Royal Institute of Technology

Master's Thesis Applied Mathematics SF299X

Hedging Interest Rate Swaps

Author
Lovisa Jangenstål

Supervisors
Boualem Djechiche
KTH
Fredrik Hesseborn
Handelsbanken

June 11, 2015

Abstract

This thesis investigates hedging strategies for a book of interest rate swaps of the currencies EUR and SEK. The aim is to minimize the variance of the portfolio and keep the transaction costs down. The analysis is performed using historical simulation for two different cases. First, with the real changes of the forward rate curve and the discount curve. Then, with principal component analysis to reduce the dimension of the changes in the curves. These methods are compared with a method using the principal component variance to randomize new principal components.

Sammanfattning

Den här uppsatsen undersöker hedgingstrategier för en portfölj bestående av ränteswapar i valutorna EUR och SEK. Syftet är att minimera portföljens varians och samtidigt minimera transaktionskostnaderna. Analysen genomförs med historisk simulering för två olika fall. Först med de verkliga förändringarna i forward- och diskonteringskurvorna. Sedan med hjälp av principalkomponentanalys för att reducera dimensionen av förändringarna i kurvorna. Dessa metoder jämförs med en metod som använder principalkomponenternas varians för att slumpa ut nya principalkomponenter.

Acknowledgements

I would like to thank my supervisors, Boualem Djechiche at KTH and Fredrik Hesseborn at Handelsbanken.

I would also like to thank Björn Löfdahl at Handelsbanken.

Contents

Chapter 1

Introduction

In finance, a derivative is a contract whose price is derived from an underlying entity. This underlying entity can be an asset, index or interest rate. A swap is a derivative in which two counterparts exchange sequences of cash flows for a set period of time. Interest rate swaps are swaps based on interest rate specifications. These swaps often exchange a fixed payment for a floating payment that is linked to an interest rate, both payments are based on the same nominal amount. Interest rate swaps are traded over-the-counter and are just contracts that are set up between one or more parties. Therefore they can be customized in many different ways. (cf. [2])

Larger banks trade interest rate swaps, mostly over the counter and with large corporations. A bank's portfolio, often referred to as the book, changes constantly and contains a large set of interest rate swaps issued at different dates and with different maturities. These interest rate swaps expose the bank to interest rate risk and credit risk. Interest rate risk is the risk that arises from fluctuating interest rates and is associated to changes in the forward curve. Credit risk is the risk that the counterpart will default on any type of debt by failing to make required payments. The banks want to reduce their interest rate risk and by hedging their books every day by interbank trading. (cf. [2])

Of course, the banks want to hedge in an optimal way. They want to minimize the interest rate risk of their books and in the same time keep transaction costs down. Hedging methods today are often performed using bumping, waves and boxes. These methods simulate changes in the forward rate curve either by adding a change directly to the forward or spot rate curve, or by changing input rate for contracts building the curve. (cf. [4])

The aim of this thesis is to find a strategy to hedge a specific day by minimizing the portfolios variance and the transaction costs.

Only interest rate swaps in the currencies SEK and EUR are considered in this report. In reality hedging is carried out several times a day, but in this report it is assumed that hedging is carried out at the end of the trading day. It is assumed that trading can be done between banks for any volume at the closing price.

The result shows that it is optimal to hedge with the shorter tenors if only the variance of the portfolio is considered. This result can be according to more noise in the curve construction for shorter tenors. When including the cost function a mix of both shorter and longer tenors is optimal. Another result is that it is beneficial to hedge with a little more SEK swaps than EUR swaps for a portfolio with swaps of both currencies.

A suggestion of further research is to perform the analysis with curves constructed in another way than cubic splines, hermite interpolation for example, since the cubic splines may have a oscillatory behaviour that affects the results.

In chapter 2, we review the theoretical tools used in the sequel. In chapter 3 the method is presented along with the data used in this report. Chapter 4 contains the result and in chapter 5 the results are analysed.

Chapter 2

Theory

This chapter presents the mathematics of the entities affecting the swap value. Interest rates, bonds, coupons, and swaps are explained along with principal component analysis and Nedler-Mead simplex algorithm.

2.1 The yield curve

The yield curve plots interest rates of different maturities, $r(t, t_1), ..., r(t, t_n)$ on the y-axis and maturities $t_1, ..., t_n$ on the x-axis. Since the maturity is the only difference between the bonds, this curve can be used to draw conclusions about future interest rates as seen from the bond market today. Normally, a larger premium is expected for lending money over a longer period of time, resulting in a positive slope the yield curve. Therefore longer term rates are expected to be higher than shorter term rates. Changes in the yield curve give rise to interest rate risk. (cf. [4])

2.2 Forward rates

From the yield curve, the current market rates can be deduced depending of the time you want to lend or borrow money. At time $t < t_1 < t_2 < ... < t_n$ the forward rate $f(t_1, t_2)$ is a best guess of the rate for which it will be able to borrow money at for some future time period (t_1, t_2). This forward rate is derived by the fact that an investment in two risk-free strategies over the same time period should yield in the same returns. Let the first strategy be to invest a dollar in a two-year zero-coupon bond $B(t, t_2)$ today and holding it for two years. The second strategy is to invest one dollar in a one-year zero-coupon bond $B(t, t_1)$ and reinvest the money received at time t_1 in a new one-year zero coupon bond $B(t_1, t_2)$ at the rate $f(t_1, t_2)$. The value at time t_2 for these two strategies should be equal, which yields

$$(1 + r(t, t_2))^2 = (1 + r(t, t_1))(1 + f(t_1, t_2)), \tag{2.1}$$

or equivalently

$$f(t_1, t_2) = \frac{(1 + r(t, t_2))^2}{(1 + r(t, t_1))} - 1. \tag{2.2}$$

As long as $r(t, t_i)$ and $r(t, t_j)$ are known, all forward rates $f(t_i, t_j)$ for time periods $t < t_i < t_j$ can be deduced in the same way. (cf. [5])

2.3 Coupon bonds and swaps

2.3.1 Fixed coupon bonds

A fixed coupon bond is a bond that pays the same amount of interest for its entire term. The bond will provide predetermined payments (coupons) to the holder of the bond. The fixed coupon bond can be described in the following way:

- Let $T_0, ..., T_n$ be a fixed number of points in time, i.e. dates. $T_i, i = 1, ..., n$ are the coupon dates while T_0 is the emission date of the bond.
- The bond owner receives the deterministic coupon c_i at the coupon date $T_i, i = 1, ..., n$ and receives the face value K at time T_n.

The return for the ith coupon c_i is typically noted as a simple rate acting on the face value K, over the time period $[T_{i-1}, T_i]$. For a standardized coupon, the coupon rates $r_i, i = 1, ..., n$ will be equal to a common coupon rate r and time intervals are equally spaced, $T_i - T_{i-1} = \delta$.

The price for such a bond is denoted $p(t)$, and will for $t \leq T_1$ be given by (cf. [3])

$$p(t) = K[p(t, T_n) + r\delta \sum_{i=1}^{n} p(t, T_i)]. \tag{2.3}$$

2.3.2 Floating rate bonds

A floating rate bond is a coupon bond for which the value of the coupon is not fixed at the time the bond is issued and is reset for every coupon period. The resetting can be benchmarked against a nonfinancial index, but most often the resetting is determined by some financial benchmark like a market interest rate.

One example is when the coupon rate r_i is set to the spot LIBOR rate $L(T_{i-1}, T_i)$, which gives that

$$c_i = (T_i - T_{i-1})L(T_{i-1}, T_i)K. \tag{2.4}$$

Note that c_i is delivered at time T_i, but $L(T_{i-1}, T_i)$ is determined already at time T_{i-1}. Assume that the principal is $K = 1$ and that the coupon dates are equally spaced, i.e. $T_i - T_{i-1} = \delta$. The LIBOR spot rate for $[T_{i-1}, T_i]$ is defined as

$$L(T_{i-1}, T_i) = -\frac{p(T_{i-1}, T_i) - 1}{(T_i - T_{i-1})p(T_{i-1}, T_i)}. \tag{2.5}$$

The goal is to compute the value of this bond at some time $t < T_0$. For the coupon c_i the following holds

$$c_i = \delta \frac{1 - p(T_{i-1}, T_i)}{\delta p(T_{i-1}, T_i)} = \frac{1}{p(T_{i-1}, T_i)} - 1. \tag{2.6}$$

The value at t, of the term -1 (paid out at T_i) is equal to $-p(t, T_i)$ and the value of the term $\frac{1}{p(T_{i-1},T_i)}$ (paid out at T_i) is $p(t, T_{i-1})$. Thereby, the value at t of the coupon bond c_i is

$$p(t, T_{i-1}) - p(t, T_i). \tag{2.7}$$

This gives the price $p(t)$ of the floating rate bond as follows (cf. [3])

$$p(t) = p(t, T_n) + \sum_{i=1}^{n} [p(t, T_{i-1}) - p(t, T_i)] = p(t, T_0). \tag{2.8}$$

2.3.3 Interest rate swap

The interest rate swap is a financial contract were two parties exchanges future cash flows. This is basically a scheme where the two parties exchange a fixed rate, known as the swap rate, for a floating rate as stream of future interest payments. Denote the principal by K and the swap rate by R. For each date $T_i, i = 1, ..., n$, the fixed leg pays the amount

$$K\delta R, \tag{2.9}$$

and the floating leg pays the amount

$$K\delta L(T_{i-1}, T_i). \tag{2.10}$$

If you swap a fixed rate for a floating rate, the net cash flow at T_i is given by

$$K\delta[L(T_{i-1}, T_i) - R]. \tag{2.11}$$

The value of the cash flow at $t < T_0$ is

$$Kp(t, T_{i-1}) - K(1 + \delta R)p(t, T_i). \tag{2.12}$$

The price $\Pi(t)$ for $t < T_0$ of the swap is thus given by

$$\Pi(t) = Kp(t, T_0) - K \sum_{i=1}^{n} d_i p(t, T_i). \tag{2.13}$$

where $d_i = R\delta, i = 1, ..., n-1$, and $d_n = 1 + R\delta$. Assume that the contract is written at $t = 0$, then the swap rate is given by (cf. [3])

$$R = \frac{p(0, T_0) - p(0, T_n)}{\delta \sum_{i=1}^{n} p(0, T_i)}. \tag{2.14}$$

2.4 Curve construction with cubic splines

Different interpolation methods can be used to construct yield curves. A continuous spot rate can be modelled in a subinterval $[t_i, t_{i+1}]$ with a cubic polynomial

$$r_i(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + d_i, \tag{2.15}$$

where a_i, b_i, c_i, d_i are parameters determined for each subinterval $[t_i, t_{i+1}], i = 1, ..., n-1$. Since the spot rate curves and its first and second order derivative are continuous at the knot points, constraints have to be set for $i = 1, ..., n-2$

$$r_i(t_{i+1}) = r_{i+1}(t_{i+1}), \tag{2.16}$$

$$r_i'(t_{i+1}) = r_{i+1}'(t_{i+1}), \tag{2.17}$$

$$r_i''(t_{i+1}) = r_{i+1}''(t_{i+1}). \tag{2.18}$$

Additional constraints are added for natural cubic splines ($r_0''(t_0) = 0$ and $r_n''(t_n) = 0$) and for financial cubic splines ($r_0''(t_0) = r_n'(t_n) = 0$). (cf. [5])

2.5 Linear regression and conditional expectations

Let L be a random variable with finite variance, $E[L^2] < \infty$, and $\mathbf{Z}$ be a random vector. The covariance matrix of $\mathbf{Z}$ is

$$\mathbf{\Sigma_Z} = \begin{pmatrix} Cov(Z_1, Z_1) & Cov(Z_1, Z_2) & \cdots & Cov(Z_1, Z_m) \\ Cov(Z_2, Z_1) & Cov(Z_2, Z_2) & \cdots & Cov(Z_2, Z_m) \\ \vdots & \ddots & \ddots & \vdots \\ Cov(Z_m, Z_1) & \cdots & \cdots & Cov(Z_m, Z_m) \end{pmatrix}, \tag{2.19}$$

and the covariance vector between L and $\mathbf{Z}$ is

$$\mathbf{\Sigma}_{L,\mathbf{Z}} = \begin{pmatrix} Cov(L, Z_1) \\ Cov(L, Z_2) \\ \vdots \\ Cov(L, Z_m) \end{pmatrix}. \tag{2.20}$$

Then it holds that

$$\begin{aligned} Var(\mathbf{w}^T\mathbf{Z} - L) &= Var(\mathbf{w}^T\mathbf{Z}) + Var(L) - 2Cov(\mathbf{w}^T\mathbf{Z}, L) \\ &= \mathbf{w}^T\mathbf{\Sigma}_{\mathbf{Z}}\mathbf{w} + Var(L) - 2\mathbf{w}^T\mathbf{\Sigma}_{L,\mathbf{Z}}. \end{aligned} \tag{2.21}$$

Lemma. *Let* $(\mathbf{A}^T, B)^T$ *be a random vector in* $\mathbb{R}^{m+1}$, *where* $\mathbf{A}$ *is m-dimensional and* L *is one-dimensional. Let* g *be a strictly convex function from* $\mathbb{R}$ *to* $\mathbb{R}$ *such that the function* h *from* $\mathbb{R}^m$ *to* $\mathbb{R}$ *given by* $h(\mathbf{x}) = E[g(\mathbf{A}^T\mathbf{x}+B)]$ *exists and is finite. Then* h *is strictly convex.* (Lindskog et al., 2006)

This Lemma with $\mathbf{A} = \mathbf{Z}$, $B = -L$, $g(y) = y^2$ and $\mathbf{x} = \mathbf{w}$ gives that $Var(\mathbf{w}^T\mathbf{Z}-L)$ is a strictly convex function of $\mathbf{w}$. Differentiating with respect to $\mathbf{w}$ we get

$$\nabla Var(\mathbf{w}^T\mathbf{Z} - L) = \mathbf{\Sigma}_{\mathbf{Z}}\mathbf{w} - \mathbf{\Sigma}_{L,\mathbf{Z}} = \mathbf{0}. \tag{2.22}$$

This means that the variance $Var(\mathbf{w}^T\mathbf{Z} - L)$ reaches its unique minimum for $\mathbf{w} = \mathbf{\Sigma}_{\mathbf{Z}}^{-1}\mathbf{\Sigma}_{\boldsymbol{L},\mathbf{Z}}$ assuming that $\mathbf{\Sigma}_{\mathbf{Z}}$ is invertible. (cf. [1])

2.6 Principal component analysis

Principal Component Analysis (PCA) is a technique of using mathematical principles to transform a number of possibly correlated variables into a smaller number of variables called principal components.

PCA uses a vector space transform to reduce the dimensionality of large data sets. The original data set, which may have involved many variables, can often be interpreted in just a few variables (the principal components).

Let $\mathbf{x}$ be a random vector

$$\mathbf{x} = (x_1, ..., x_p)^T. \tag{2.23}$$

Denote the mean of this data set by

$$\boldsymbol{\mu}_{\mathbf{x}} = E[\mathbf{x}], \tag{2.24}$$

and the covariance matrix of the same data by

$$\mathbf{C_x} = E[(\mathbf{x} - \boldsymbol{\mu}_\mathbf{x})(\mathbf{x} - \boldsymbol{\mu}_\mathbf{x})^T]. \tag{2.25}$$

The components of $\mathbf{C_X}$ are denoted by $\mathbf{c_{ij}}$ and represent the covariance between the components x_i and x_j and c_{ii} is the variance of x_i.

Since the covariance matrix is symmetric, an orthogonal matrix can be found by finding the eigenvalues and eigenvectors of this matrix. The eigenvectors $\mathbf{e_i}$ and corresponding eigenvalues $\boldsymbol{\lambda}_i$ are the solutions of

$$\mathbf{C_x v_i} = \boldsymbol{\lambda}_i \mathbf{v_i} \quad i = 1, ..., p. \tag{2.26}$$

The next step is to order the eigenvectors in the order of decreasing eigenvalues and thereby create an ordered orthogonal basis where the first eigenvector has the direction of largest variance in the data. This calculation gives the directions in which the data set has the most significant amounts of energy.

Assume that the covariance matrix and sample mean of a data set have been calculated and let $\mathbf{W}$ be a matrix with the eigenvectors of the covariance matrix as the row vectors. (cf. [6])

Then the coordinates $\mathbf{y}$ in the orthogonal base of a data vector $\mathbf{x}$ can be written

$$\mathbf{y} = \mathbf{W}(\mathbf{x} - \boldsymbol{\mu}_\mathbf{x}). \tag{2.27}$$

Using that $W^T W = I$ the original data vector $\mathbf{x}$ can be reconstructed from $\mathbf{y}$

$$\mathbf{x} = \mathbf{W^T y} + \boldsymbol{\mu}_\mathbf{x}. \tag{2.28}$$

Let $\mathbf{W_L}$ have the first L eigenvectors as row vectors. Then a similar transformation be performed

$$\mathbf{y} = \mathbf{W_L}(\mathbf{x} - \boldsymbol{\mu}_\mathbf{x}), \tag{2.29}$$

$$\mathbf{x} = \mathbf{W_L^T y} + \boldsymbol{\mu}_\mathbf{x}. \tag{2.30}$$

This transformation projects the original data vector on the coordinate axes having the dimension L and transforming the vector back, which minimizes the mean-square error between the data and this representation with L eigenvectors. (cf. [7])

2.7 Nelder-Mead simplex algorithm

The Nelder-Mead algorithm is a method of minimizing a real-valued function $f(\mathbf{y})$ for $\mathbf{y} \in \mathbb{R}^n$. In order to define a complete Nelder-Mead method, four parameters must be specified; *reflection* (ρ), *expansion* (χ), *contraction* (γ) and *shrinkage* (σ). These parameters should satisfy

$$\rho > 0, \quad \chi > 1, \quad \chi > \rho, \quad 0 < \gamma < 1, \quad 0 < \sigma < 1. \tag{2.31}$$

At the kth iteration, a nondegenerate simplex Δ_k is given together with its $n+1$ vertices. It is assumed that the kth iteration begins with ordering these vertices as $\mathbf{y}_1^{(k)}, ..., \mathbf{y}_{n+1}^{(k)}$ as

$$f_1^{(k)} \leq f_2^{(k)} \leq ... \leq f_n^{(k)} \leq f_{n+1}^{(k)}, \tag{2.32}$$

where $f_i^{(k)}$ is the same as $f(\mathbf{y}_i^{(k)})$. A set of $n+1$ vertices are generated by iteration k. These vertices define a different simplex for the next iteration so that $\Delta_{k+1} \neq \Delta_k$. Since the goal is to minimize f, $\mathbf{y}_1^{(k)}$ is referred to as the best point or the vertex, $\mathbf{y}_{n+1}^{(k)}$ is referred to as the worst point and $\mathbf{y}_n^{(k)}$ as the next-worst point and so on. An iteration k of the Nelder-Mead algorithm is specified below. Note that the subscript k is omitted. The result of an iteration is either (i) the accepted point, a single new vertex that replaces $\mathbf{y}_{n+1}$ in the next iteration, or (ii) a set of n new points that form the simplex at the next iteration together with $\mathbf{x}_1$. The later is the result if a shrink is performed.

1. **Order.** The first step is to order the $n+1$ vertices so they satisfy

$$f(\mathbf{y}_1) \leq ... \leq f(\mathbf{y}_{n+1}).$$

2. **Reflect.** Let $\bar{\mathbf{y}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{y}_i$ be the centroid of the n best points. These points are all the vertices except for $\mathbf{y}_{n+1}$. Then compute the reflection point denoted $\mathbf{y}_r$

$$\mathbf{y}_r = \bar{\mathbf{y}} + \rho(\bar{\mathbf{y}} - \mathbf{y}_{n+1}) = (1+\rho)\bar{\mathbf{y}} - \rho\mathbf{y}_{n+1}, \tag{2.33}$$

and calculate $f(\mathbf{y}_r) := f_r$.
If $f_i \leq f_r < f_n$, accept the reflected point $\mathbf{x}_r$ and stop the iteration.

3. **Expand.** If $f_r < f_1$ calculate the expansion point $\mathbf{y}_e$

$$\mathbf{y}_e = \bar{\mathbf{y}} + \chi(\mathbf{y}_r - \bar{\mathbf{y}}) = \bar{\mathbf{y}} + \rho\chi(\bar{\mathbf{x}} - \mathbf{x}_{n+1}) = (1+\rho\chi)\bar{\mathbf{y}} - \rho\chi\mathbf{y}_{n+1} \tag{2.34}$$

Then compute $f(\mathbf{y}_e) := f_e$. If $f_e < f_r$, accept $\mathbf{y}_e$ and stop the iteration. Otherwise, accept $\mathbf{y}_r$ and stop the iteration.

4. **Contract.** If $f_r \geq f_n$, perform a contraction between $\bar{\mathbf{y}}$ and the better of $\mathbf{y}_r$ and $\mathbf{y}_{n+1}$.

   (a) **Outside.** If $f_n \leq f_r < f_{n+1}$, perform an outside contraction

   $$\mathbf{y}_c = \bar{\mathbf{y}} + \gamma(\mathbf{y}_r - \bar{\mathbf{y}}) = \bar{\mathbf{y}} + \gamma\rho(\bar{\mathbf{y}} - \mathbf{y}_{n+1}) = (1 + \rho\gamma)\bar{\mathbf{y}} - \rho\gamma\mathbf{y}_{n+1}. \tag{2.35}$$

   Then compute $f(\mathbf{y}_c) := f_c$. If $f_c \leq f_r$, accept $\mathbf{y}_c$ and end the iteration. Otherwise, perform a shrink step (go to step 5).

   (b) **Inside.** If $f_r \geq f_{n+1}$, perform an inside contraction

   $$\mathbf{y}_{cc} = \bar{\mathbf{y}} - \gamma(\bar{\mathbf{y}} - \mathbf{y}_{n+1}) = (1 - \gamma)\bar{\mathbf{y}} + \gamma\mathbf{y}_{n+1}, \tag{2.36}$$

   and compute $f(\mathbf{y}_{cc}) := f_{cc}$. If $f_{cc} < f_{n+1}$, accept $\mathbf{y}_{cc}$ and stop the iteration. Otherwise, perform a shrink step (go to step 5).

5. **Perform a shrink step.** Compute f at $\mathbf{v}_i = \mathbf{y}_1 + \sigma(\mathbf{y}_i - \mathbf{y}_1)$ for $i = 2, ..., n+1$. Then the unordered vertices of the simplex at the next iteration are $\mathbf{y}_1, \mathbf{v}_2, ..., \mathbf{v}_{n+1}$.

(cf. [8])

Chapter 3

Data and Methodology

In this chapter the four different methods used in the study are described in detail.

The analysis is performed using swaps and curves of two different currencies, EUR and SEK. The curves are the three month forward rate curve and the discount curve constructed using cubic splines; $SEK_{DSC}, EUR_{DSC}, SEK_{3M}$ and EUR_{3M}. There is one variable for each point on the curve, resulting in 40 variables on each curve and 161 variables in total including the EURSEK exchange rate.

In this report, these curves at time t are denoted

$$\mathbf{X}_t = \begin{pmatrix} SEKDSC_{1,t} \\ \vdots \\ SEKDSC_{40,t} \\ EURDSC_{1,t} \\ \vdots \\ EURDSC_{40,t} \\ SEK3M_{1,t} \\ \vdots \\ SEK3M_{40,t} \\ EUR3M_{1,t} \\ \vdots \\ EUR3M_{40,t} \\ EURSEK_t \end{pmatrix}. \tag{3.1}$$

There are swaps of 14 different tenors traded on the interbank market, see Table 3.1 below.

SEK swaps	EUR swaps
1YSEK	1YEUR
2YSEK	2YEUR
3YSEK	3YEUR
4YSEK	4YEUR
5YSEK	5YEUR
6YSEK	6YEUR
7YSEK	7YEUR
8YSEK	8YEUR
9YSEK	9YEUR
10YSEK	10YEUR
12YSEK	12YEUR
15YSEK	15YEUR
20YSEK	20YEUR
25YSEK	25YEUR

Table 3.1: Swaps of 14 different tenors are traded on the interbank market

Let $\mathbf{s}(t, \mathbf{X}_t))$ be the swaps valued at time t on the curves denoted by $\mathbf{X}_t$ and let $K(t, \mathbf{X}_t)$ be the customer portfolio valued at time t on the same curves. When the subscripts EUR and SEK are added to K it means that the analysis is performed only on the SEK or EUR part of the costumer portfolio. If the subscript is omitted swaps from both currencies are used. Let w_i be the nominal amount in swap s_i. When $w_i > 0$ it corresponds to a payer swap, and $w_i < 0$ corresponds to a receiver swap.

The cost of swap i, denoted c_i, is chosen to be the swap valued one basis point above the curves, one basis point is equivalent to 0.01 %.

The hedging problem can be formulated as follows

$$\min_{\mathbf{w}} \quad E\{[\mathbf{w}^T \mathbf{s}(t, X_t) - (\mathbf{K}(t, X_t) - \mathbf{K}(0, X_0))]^2 + \lambda g(\mathbf{w})\}, \tag{3.2}$$

where $g(\mathbf{w}) = \mathbf{c}^T \mathbf{w}$ and $\lambda \geq 0$ is a constant.

The problem in equation 3.2 is central in this thesis and studied for two different cases:

- $\lambda = 0$, only minimizing the portfolio variance,
- $\lambda > 0$, including the cost function.

Four different methods are performed and presented in the sections below.

3.1 Historical simulation absolute changes

The first method is based on 250 historical scenarios of the interest rate, with the curves from one specific day chosen as the base curves; $SEKDSC_{base}$, $EURDSC_{base}$, $SEK3M_{base}$, $EUR3M_{base}$.

By adding the daily changes of the curves, denoted δ_i, to the base curves $n = 250$ scenarios of possible daily changes of the base curves are constructed:

$$SEKDSC_i = SEKDSC_{base} + \delta_i^{SEKDSC}, \quad i = 1, ..., n, \tag{3.3}$$

$$EURDSC_i = EURDSC_{base} + \delta_i^{EURDSC}, \quad i = 1, ..., n, \tag{3.4}$$

$$SEK3M_i = SEK3M_{base} + \delta_i^{SEK3M}, \quad i = 1, ..., n, \tag{3.5}$$

$$EUR3M_i = EUR3M_{base} + \delta_i^{EUR3M}, \quad i = 1, ..., n. \tag{3.6}$$

The customer portfolio is valued for every scenario on these new curves, and the difference against the base curves is computed as follows

$$k_i = K_i - K_{base} \quad i = 1, ..., n. \tag{3.7}$$

The swaps traded on the interbank market, presented in Table 3.1, are denoted $S_j^i, j = 1, ..., 14$, where i is the trading day. Similarly to the costumer portfolio, the swaps are valued for every constructed curve scenario.

This gives us three matrices of swaps:

$$\begin{pmatrix} S_1^{1,SEK} & S_1^{2,SEK} & \cdots & S_1^{250,SEK} \\ S_2^{1,SEK} & S_2^{2,SEK} & \cdots & S_2^{250,SEK} \\ \vdots & \ddots & \ddots & \vdots \\ S_{14}^{1,SEK} & \cdots & \cdots & S_{14}^{250,SEK} \end{pmatrix}, \tag{3.8}$$

$$\begin{pmatrix} S_1^{1,EUR} & S_1^{2,EUR} & \cdots & S_1^{250,EUR} \\ S_2^{1,EUR} & S_2^{2,EUR} & \cdots & S_2^{250,EUR} \\ \vdots & \ddots & \ddots & \vdots \\ S_{14}^{1,EUR} & \cdots & \cdots & S_{14}^{250,EUR} \end{pmatrix}, \tag{3.9}$$

$$\begin{pmatrix} S_1^{1} & S_1^{2} & \cdots & S_1^{250} \\ S_2^{1} & S_2^{2} & \cdots & S_2^{250} \\ \vdots & \ddots & \ddots & \vdots \\ S_{14}^{1} & \cdots & \cdots & S_{14}^{250} \end{pmatrix}, \tag{3.10}$$

and three different vectors of changes in the costumer portfolio

$$k_1^{SEK}, ..., k_{250}^{SEK}, \tag{3.11}$$

$$k_1^{EUR}, ..., k_{250}^{EUR}, \tag{3.12}$$

$$k_1, ..., k_{250}. \tag{3.13}$$

If the cost function is excluded, i.e. $\lambda = 0$ in equation 3.2, the perfect hedge of the portfolio would be to solve the following equation system exactly

$$\begin{pmatrix} S_1^1 & S_2^1 & \cdots & S_{14}^1 \\ S_1^2 & S_2^2 & \cdots & S_{14}^2 \\ \vdots & \ddots & \ddots & \vdots \\ S_1^{250} & \cdots & \cdots & S_{14}^{250} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{14} \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_{250} \end{pmatrix}. \tag{3.14}$$

Since this is an overdetermined equation system it cannot be solved exactly, instead we want to find

$$\underset{\mathbf{w}}{min} E[(\mathbf{Sw} - \mathbf{k})^2]. \tag{3.15}$$

If the cost function is included, $\lambda > 0$ in equation 3.2, the Nedler-Mead simplex algorithm is used for minimization of the objective function using the following procedure.

The first step of the algorithm is to make an initial guess, $\mathbf{y}_0$. Then to make a simplex around $\mathbf{y}_0$ by adding 5% of each component $y_0(i)$ to $\mathbf{y}_0$. In addition to $\mathbf{y}_0$ it uses these vectors as elements of the simplex, but if $y_0(i) = 0$ it uses 0.00025 instead. The simplex is then modified repeatedly according to the steps described below.

1. Denote the points in the current simplex by $y(1), ..., y(n), y(n+1)$.

2. Order the points in the simplex in increasing function value, $f(y(1)), ..., f(y(n)), f(y(n+1))$. Discard the current worst point, $y(n+1)$, at each step of the iteration and and accept another point in to the simplex. An exception is in the case of step 7 below, where the algorithm changes all n points with values above $f(y(1))$.

3. Let r be the reflected point defined by

$$r = 2u - y(n+1), \tag{3.16}$$

   where

$$u = \frac{\sum_{i=1}^{n} y(i)}{n}. \tag{3.17}$$

Then calculate $f(r)$.

4. If $f(y(1)) \leq f(r) < f(y(n))$, accept r and end this iteration.

5. If $f(r) < f(y(1))$, calculate the expansion point s defined by

$$s = u + 2(u - y(n+1)). \tag{3.18}$$

Then calculate $f(s)$.

   (a) If $f(s) < f(r)$, accept s and end the iteration.

   (b) Otherwise, accept r and end the iteration.

6. If $f(y(n)) \leq f(r)$, perform a contraction between u and the the smallest of $f(r)$ and $f(y(n+1))$.

   (a) If $f(r) < f(y(n+1))$, calculate c defined by

$$c = u + \frac{r - u}{2}. \tag{3.19}$$

   If $f(c) < f(r)$, accept c and stop the iteration. Otherwise, continue to step 7.

   (b) If $f(y(n+1)) \leq f(r)$, calculate $\tilde{c}$

$$\tilde{c} = u + \frac{y(n+1) - u}{2}. \tag{3.20}$$

   If $f(\tilde{c}) < f(y(n+1))$, accept $\tilde{c}$ and stop the iteration. Otherwise, continue to step 7.

7. Compute

$$v(i) = y(1) + \frac{y(i) - y(1)}{2} \quad i = 1, ..., n. \tag{3.21}$$

Then, calculate $f(v(i))$ for $i = 2, ..., n+1$. The simplex is $y(1), v(2), ..., v(n+1)$ in the next iteration.

The iteration continues until it meets a stopping criterion.

3.2 Historical simulation PCA of changes

Since the curves are constructed using cubic splines it is possible that the changes in the curves is affected by noise in the curve construction. Therefore, the number of variables are reduced into a smaller number of variables describing the largest variance using principal component analysis.

The vector of size $p \times 1$ representing the daily changes in the data, absolute changes for the curves and log relative changes for the exchange rate, is denoted $\mathbf{x}$.

A data set of $\mathbf{x_1}, ..., \mathbf{x_n}$ is constructed with the daily changes in the curves and the exchange rate for $n = 250$ days. Then principal component analysis is performed constructing a basis of $L = 4$ vectors describing more than 90 % of the variance in the data.

The vectors $\mathbf{x_1}, ..., \mathbf{x_n}$ are projected on this new basis of principal components

$$\mathbf{y_i} = \mathbf{W_L}(\mathbf{x_i} - \boldsymbol{\mu}_{\mathbf{x_i}}). \tag{3.22}$$

Using the fact that $\mathbf{W}^T\mathbf{W} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix, the vectors are transformed back using

$$\mathbf{x_i} = \mathbf{W_L^T}\mathbf{y_i} + \boldsymbol{\mu}_{\mathbf{x_i}}. \tag{3.23}$$

Now, the historical simulation method in the previous section is repeated using

$$SEKDSC_i = SEKDSC_{base} + \mathbf{x}_i^{SEKDSC}, \quad i = 1, ..., n, \tag{3.24}$$

$$EURDSC_i = EURDSC_{base} + \mathbf{x}_i^{EURDSC}, \quad i = 1, ..., n, \tag{3.25}$$

$$SEK3M_i = SEK3M_{base} + \mathbf{x}_i^{SEK3M}, \quad i = 1, ..., n, \tag{3.26}$$

$$EUR3M_i = EUR3M_{base} + \mathbf{x}_i^{EUR3M}, \quad i = 1, ..., n. \tag{3.27}$$

3.3 Random principal components

From the PCA in the previous section we get the principal component variances as the eigenvalues of the covariance matrix of the data set $\mathbf{x_1}, ..., \mathbf{x_n}$.

Since the principal components are normally distributed it is able to randomize new principal components. This is performed by generating a normally distributed random number and multiplying with the standard deviation of the principal components.

Then, the randomized principal components of the actual daily changes in the curves can be used in order to construct new possible daily changes in the curves. This is done by projecting $\mathbf{x_1}, ..., \mathbf{x_n}$ onto this new basis $\widetilde{\mathbf{W_L}}$

$$\tilde{\mathbf{y_i}} = \widetilde{\mathbf{W_L}}(\mathbf{x_i} - \boldsymbol{\mu}_{\mathbf{x_i}}), \tag{3.28}$$

and transform back using the fact that $\widetilde{\mathbf{W}}^T\widetilde{\mathbf{W}} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix

$$\mathbf{x_i} = \tilde{\mathbf{W_L}}^{\mathbf{T}}\tilde{\mathbf{y_i}} + \boldsymbol{\mu}_{\mathbf{x_i}}. \tag{3.29}$$

These new possible daily changes are added to the base curves (x_{base}), constructing $n = 250$ new scenarios of curves $x_1, ..., x_n$

$$\begin{array}{lll} x_{base} & s(x_{base}) & k(x_{base}) \\ x_1 & s(x_1) & k(x_1) \\ \vdots & \vdots & \vdots \\ x_n & s(x_n) & k(x_n). \end{array}$$

As before, problem in equation 3.2 is solved for $\lambda = 0$ and $\lambda > 0$. The solution for $\lambda = 0$ is given by $\mathbf{w} = \boldsymbol{\Sigma}_{\mathbf{s}}^{-1}\boldsymbol{\Sigma}_{k,s}$, the proof is presented in Section 2.7. For $\lambda > 0$ the problem is solved by the Nedler-Mead algorithm as before.

3.4 Changing the cost function

For the previous methods the cost function is the same for both EUR and SEK swaps.

Now, the swaps are valued one basis point (bp) above the curves for the SEK swaps and $\varphi \cdot 1bp$ above the curves for the EUR swaps, where $0 \leq \varphi \leq 1$. Then the historical simulation with PCA of changes is repeated.

Chapter 4

Result

The numerical results of the methods described in chapter 3 are presented in this chapter.

4.1 Historical simulation absolute changes

Following the method described in Section 3.1 the analysis is performed twice with the curves from 2015-04-27 and 2014-11-27 chosen as base curves.

4.1.1 Not including the cost function ($\lambda = 0$)

The results for the analysis of the EUR and SEK portfolio separately is presented in Table 4.1 and 4.2 below.

	$\mathbf{w_{SEK}}$		$\mathbf{w_{EUR}}$
1YSEK	33.44 %	1YEUR	7.95 %
2YSEK	18.05 %	2YEUR	28.73 %
3YSEK	3.58 %	3YEUR	21.51 %
4YSEK	6.52 %	4YEUR	18.09 %
5YSEK	8.63 %	5YEUR	13.31 %
6YSEK	7.12 %	6YEUR	2.42 %
7YSEK	6.19 %	7YEUR	2.11 %
8YSEK	3.56 %	8YEUR	0.15 %
9YSEK	2.60 %	9YEUR	1.61 %
10YSEK	5.76 %	10YEUR	2.51 %
12YSEK	-1.46 %	12YEUR	1.44 %
15YSEK	1.93 %	15YEUR	-0.02 %
20YSEK	1.11 %	20YEUR	-0.07 %
25YSEK	0.04 %	25YEUR	0.08 %

Table 4.1: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. A separate analysis for the currencies EUR and SEK

	$\mathbf{w_{SEK}}$		$\mathbf{w_{EUR}}$
1YSEK	30.75 %	1YEUR	14.09 %
2YSEK	12.81 %	2YEUR	19.67 %
3YSEK	13.41 %	3YEUR	24.69 %
4YSEK	6.48 %	4YEUR	13.74 %
5YSEK	9.47 %	5YEUR	16.86 %
6YSEK	10.08 %	6YEUR	2.55 %
7YSEK	7.64 %	7YEUR	1.89 %
8YSEK	3.52 %	8YEUR	0.81 %
9YSEK	1.21 %	9YEUR	0.05 %
10YSEK	0.96 %	10YEUR	3.70 %
12YSEK	2.03 %	12YEUR	1.34 %
15YSEK	0.55 %	15YEUR	-0.30 %
20YSEK	0.68 %	20YEUR	0.27 %
25YSEK	0.41 %	25YEUR	0.05 %

Table 4.2: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2014-11-27. A separate analysis for the currencies EUR and SEK

The results of combining the currencies EUR and SEK in the portfolio are presented in Table 4.3 and 4.4 below.

	w		**w**
1YSEK	17.53 %	1YEUR	-18.47 %
2YSEK	10.77 %	2YEUR	14.96 %
3YSEK	1.93 %	3YEUR	-0.51 %
4YSEK	4.10 %	4YEUR	2.50 %
5YSEK	4.87 %	5YEUR	1.27 %
6YSEK	3.66 %	6YEUR	0.91 %
7YSEK	4.90 %	7YEUR	-0.22 %
8YSEK	2.24 %	8YEUR	0.26 %
9YSEK	1.05 %	9YEUR	0.41 %
10YSEK	3.52 %	10YEUR	-0.71 %
12YSEK	-0.86 %	12YEUR	1.26 %
15YSEK	1.19 %	15YEUR	-0.06 %
20YSEK	0.65 %	20YEUR	-1.33 %
25YSEK	-0.01 %	25YEUR	0.81 %

Table 4.3: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is for both EUR and SEK together

	w		**w**
1YSEK	20.62 %	1YEUR	-6.42 %
2YSEK	8.42 %	2YEUR	9.30 %
3YSEK	9.11 %	3YEUR	2.46 %
4YSEK	4.67 %	4YEUR	0.38 %
5YSEK	6.26 %	5YEUR	3.44 %
6YSEK	6.70 %	6YEUR	-1.75 %
7YSEK	5.16 %	7YEUR	3.16 %
8YSEK	2.63 %	8YEUR	0.37 %
9YSEK	0.32 %	9YEUR	-1.33 %
10YSEK	1.36 %	10YEUR	1.16 %
12YSEK	0.75 %	12YEUR	0.61 %
15YSEK	0.61 %	15YEUR	-0.42 %
20YSEK	0.48 %	20YEUR	-1.05 %
25YSEK	0.29 %	25YEUR	0.78 %

Table 4.4: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2014-11-27. The analysis is for both EUR and SEK together

Figure 4.1 and 4.2 below presents the daily changes in value of the unhedged portfolio compared with the hedged portfolio. Figure 4.3 and 4.4 presents the results in histograms.

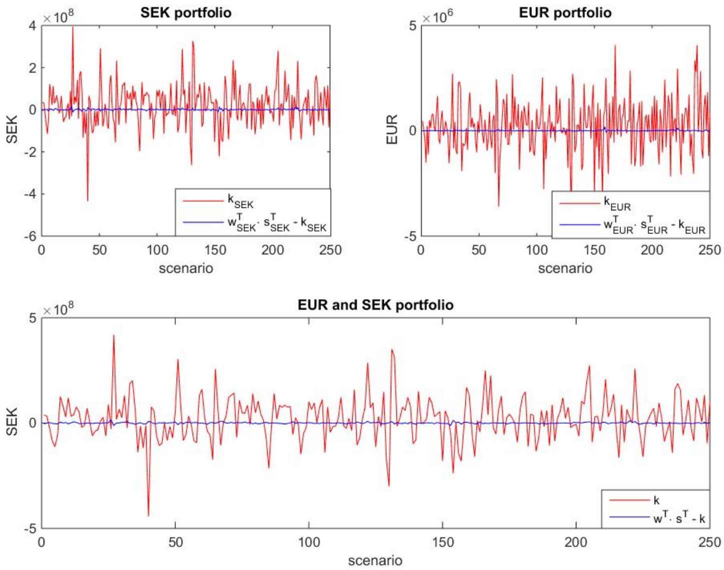

Figure 4.1: Daily changes in value of the unhedged portfolio (red) and the hedged portfolio (blue). The base curves are from 2015-04-27

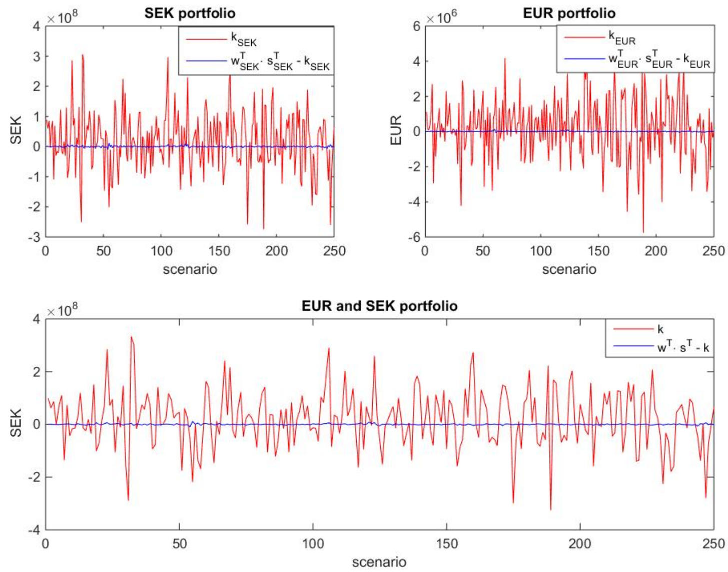

Figure 4.2: Daily changes in value of the unhedged portfolio (red) and the hedged portfolio (blue). The base curves are from 2014-11-27

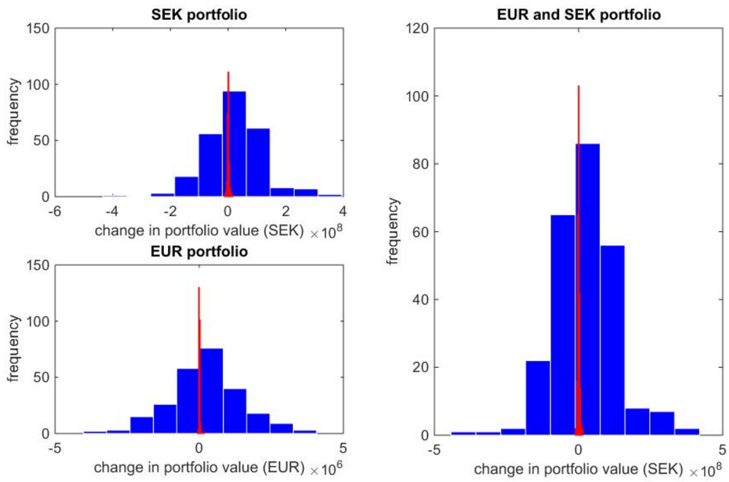

Figure 4.3: Histograms of the daily changes in value of the unhedged portfolio (blue) and the hedged portfolio (red). The base curves are from 2015-04-27

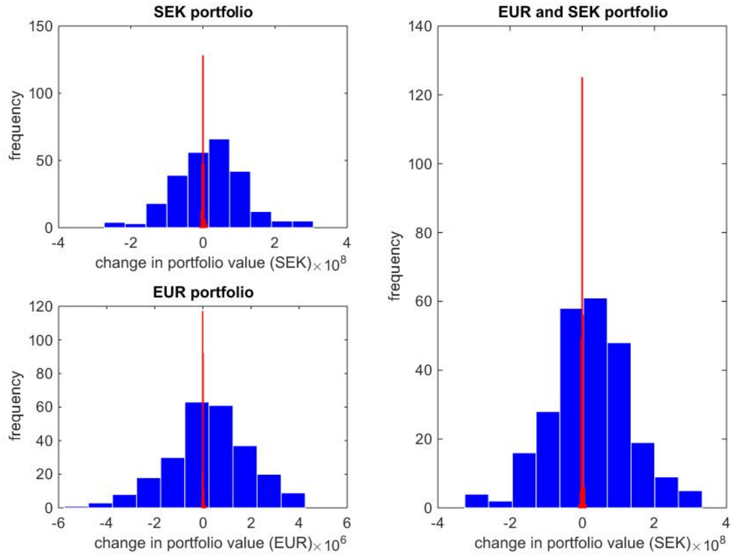

Figure 4.4: Histogram of the daily change in value of the unhedged portfolio (blue) and the hedged portfolio (red) for EUR and SEK separately and for the two currencies combined. The base curves are from 2015-04-27

4.1.2 Including the cost function ($\lambda = 1$)

The result of letting $\lambda = 1$ and thereby including the cost function $\mathbf{c}$ in the minimization are presented in this section.

Table 4.5 and 4.6 below show the results for a separate analysis of the currencies while Table 4.7 and 4.8 present the results for both EUR and SEK together.

	$\mathbf{w}_{\mathbf{SEK}}$		$\mathbf{w}_{\mathbf{EUR}}$
1YSEK	-16.34 %	1YEUR	-9.58 %
2YSEK	-12.25 %	2YEUR	2.59 %
3YSEK	6.24 %	3YEUR	-3.41 %
4YSEK	0.09 %	4YEUR	28.72 %
5YSEK	18.10 %	5YEUR	-9.30 %
6YSEK	-6.74 %	6YEUR	-4.26 %
7YSEK	-0.84 %	7YEUR	6.57 %
8YSEK	-3.82 %	8YEUR	1.49 %
9YSEK	5.36 %	9YEUR	7.48 %
10YSEK	1.13 %	10YEUR	-2.98 %
12YSEK	-10.25 %	12YEUR	6.54 %
15YSEK	11.40 %	15YEUR	-8.32 %
20YSEK	4.21 %	20YEUR	-4.12 %
25YSEK	-3.24 %	25YEUR	4.64 %

Table 4.5: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. A separate analysis for EUR and SEK

	$\mathbf{w}_{\mathbf{SEK}}$		$\mathbf{w}_{\mathbf{EUR}}$
1YSEK	-12.44 %	1YEUR	0.54 %
2YSEK	1.04 %	2YEUR	-11.50 %
3YSEK	-6.77 %	3YEUR	-4.68 %
4YSEK	3.09 %	4YEUR	-0.03 %
5YSEK	-6.87 %	5YEUR	0.93 %
6YSEK	13.50 %	6YEUR	11.65 %
7YSEK	10.16 %	7YEUR	5.16 %
8YSEK	1.23 %	8YEUR	14.62 %
9YSEK	4.08 %	9YEUR	-5.60 %
10YSEK	7.70 %	10YEUR	9.04 %
12YSEK	14.67 %	12YEUR	3.75 %
15YSEK	5.36 %	15YEUR	2.64 %
20YSEK	1.56 %	20YEUR	-20.36 %
25YSEK	-11.53 %	25YEUR	9.50 %

Table 4.6: The optimal nominal amount for each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2014-11-27. A separate analysis for EUR and SEK

	$\mathbf{w}$		$\mathbf{w}$
1YSEK	2.58 %	1YEUR	-1.22 %
2YSEK	-3.46 %	2YEUR	-6.83 %
3YSEK	-7.64 %	3YEUR	3.52 %
4YSEK	-3.26 %	4YEUR	-3.64 %
5YSEK	4.77 %	5YEUR	-3.04 %
6YSEK	-4.17 %	6YEUR	-1.37 %
7YSEK	12.16 %	7YEUR	-1.68 %
8YSEK	6.49 %	8YEUR	-0.51 %
9YSEK	5.02 %	9YEUR	1.11 %
10YSEK	4.29 %	10YEUR	-7.65 %
12YSEK	1.90 %	12YEUR	-0.59 %
15YSEK	1.19 %	15YEUR	3.32 %
20YSEK	3.51 %	20YEUR	1.43 %
25YSEK	-3.63 %	25YEUR	0.02 %

Table 4.7: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is for the both currencies together

	$\mathbf{w_{SEK}}$		$\mathbf{w_{EUR}}$
1YSEK	-15.57 %	1YEUR	-0.55 %
2YSEK	1.93 %	2YEUR	7.50 %
3YSEK	-1.95 %	3YEUR	3.51 %
4YSEK	-0.25 %	4YEUR	2.21 %
5YSEK	0.79 %	5YEUR	1.65 %
6YSEK	3.18 %	6YEUR	-6.17 %
7YSEK	-0.43 %	7YEUR	3.56 %
8YSEK	-3.49 %	8YEUR	4.23 %
9YSEK	5.28 %	9YEUR	-1.00 %
10YSEK	7.40 %	10YEUR	-3.15 %
12YSEK	2.39 %	12YEUR	-0.60 %
15YSEK	3.64 %	15YEUR	-2.88 %
20YSEK	-8.77 %	20YEUR	-1.07 %
25YSEK	4.13 %	25YEUR	2.72 %

Table 4.8: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2014-11-27. The analysis is for both currencies together

Figure 4.5 and 4.5 below present the daily changes in value of the unhedged portfolio compared to the hedged portfolio. Figure 4.7 and 4.8 show the same result in histograms.

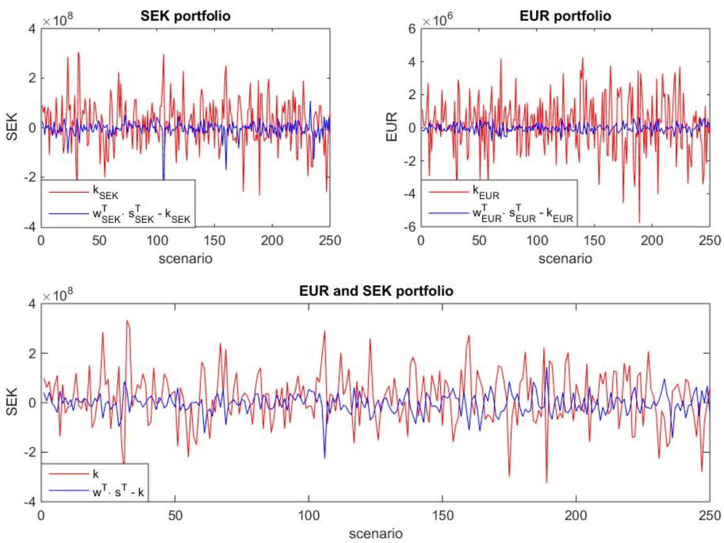

Figure 4.5: Value of the unhedged portfolio (red) and the hedged portfolio (blue) or EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

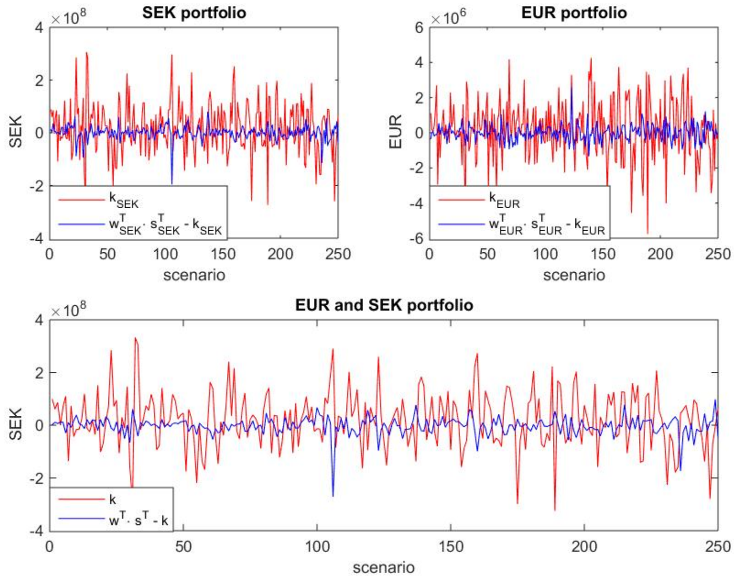

Figure 4.6: Value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the two currencies combined. The base curves are from 2014-11-27

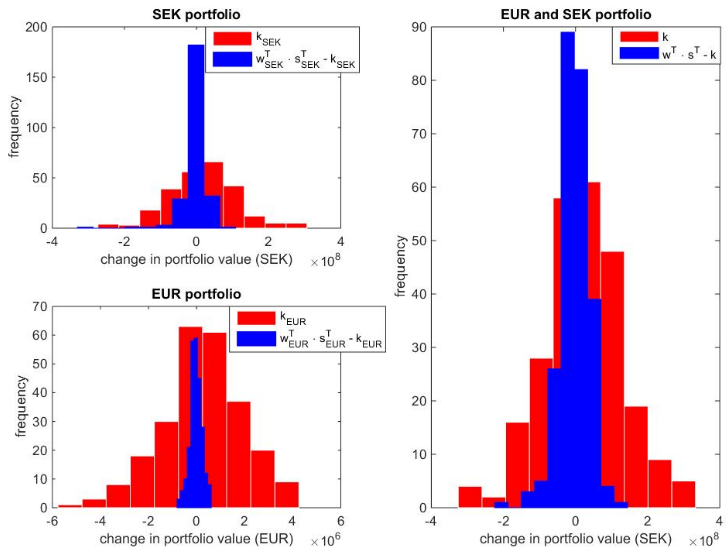

Figure 4.7: Value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

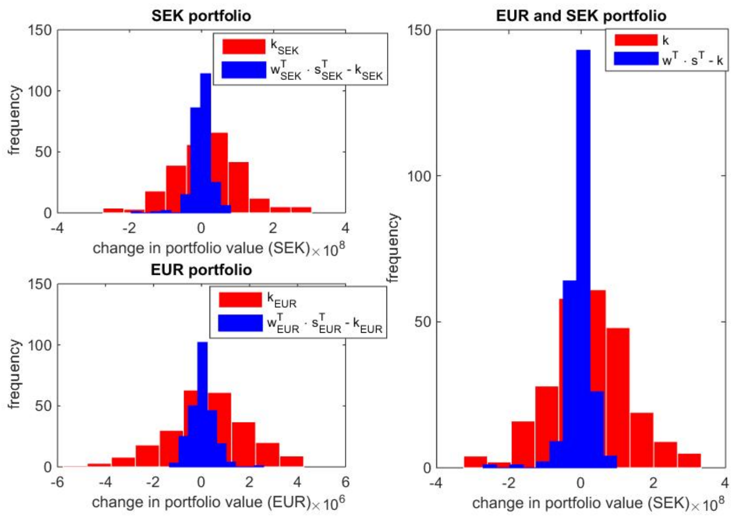

Figure 4.8: Value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2014-11-27

4.2 Historical simulation, PCA

This analysis is performed using the method described in Section 3.2 with the curves from 2015-04-27 chosen as base curves. First for $\lambda = 0$, i.e. only considering the minimization of the portfolio variance. Then for $\lambda = 1$, which means that the cost function is included.

4.2.1 Not including the cost function ($\lambda = 0$)

The results for the separate analysis of the currencies are presented in Table 4.9 below. Table 4.10 show the result for analysis of the both currencies.

	$\mathbf{w}_{\mathbf{SEK}}$		$\mathbf{w}_{\mathbf{EUR}}$
1YSEK	-11.70 %	1YEUR	-15.98 %
2YSEK	37.91 %	2YEUR	-3.54 %
3YSEK	-12.13 %	3YEUR	40.36 %
4YSEK	13.89 %	4YEUR	23.00 %
5YSEK	2.53 %	5YEUR	-0.87 %
6YSEK	-1.64 %	6YEUR	-3.97 %
7YSEK	4.66 %	7YEUR	3.66 %
8YSEK	3.60 %	8YEUR	3.22 %
9YSEK	4.63 %	9YEUR	0.65 %
10YSEK	1.41 %	10YEUR	3.84 %
12YSEK	0.51 %	12YEUR	-0.47 %
15YSEK	-3.64 %	15YEUR	0.05 %
20YSEK	0.59 %	20YEUR	0.25 %
25YSEK	1.16 %	25YEUR	-0.14 %

Table 4.9: The optimal nominal amount for each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is separate for the currencies EUR and SEK

	$\mathbf{w}$		$\mathbf{w}$
1YSEK	16.54 %	1YEUR	17.69 %
2YSEK	11.14 %	2YEUR	2.32 %
3YSEK	-1.52 %	3YEUR	-5.88 %
4YSEK	-15.75 %	4YEUR	-6.04 %
5YSEK	-1.71 %	5YEUR	2.01 %
6YSEK	3.11 %	6YEUR	1.14 %
7YSEK	3.21 %	7YEUR	-1.34 %
8YSEK	1.17 %	8YEUR	0.30 %
9YSEK	1.16 %	9YEUR	1.94 %
10YSEK	0.49 %	10YEUR	1.17 %
12YSEK	1.21 %	12YEUR	-0.25 %
15YSEK	0.09 %	15YEUR	-0.89 %
20YSEK	-0.35 %	20YEUR	-1.12 %
25YSEK	0.03 %	25YEUR	0.43 %

Table 4.10: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27 for the portfolio of both currencies

The daily changes in value of the unhedged portfolio compared to the hedged portfolio are presented in Figure 4.9 and the corresponding histograms are presented in Figure 4.10 below.

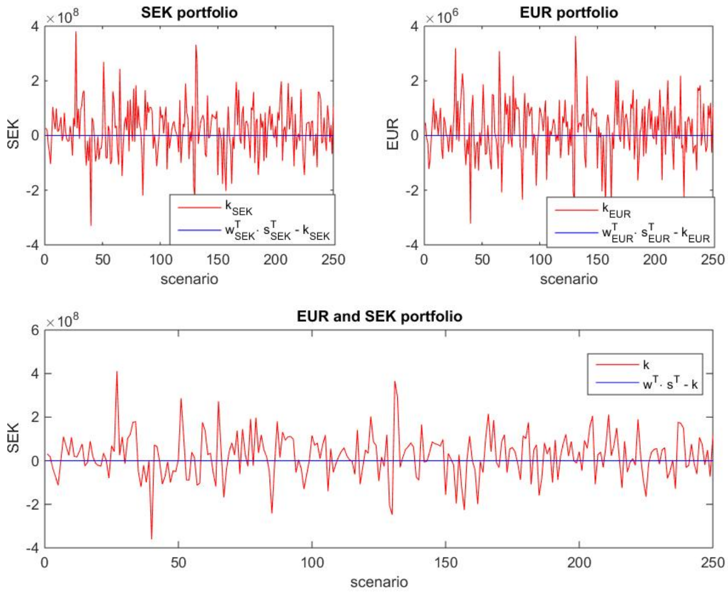

Figure 4.9: Daily change in value of the unhedged portfolio (red) and the hedged portfolio (blue) for the EUR portfolio and the SEK portfolio separately and for the total portfolio of the both currencies. The base curves are from 2015-04-27

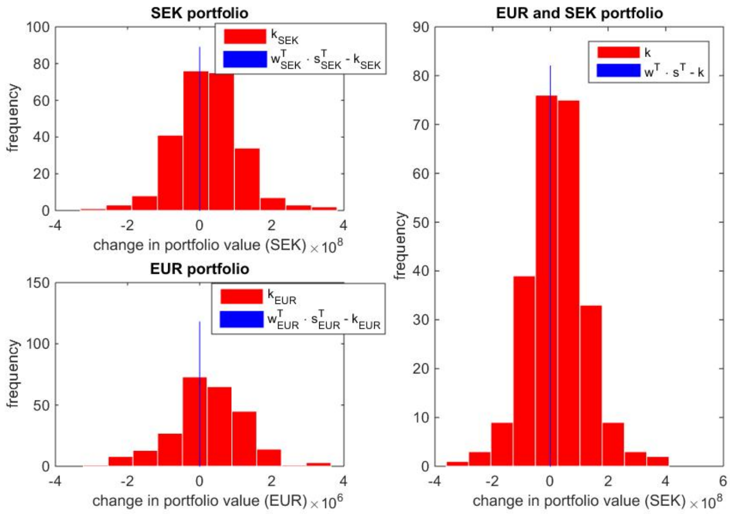

Figure 4.10: Histograms of the daily changes in value of the unhedged portfolio (red) and the hedged portfolio (blue) for the EUR portfolio and the SEK portfolio separately, and for the portfolio of the both currencies. The base curves are from 2015-04-27

4.2.2 Including the cost function ($\lambda = 1$)

The results for $\lambda = 1$ are presented in this section.

The results from a separate analysis of the both currencies are presented in Table 4.11 below. Table 4.12 shows the result for the analysis of the both currencies.

	$\mathbf{w}_{\mathbf{SEK}}$		$\mathbf{w}_{\mathbf{EUR}}$
1YSEK	0.15 %	1YEUR	-15.77 %
2YSEK	-3.83 %	2YEUR	-2.87 %
3YSEK	-3.44 %	3YEUR	4.04 %
4YSEK	11.62 %	4YEUR	3.62 %
5YSEK	13.39 %	5YEUR	10.61 %
6YSEK	0.99 %	6YEUR	5.94 %
7YSEK	-0.19 %	7YEUR	7.36 %
8YSEK	-11.67 %	8YEUR	-0.24 %
9YSEK	4.01 %	9YEUR	-5.63 %
10YSEK	-6.83 %	10YEUR	6.39 %
12YSEK	1.56 %	12YEUR	-24.49 %
15YSEK	6.39 %	15YEUR	9.22 %
20YSEK	-18.46 %	20YEUR	3.19 %
25YSEK	17.46 %	25YEUR	0.62 %

Table 4.11: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is separate for the two currencies

	$\mathbf{w}$		$\mathbf{w}$
1YSEK	-4.48 %	1YEUR	-0.69 %
2YSEK	4.76 %	2YEUR	-2.93 %
3YSEK	1.79 %	3YEUR	2.24 %
4YSEK	5.11 %	4YEUR	-5.40 %
5YSEK	0.21 %	5YEUR	-14.43 %
6YSEK	3.85 %	6YEUR	1.62 %
7YSEK	6.49 %	7YEUR	-4.02 %
8YSEK	0.64 %	8YEUR	-1.64 %
9YSEK	2.50 %	9YEUR	-1.07 %
10YSEK	3.72 %	10YEUR	6.92 %
12YSEK	-5.79 %	12YEUR	2.65 %
15YSEK	6.32 %	15YEUR	4.01 %
20YSEK	-2.20 %	20YEUR	0.11 %
25YSEK	-3.82 %	25YEUR	-0.58 %

Table 4.12: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is for the both currencies together

Figure 4.11 and 4.12 present the daily changes in value of the unhedged portfolio compared to the hedged portfolio and the corresponding histograms.

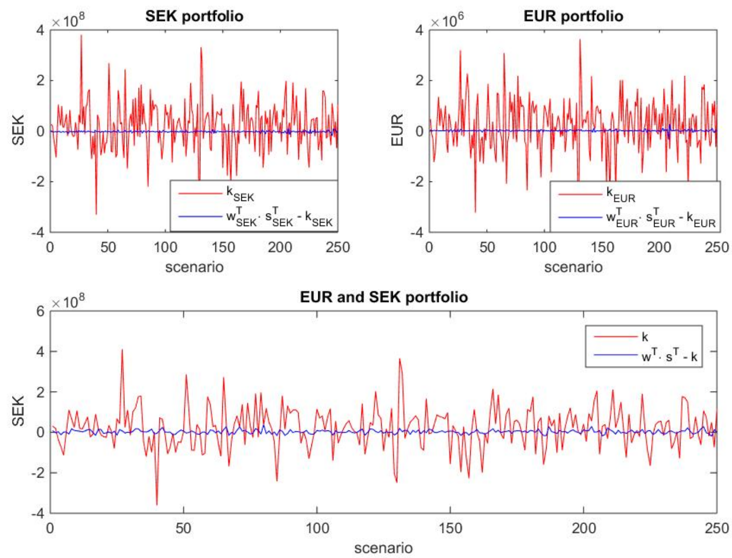

Figure 4.11: Value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

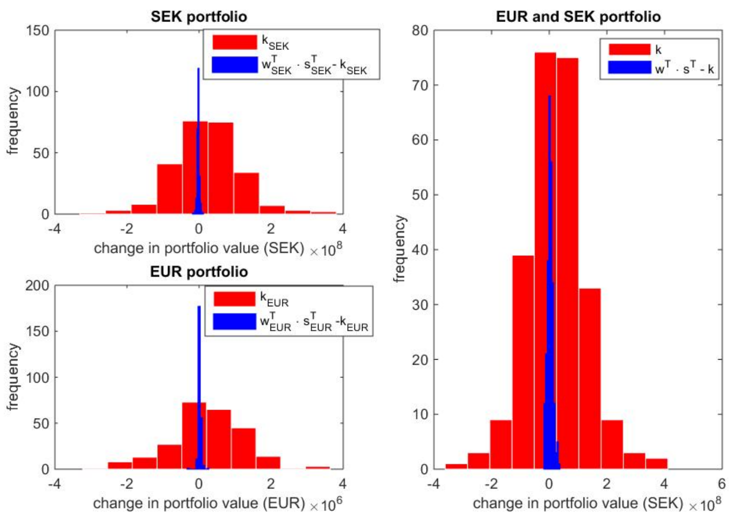

Figure 4.12: Histograms of the values of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

4.3 Random principal components

The results from the method in Section 3.3 are presented in this section. The curves from 2015-04-27 are chosen as the base curves.

4.3.1 Not including the cost function ($\lambda = 0$)

The results from the separate analysis of the currencies are presented in Table 4.13, while the result of the combined analysis with the both currencies is presented in Table 4.14.

	$\mathbf{w_{SEK}}$		$\mathbf{w_{EUR}}$
1YSEK	-8.54 %	1YEUR	1.25 %
2YSEK	31.38 %	2YEUR	0.36 %
3YSEK	-2.95 %	3YEUR	0.59 %
4YSEK	5.59 %	4YEUR	0.71 %
5YSEK	3.00 %	5YEUR	0.64 %
6YSEK	-3.25 %	6YEUR	0.06 %
7YSEK	-3.09 %	7YEUR	-0.42 %
8YSEK	12.99 %	8YEUR	12.62 %
9YSEK	-7.39 %	9YEUR	-43.08 %
10YSEK	-10.55 %	10YEUR	33.20 %
12YSEK	10.09 %	12YEUR	0.53 %
15YSEK	0.78 %	15YEUR	-4.53 %
20YSEK	-0.40 %	20YEUR	2.02 %
25YSEK	0.00 %	25YEUR	-0.00 %

Table 4.13: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is separate for the currencies EUR and SEK

	w		**w**
1YSEK	5.18 %	1YEUR	-1.46 %
2YSEK	-12.04 %	2YEUR	0.46 %
3YSEK	2.71 %	3YEUR	-0.27 %
4YSEK	1.03 %	4YEUR	-0.28 %
5YSEK	0.35 %	5YEUR	0.48 %
6YSEK	0.16 %	6YEUR	0.00 %
7YSEK	0.02 %	7YEUR	1.44 %
8YSEK	0.21 %	8YEUR	-9.34 %
9YSEK	0.46 %	9YEUR	0.00 %
10YSEK	3.77 %	10YEUR	-0.00 %
12YSEK	3.33 %	12YEUR	22.15 %
15YSEK	-8.22 %	15YEUR	-19.57 %
20YSEK	1.70 %	20YEUR	4.01 %
25YSEK	0.00 %	25YEUR	1.35 %

Table 4.14: The optimal nominal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is for the currencies EUR and SEK together

The daily changes in value of the unhedged portfolio compared to the hedged portfolio are presented in Figure 4.13. Figure 4.14 show the corresponding histograms.

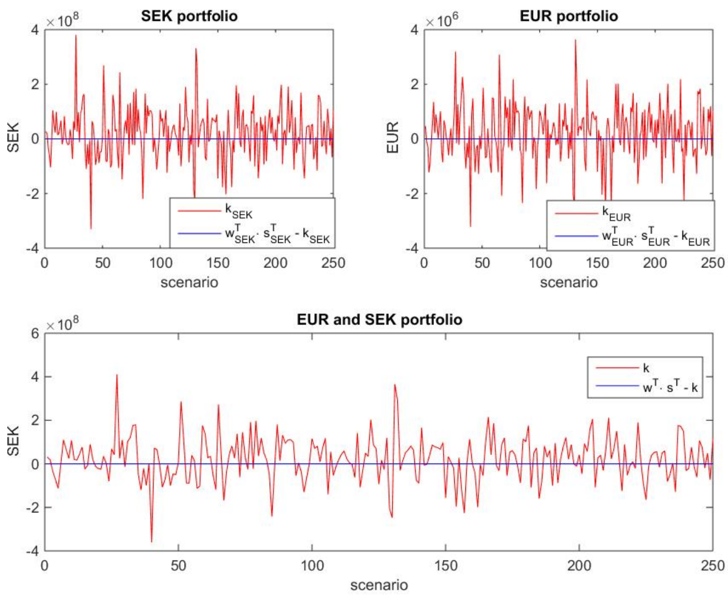

Figure 4.13: Daily change in value of the unhedged portfolio (red) and the hedged portfolio (blue) for the EUR portfolio and the SEK portfolio separately and for the total portfolio of the both currencies. The base curves are from 2015-04-27

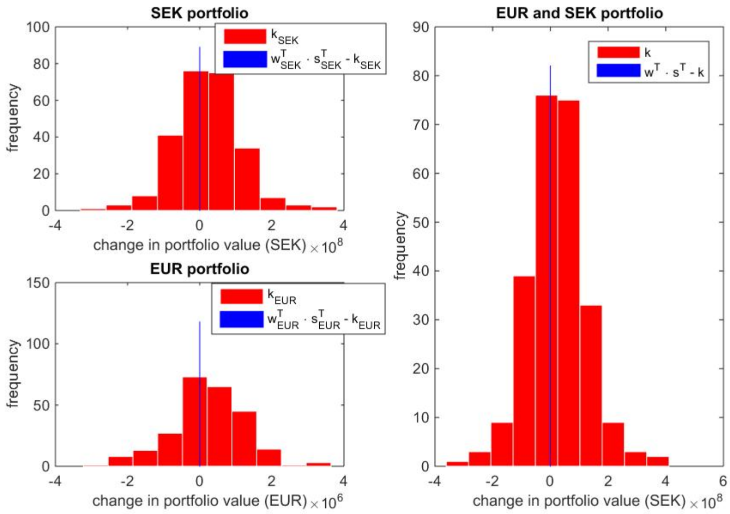

Figure 4.14: Histogram of the daily change in value of the unhedged portfolio (red) and the hedged portfolio (blue) for the EUR portfolio and the SEK portfolio separately and for the total portfolio of the both currencies. The base curves are from 2015-04-27

4.3.2 Including the cost function ($\lambda = 1$)

In this section the cost function is included by letting $\lambda = 1$.

The results from the separate analysis of the currencies are presented Table 4.15 and the results from the analysis of both currencies is presented in Table 4.14. The daily changes in value of the unhedged portfolio compared to the hedged portfolio are presented in Figure 4.15 and the corresponding histograms are presented in Figure 4.16.

	$\mathbf{w}_{\mathbf{SEK}}$		$\mathbf{w}_{\mathbf{EUR}}$
1YSEK	-15.84 %	1YEUR	-19.74 %
2YSEK	8.91 %	2YEUR	21.57 %
3YSEK	-4.53 %	3YEUR	0.80 %
4YSEK	8.01 %	4YEUR	-2.39 %
5YSEK	0.88 %	5YEUR	0.75 %
6YSEK	4.51 %	6YEUR	0.10 %
7YSEK	9.95 %	7YEUR	-8.83 %
8YSEK	2.02 %	8YEUR	14.22 %
9YSEK	-8.37 %	9YEUR	0.49 %
10YSEK	-5.59 %	10YEUR	-11.83 %
12YSEK	0.16 %	12YEUR	-5.78 %
15YSEK	-4.33 %	15YEUR	-0.70 %
20YSEK	16.94 %	20YEUR	12.68 %
25YSEK	-9.97 %	25YEUR	0.13 %

Table 4.15: The optimal nominal amount for each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27. The analysis is separate for the currencies EUR and SEK

	w		w
1YSEK	-4.39 %	1YEUR	-2.01 %
2YSEK	-1.69 %	2YEUR	-2.88 %
3YSEK	-3.83 %	3YEUR	0.59 %
4YSEK	1.83 %	4YEUR	2.82 %
5YSEK	-1.48 %	5YEUR	-2.25 %
6YSEK	1.54 %	6YEUR	0.09 %
7YSEK	6.61 %	7YEUR	-1.97 %
8YSEK	10.98 %	8YEUR	-1.36 %
9YSEK	1.03 %	9YEUR	1.07 %
10YSEK	-9.28 %	10YEUR	3.71 %
12YSEK	8.71 %	12YEUR	-9.18 %
15YSEK	7.58 %	15YEUR	2.24 %
20YSEK	0.47 %	20YEUR	-3.80 %
25YSEK	1.37 %	25YEUR	5.28 %

Table 4.16: The optimal amount for each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27 for the both currencies together

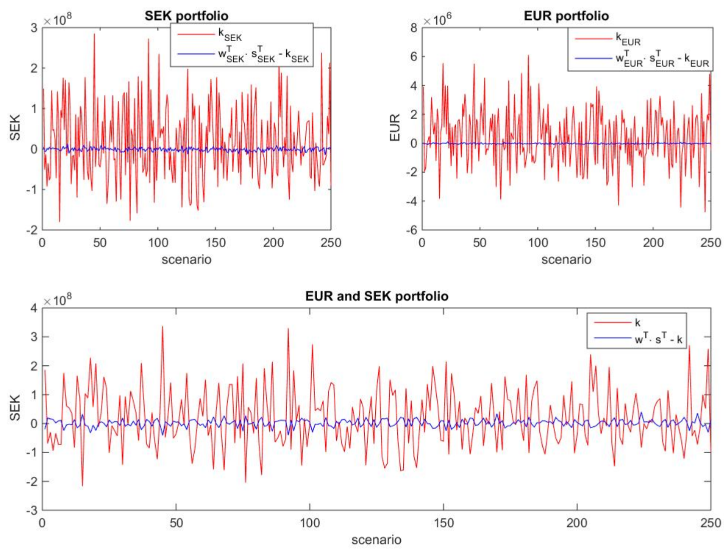

Figure 4.15: Daily change in value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

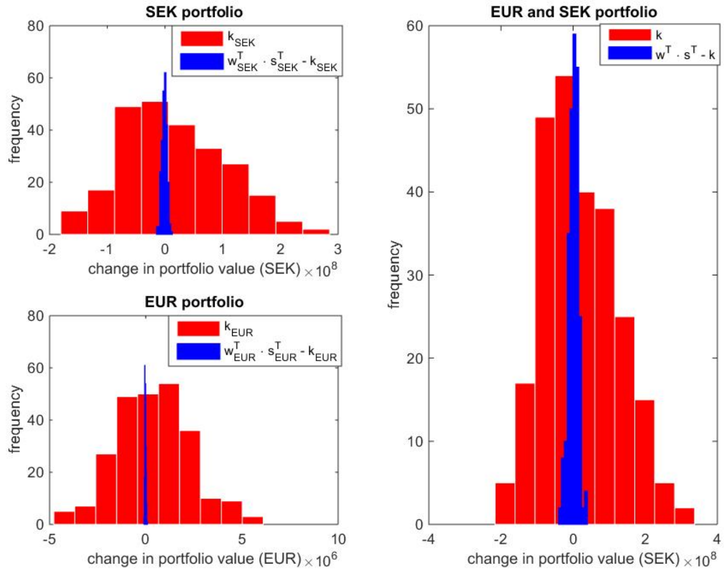

Figure 4.16: Histogram of the daily change in value of the unhedged portfolio (red) and the hedged portfolio (blue) for EUR and SEK separately and for the both currencies combined. The base curves are from 2015-04-27

4.4 Changing the cost function

The analysis is performed following the method described in Section 3.4. First for $\lambda = 1$, resulting in very small changes for different values of φ. Therefore the analysis is repeated with $\lambda = 10^6$ and the results for different values of φ are presented in the tables below.

$\varphi = 1$

	w		**w**
1YSEK	-3.75 %	1YEUR	-0.62 %
2YSEK	5.48 %	2YEUR	-4.57 %
3YSEK	-0.02 %	3YEUR	2.27 %
4YSEK	4.44 %	4YEUR	-5.91 %
5YSEK	0.10 %	5YEUR	-13.84 %
6YSEK	3.79 %	6YEUR	1.57 %
7YSEK	6.58 %	7YEUR	-4.64 %
8YSEK	0.97 %	8YEUR	-1.88 %
9YSEK	2.92 %	9YEUR	-0.90 %
10YSEK	3.73 %	10YEUR	7.51 %
12YSEK	-5.62 %	12YEUR	2.46 %
15YSEK	6.19 %	15YEUR	4.25 %
20YSEK	-0.89 %	20YEUR	0.51 %
25YSEK	-4.45 %	25YEUR	0.14 %

Table 4.17: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 1$

$\varphi = 0.9$

	w		**w**
1YSEK	-5.82 %	1YEUR	-0.61 %
2YSEK	5.45 %	2YEUR	-1.37 %
3YSEK	1.49 %	3YEUR	1.88 %
4YSEK	4.51 %	4YEUR	-6.12 %
5YSEK	-0.90 %	5YEUR	-14.08 %
6YSEK	4.60 %	6YEUR	1.73 %
7YSEK	6.97 %	7YEUR	-2.94 %
8YSEK	0.02 %	8YEUR	-1.34 %
9YSEK	2.31 %	9YEUR	-1.44 %
10YSEK	3.20 %	10YEUR	7.91 %
12YSEK	-5.73 %	12YEUR	2.67 %
15YSEK	5.51 %	15YEUR	4.23 %
20YSEK	-1.91 %	20YEUR	0.33 %
25YSEK	-4.63 %	25YEUR	-0.30 %

Table 4.18: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.9$

$\varphi = 0.8$

	w		**w**
1YSEK	-5.56 %	1YEUR	-0.60 %
2YSEK	5.28 %	2YEUR	-1.48 %
3YSEK	1.62 %	3YEUR	1.95 %
4YSEK	4.68 %	4YEUR	-6.02 %
5YSEK	-0.69 %	5YEUR	-14.21 %
6YSEK	4.45 %	6YEUR	1.74 %
7YSEK	6.83 %	7YEUR	-3.10 %
8YSEK	0.14 %	8YEUR	-1.34 %
9YSEK	2.32 %	9YEUR	-1.35 %
10YSEK	3.30 %	10YEUR	7.78 %
12YSEK	-5.76 %	12YEUR	2.66 %
15YSEK	5.66 %	15YEUR	4.15 %
20YSEK	-2.02 %	20YEUR	0.25 %
25YSEK	-4.57 %	25YEUR	-0.48 %

Table 4.19: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.8$

$\varphi = 0.7$

	w		**w**
1YSEK	-6.20 %	1YEUR	-0.79 %
2YSEK	5.24 %	2YEUR	-1.29 %
3YSEK	1.90 %	3YEUR	1.87%
4YSEK	4.56 %	4YEUR	-6.04 %
5YSEK	-0.93 %	5YEUR	-14.16 %
6YSEK	4.57 %	6YEUR	1.67 %
7YSEK	7.20 %	7YEUR	-2.79 %
8YSEK	-0.10 %	8YEUR	-1.22 %
9YSEK	2.23 %	9YEUR	-1.50 %
10YSEK	3.17 %	10YEUR	7.45 %
12YSEK	-5.79 %	12YEUR	2.79 %
15YSEK	5.61 %	15YEUR	4.26 %
20YSEK	-2.06 %	20YEUR	0.34 %
25YSEK	-4.21 %	25YEUR	-0.07 %

Table 4.20: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.7$

$\varphi = 0.6$

	w		**w**
1YSEK	-3.86 %	1YEUR	-0.96 %
2YSEK	4.76 %	2YEUR	-4.14 %
3YSEK	0.99 %	3YEUR	2.65 %
4YSEK	4.91 %	4YEUR	-5.49 %
5YSEK	0.57 %	5YEUR	-14.18 %
6YSEK	3.51 %	6YEUR	1.41 %
7YSEK	6.54 %	7YEUR	-4.90 %
8YSEK	0.95 %	8YEUR	-1.70 %
9YSEK	2.73 %	9YEUR	-0.85 %
10YSEK	3.86 %	10YEUR	6.52 %
12YSEK	-5.80 %	12YEUR	2.60 %
15YSEK	6.62 %	15YEUR	4.16 %
20YSEK	-1.44 %	20YEUR	0.20 %
25YSEK	-3.61 %	25YEUR	-0.07 %

Table 4.21: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.6$

$\varphi = 0.5$

	w		**w**
1YSEK	-1.93 %	1YEUR	-1.17 %
2YSEK	4.03 %	2YEUR	-4.43 %
3YSEK	0.39 %	3YEUR	3.60 %
4YSEK	5.22 %	4YEUR	-5.86 %
5YSEK	1.54 %	5YEUR	-14.60 %
6YSEK	2.68 %	6YEUR	1.41 %
7YSEK	5.74 %	7YEUR	-6.35 %
8YSEK	1.82 %	8YEUR	-1.34 %
9YSEK	2.86 %	9YEUR	-0.07 %
10YSEK	4.25 %	10YEUR	6.20 %
12YSEK	-5.90 %	12YEUR	2.29 %
15YSEK	6.98 %	15YEUR	3.90 %
20YSEK	-0.33 %	20YEUR	-0.16 %
25YSEK	-3.94 %	25YEUR	-1.02 %

Table 4.22: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.5$

$\varphi = 0.4$

	w		**w**
1YSEK	-3.69 %	1YEUR	-1.10 %
2YSEK	4.54 %	2YEUR	-3.51 %
3YSEK	0.96 %	3YEUR	2.94 %
4YSEK	4.99 %	4YEUR	-6.02 %
5YSEK	0.61 %	5YEUR	-14.52 %
6YSEK	3.36 %	6YEUR	1.49 %
7YSEK	6.46 %	7YEUR	-5.02 %
8YSEK	1.11 %	8YEUR	-1.24 %
9YSEK	2.67 %	9YEUR	-0.59 %
10YSEK	3.88 %	10YEUR	6.61 %
12YSEK	-5.92 %	12YEUR	2.55 %
15YSEK	6.58 %	15YEUR	4.10 %
20YSEK	-0.98 %	20YEUR	0.07 %
25YSEK	-4.01 %	25YEUR	-0.45 %

Table 4.23: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.4$

$\varphi = 0.3$

	w		**w**
1YSEK	-5.04 %	1YEUR	-0.67 %
2YSEK	5.37 %	2YEUR	-2.17 %
3YSEK	1.04 %	3YEUR	2.20 %
4YSEK	4.58 %	4YEUR	-6.08 %
5YSEK	-0.46 %	5YEUR	-14.11 %
6YSEK	4.30 %	6YEUR	1.66 %
7YSEK	6.75 %	7YEUR	-3.69 %
8YSEK	0.37 %	8YEUR	-1.47 %
9YSEK	2.48 %	9YEUR	-1.22 %
10YSEK	3.41 %	10YEUR	7.77 %
12YSEK	-5.76 %	12YEUR	2.58 %
15YSEK	5.80 %	15YEUR	4.23 %
20YSEK	-1.56 %	20YEUR	0.29 %
25YSEK	-4.61 %	25YEUR	-0.34 %

Table 4.24: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.3$

$\varphi = 0.2$

	w		w
1YSEK	-5.54 %	1YEUR	-0.53 %
2YSEK	5.49 %	2YEUR	-1.28 %
3YSEK	1.30 %	3YEUR	1.94 %
4YSEK	4.53 %	4YEUR	-6.23 %
5YSEK	-0.86 %	5YEUR	-14.10 %
6YSEK	4.57 %	6YEUR	1.79 %
7YSEK	6.80 %	7YEUR	-3.05 %
8YSEK	0.13 %	8YEUR	-1.32 %
9YSEK	2.34 %	9YEUR	-1.36 %
10YSEK	3.22 %	10YEUR	8.15 %
12YSEK	-5.72 %	12YEUR	2.59 %
15YSEK	5.47 %	15YEUR	4.18 %
20YSEK	-1.78 %	20YEUR	0.29 %
25YSEK	-4.93 %	25YEUR	-0.53 %

Table 4.25: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.2$

$\varphi = 0.1$

	w		w
1YSEK	6.58 %	1YEUR	-1.81 %
2YSEK	-7.24 %	2YEUR	-6.51 %
3YSEK	0.48 %	3YEUR	0.28 %
4YSEK	-1.95 %	4YEUR	6.14 %
5YSEK	4.12 %	5YEUR	8.73 %
6YSEK	-6.45 %	6YEUR	-2.38 %
7YSEK	-4.05 %	7YEUR	-1.62 %
8YSEK	1.66 %	8YEUR	1.06 %
9YSEK	-0.98 %	9YEUR	2.35 %
10YSEK	-0.62 %	10YEUR	-12.68 %
12YSEK	3.46 %	12YEUR	-1.15 %
15YSEK	-0.46 %	15YEUR	-2.96 %
20YSEK	2.10 %	20YEUR	-0.29 %
25YSEK	9.16 %	25YEUR	2.74 %

Table 4.26: The optimal amount of each tenor of the swaps as percentage of the total amount to hedge the portfolio from 2015-04-27, $\varphi = 0.1$

A plot of the number of EUR swaps in the hedging portfolio as a function of φ is presented in Figure 4.17.

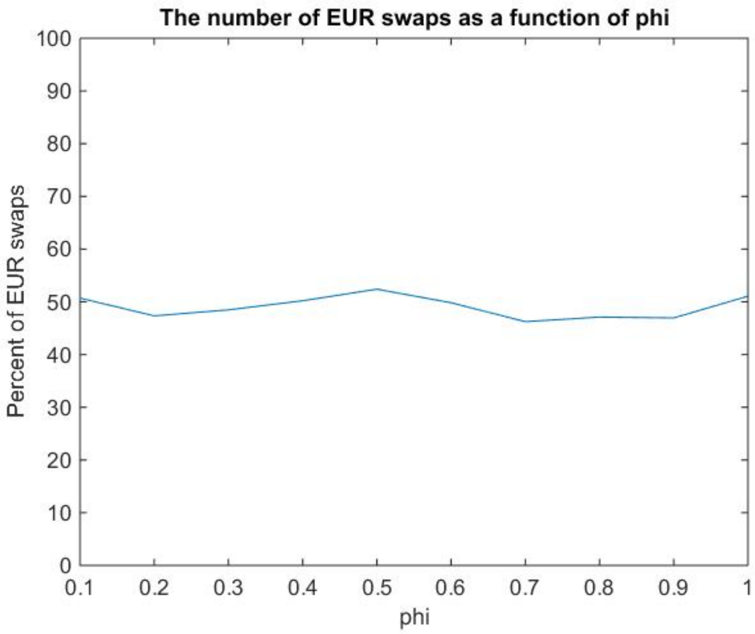

Figure 4.17: The number of EUR swaps in the hedging portfolio as a function of φ

Chapter 5

Analysis

This chapter presents a discussion of the results in the previous section

5.1 Discussion

The results from the first two methods show that it seems more optimal to hedge with swaps with shorter tenors while only considering the portfolio variance. A reason of this may be that there are more noise in the curve construction for the earlier tenors and may result in overhedging. The result with the cost function included show that it is preferable to hedge with swaps with longer tenors too, in order to keep the transactions costs down.

For the third method using random principal components show that it is preferable to hedge with longer tenors in EUR than in SEK.

As mentioned earlier the curves are constructed using cubic splines. The disadvantage with this curve construction is that they have a oscillatory behaviour. A change in one end of the curve can have an effect a distance away. (cf. [5]) This fact affects the results and the analysis would have been more correct if another interpolation method would have been used and it may be the reason that the results of the methods differ.

Notable are the results from the analysis of the both currencies together. The results show that it is a little more preferable to hedge with SEK swaps, both when the portfolio variance is considered and when it is not. As can be seen in Section 4.4, even when the cost function of the EUR swaps is decreasing it is optimal to hedge with approximately 50 % SEK swaps. The reason can be that there are more noise in the curve construction for the SEK swaps, or due to the market change.

Bibliography

[1] Lindskog F, Hult H, Hammarlid O, Rehn CJ. *Risk and Portfolio Analysis*, Stockholm: Springer; 2012

[2] Hull, J. *Fundamentals of Futures and Options Markets*, Edinburgh: Pearson; 2014

[3] Björk T. *Arbitrage Theory in Continuous Time Third Edition*, Stockholm: Oxford University Press; 2009

[4] Hagan P, West G. *Methods for Constructing a Yield Curve*, WILMOTT Magazine; May 2008, pp 70-81

[5] Hagan P, West G. *Interpolation Methods for Curve Construction*, Applied Mathematical Finance, Vol 13, No. 2; 2006 pp 89-129

[6] Hull, J. *Risk Management and Financial Institutions Third Edition*, New Jersey: John Wiley & Sons, Inc.; 2012

[7] Bishop, C. *Pattern recognition and machine learning*, New York: Springer; 2006

[8] Lagarias J, Reeds J, Wright M, Wright P. *Convergence Properties of the Nelder-Mead Simplex Method in low dimensions*, SIAM Journal of Optimization, Vol. 9, No. 1; 1998, pp. 112-147