. 2012.

[18] Jegadeesh, N. and Titman, S. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48(1):65–91, 1993.

[19] Jensen, M. C. Risk, The Pricing of Capital Assets, and The Evaluation of Investment Portfolios. *The Journal of Business*, 42(2):167–247, 1969.

[20] Kaplan, P. Let Us Not All Become Fundamental Indexers Just Yet. *Morningstar Advisor*, pages 22–25, 2008.

[21] Kirchmann, C. R. and Finnerman, E. Interview with Kristin Eckerberg, Index Analyst at Nasdaq Global Information Services. *2015-04-08*.

[22] Kirchmann, C. R. and Finnerman, E. Interview with Peter Stengård, CEO Avanza Fonder AB. *2015-04-16*.

[23] Lückoff, P. *Mutual Fund Performance and Performance Persistence*. Gabler Verlag, 2010.

[24] Maillard, S., Roncalli, T., and Teiletche, J. On the properties of equally-weighted risk contributions portfolios. *Journal of Portfolio Management*, 36(4):60–70, 2010.

[25] Malkiel, B. G. Returns from Investing in Equity Mutual Funds 1971 to 1991. *The Journal of Finance*, 50(2):549–572, 1995.

[26] MathWorks. Choosing a solver. *http://se.mathworks.com/help/optim/ug/choosing-a-solver.html*, (Date accessed: 2015-04-20).

[27] Merh, N., Saxena, V. P., and Pardasani, K. R. Next Day Stock Market Forecasting: An Application of ANN and ARIMA. *The IUP Journal of Applied Finance*, 17(1):70–84, 2011.

[28] Morningstar. Standard Deviation and Sharpe Ratio. *http://corporate.morningstar.com/US/documents/MethodologyDocuments/MethodologyPapers/Standard Deviation SharpeRatio_Definition.pdf*, (Date accessed: 2015-05-01).

[29] Nasdaq. Rules for the Construction and Maintenance of the OMX STOCKHOLM 30 INDEX. *https://indexes.nasdaqomx.com/docs/Methodology_OMXS30.pdf*, (Date accessed: 2015-04-19).

[30] Rosillo, R., de la Fuente, D., and Brugos, J. A. L. Technical analysis and the Spanish stock exchange: testing the RSI, MACD, momentum and stochastic rules using Spanish market companies. *Applied Economics*, 45(12):1541–1550, 2013.

[31] SIX Financial Information. Aktieindex. *http://www.six.se/six-index/aktieindex*, (Date accessed: 2015-05-06).

[32] SIX Financial Information. Rules for the SIX30 Return Index (SIX30RX). *http://www.six.se/hs-fs/hub/370766/file-1795345329-pdf/ SIX_Index/Rules__Regulations/ Rules_for_SIX30_Index.pdf?t=1431351210000*, (Date accessed: 2015-05-03).

[33] SIX Financial Information. SIX30 Index Factsheet. *http://www.six.se/hs-fs/hub/370766/file-1932956873-pdf/SIX_Sites/SIX_Index/Produktblad/SIX_30_Index/ 4A_0003_NOFS_SIX30_Index_A4_EN-UK_Web.pdf?t=1431351210000*, (Date accessed: 2015-04-28).

[34] Stein, M. Mean-Variance Portfolio Selection With Complex Constraints. *PhD thesis, University of Karlsruhe*, 2007.

[35] Welles Wilder, Jr., J. The Relative Strength Index. In *New Concepts in Technical Trading Systems*, pages 63–70. Trend Research, 1978.

[36] Wikander, J. Hur påverkas en fonds prestation av dess förmögenhet? *Master thesis, Uppsala University*, 2008.

[37] Zhang, J., Shan, R., and Su, W. Applying Time Series Analysis Builds Stock Price Forecast Model. *Modern Applied Science*, 3(5):152–157, 2009.

A Appendix

A.1 Portfolio concentration measures

The concentration of the portfolios is measured using both the Gini coefficient and the Herfindahl-Hirschman Index as described in [24].

Gini coefficient

The Gini coefficient is a measure of statistical dispersion using the Lorenz curve. It is used to measure the level of inequality and ranges from 0 to 1, usually in the context of the level of income inequality among a nation's residents. In the stock portfolio context, the Gini coefficient describes the value each stock contributes to the overall value of the portfolio.

It is formally defined as follows. Let X be a random variable defined on $[0, 1]$ with distribution function F. Then the Lorenz curve is given by

$$L(x) = \frac{\int_0^x \theta dF(\theta)}{\int_0^1 \theta dF(\theta)}. \tag{A.1}$$

Empirically, it is computed as follows. Let $(w_1, w_2, \ldots, w_n)$ be a sequence of n weights indexed in non-decreasing order. Then the Lorenz curve is given as the continuous piecewise linear function connecting the points (F_i, L_i), $i = 0, \ldots, n$ where $F_0 = 0$, $L_0 = 0$, $F_i = i/n$ and

$$L_i = \frac{\sum_{j=1}^{i} w_j}{\sum_{j=1}^{n} w_j} \tag{A.2}$$

for $i = 1, \ldots n$.

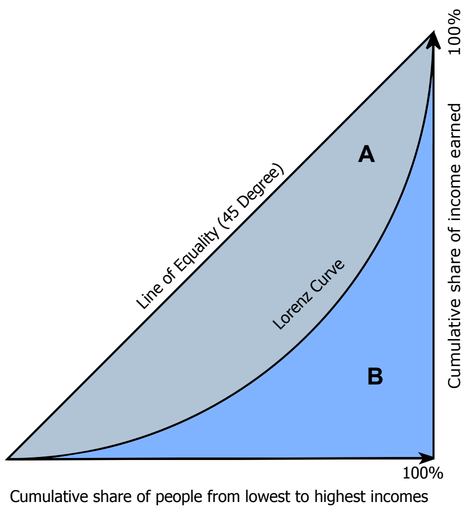

Figure 1: Lorenz curve and the Gini coefficient. The Gini coefficient is defined as the ratio between the area above the Lorenz curve and the total area below the line of inequality.

The Gini index is defined as a ratio of the areas on the Lorenz curve diagram, see figure 1 on the preceding page. If the area between the line of perfect equality and the Lorenz curve is denoted by A and the area under the Lorenz curve is denoted by B, then the Gini index is

$$G = \frac{A}{A+B}, \tag{A.3}$$

where B is calculated as

$$B = \int_0^1 L(x)dx. \tag{A.4}$$

Both axes in figure 1 on the previous page go from 0 to 1 and thus $A + B = 0.5$. Then, the Gini index can be written as

$$G = 1 - 2B. \tag{A.5}$$

Herfindahl-Hirschman Index

The Herfindahl-Hirschman Index value is a concentration measure measuring how many different types there are in a dataset and how evenly e.g. weights are distributed among those. It is defined as follows. Let $(w_1, w_2, \ldots, w_n)$ be a sequence of n weights where $w_i \epsilon [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Then the Herfindahl-Hirschman Index is defined as

$$H = \sum_{i=1}^{n} w_i^2, \tag{A.6}$$

and $H \epsilon \left[\frac{1}{n}, 1\right]$ where the $H = 1/n$ occurs when all weights are equal and $H = 1$ occurs when all weights are zero except one. To scale this onto $[0, 1]$ the modified Herfindahl-Hirschman Index was used:

$$H^* = \begin{cases} \frac{H-1/n}{1-1/n} & n > 1, \\ 1 & n = 1. \end{cases} \tag{A.7}$$

Both the Gini coefficient and the modified Herfindahl-Hirschman Index value ranges from 0 for an equally weighted portfolio to 1 for a completely concentrated portfolio but there are differences between the two. For example, a portfolio with a few large holdings and several smaller holdings might get a large Gini coefficient but the large number of holdings renders its Herfindahl-Hirschman Index value low ([5]).

A.2 Performance graphs

In the figures below the returns of each constructed portfolio are illustrated over the 25-year period. There is one figure for each group of weighting techniques; heuristic, optimisation and momentum based ones. The market capitalisation portfolio is included in all figures to enable easier comparison. Each figure is constructed to illustrate the return if an investor had invested 1 MSEK at the starting date of the constructed portfolios.

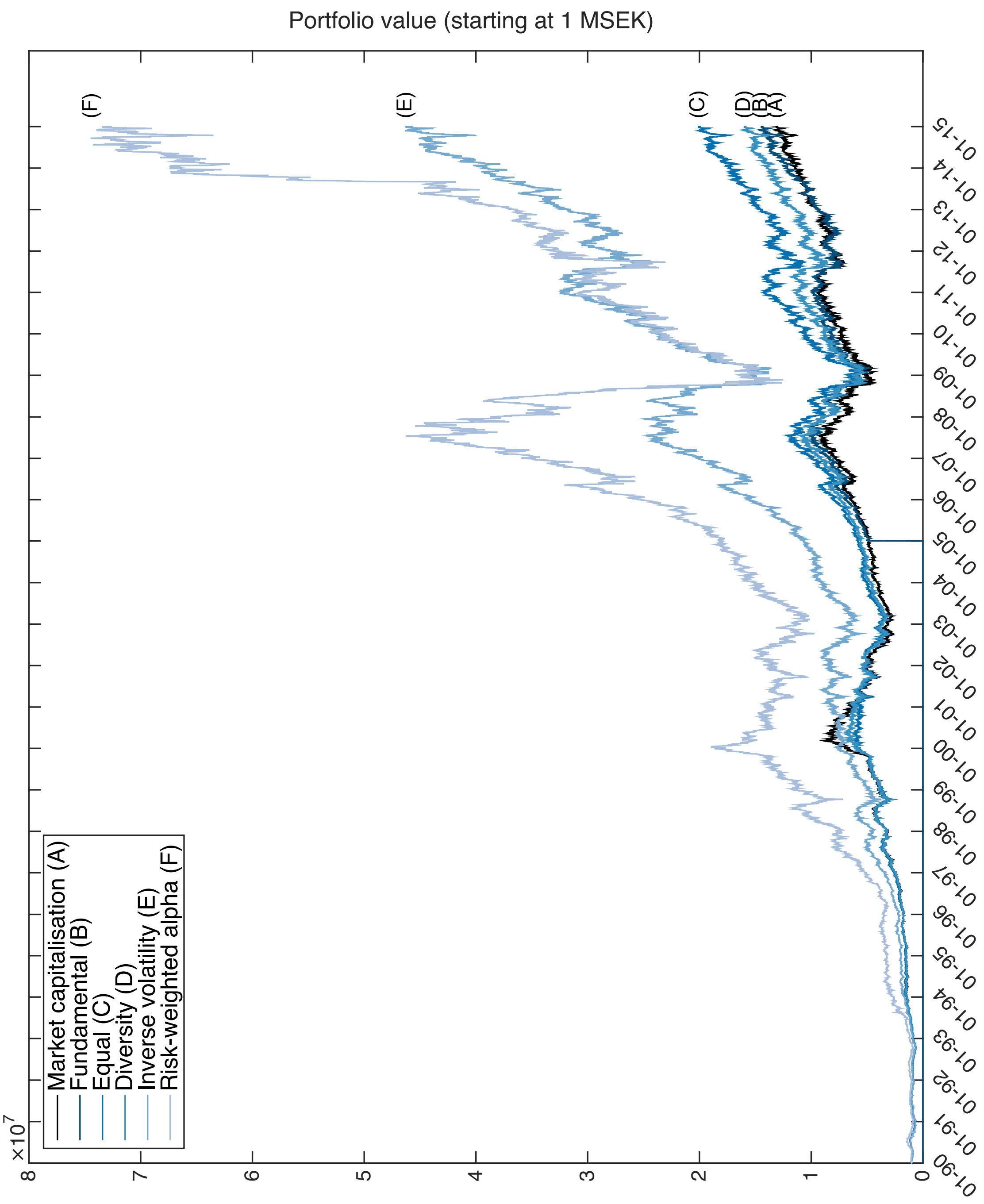

Figure 2: Performance of the heuristic based weighting techniques. This graph shows the performance over the 25-year period, 1990-01-01 until 2014-12-31, when investing 1 MSEK at the starting date in the following portfolios. The included portfolios are market capitalisation, fundamental, equal, diversity, inverse volatility and risk-weighted alpha. The fundamental weighting technique is included from 2005-01-03 until 2014-12-31 and is at its start indexed to have a value equal to the value of the market capitalisation portfolio.

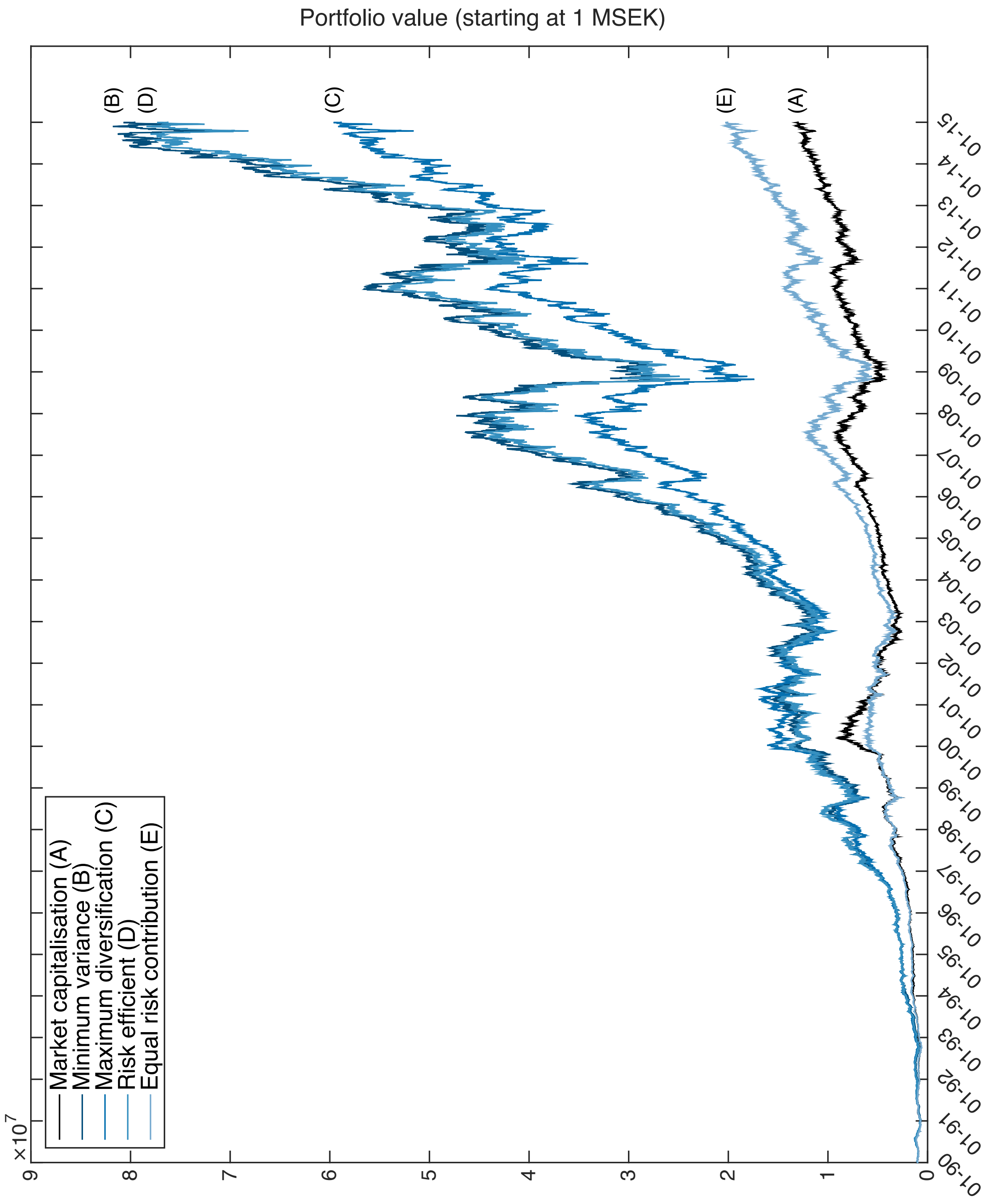

Figure 3: Performance of the optimisation based weighting techniques. This graph shows the performance over the 25-year period, 1990-01-01 until 2014-12-31, when investing 1 MSEK at the starting date in the following portfolios. The included portfolios are market capitalisation, minimum variance, maximum diversification, risk efficient and ERC.

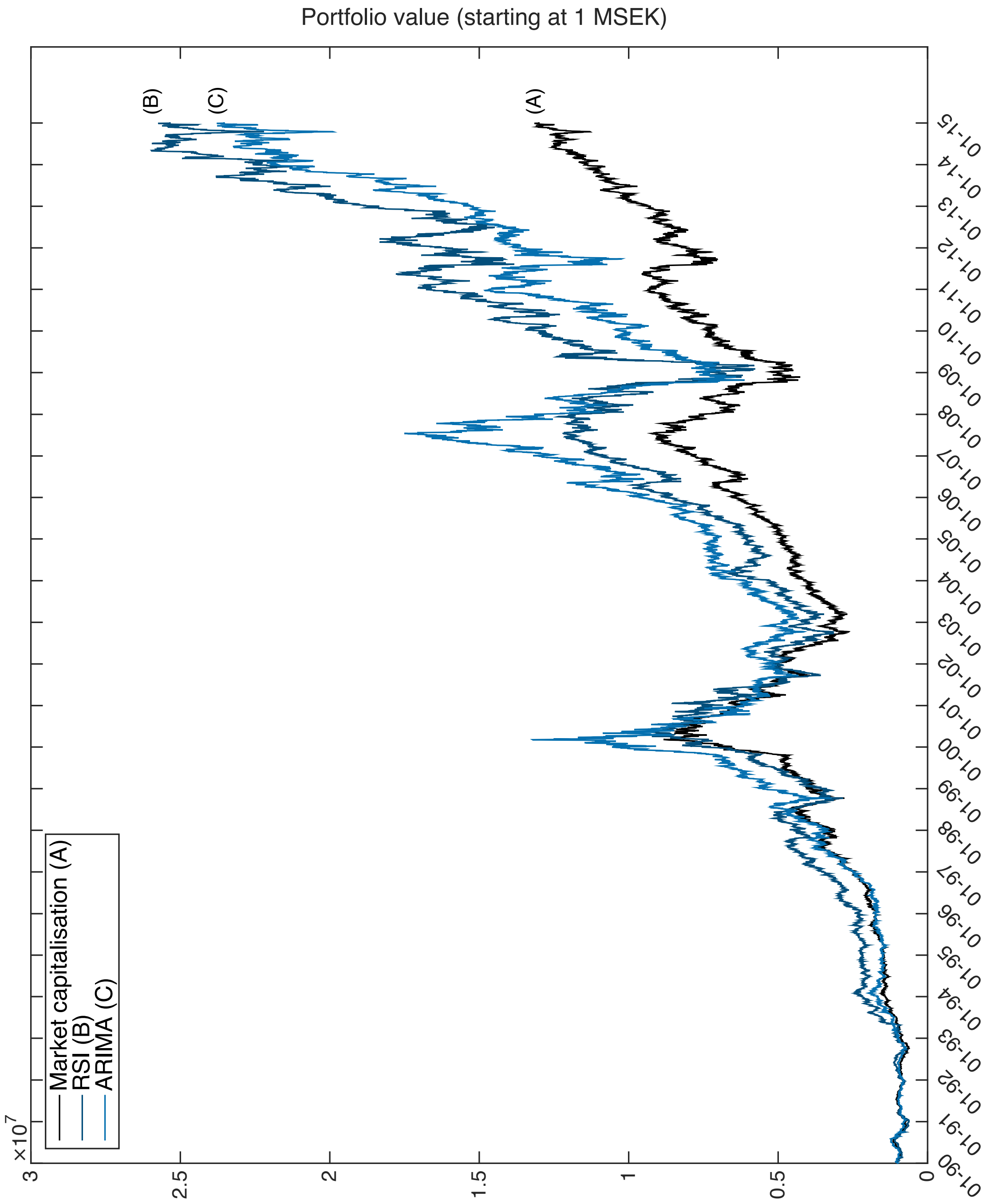

Figure 4: Performance of the momentum based weighting techniques. This graph shows the performance over the 25-year period, 1990-01-01 until 2014-12-31, when investing 1 MSEK at the starting date in the following portfolios. The included portfolios are market capitalisation, RSI and ARIMA.