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A framework for modeling the liquidity and interest rate risk of demand deposits

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Abstract

The objective of this report is to carry out a pre-study and develop a framework for how the liquidity and interest rate risk of a bank's demand deposits can be modeled. This is done by first calibrating a Vasicek short rate model and then deriving models for the bank's deposit volume and deposit rate using multiple regression.

The volume model and the deposit rate model are used to determine the liquidity and interest rate risk, which is done separately. The liquidity risk is determined by a liquidity quantile which estimates the minimum deposit volume that is expected to remain in the bank over a given time period. The interest rate risk is quantified by an arbitrage-free valuation of the demand deposit which can be used to determine the sensitivity of the net present value of the demand deposit caused by a parallel shift in the market rates. Furthermore, an immunization and a replicating portfolio are constructed and the performances of these are tested when introducing the same parallel shifts in the market rates as in the valuation of the demand deposit.

The conclusion of this thesis is that the framework for the liquidity risk management that is developed gave satisfactory results and could be used by the bank if the deposit volume is estimated on representative data and a more accurate model for the short rate is used. The interest rate risk framework did however not yield as reliable results and would be more challenging to implement as a more advanced model for the deposit rate is required.

Keywords:
Non-maturing liabilities; Liquidity risk; Interest rate risk; Vasicek short rate model; Deposit volume modeling; Deposit rate modeling; Valuation of demand deposits

Ett ramverk för att modellera likviditets- och ränterisk för inlåning

Sammanfattning

Målet med denna rapport är att utveckla ett ramverk för att bestämma likviditets- och ränterisken som är relaterad till en banks inlåningsvolym. Detta görs genom att först ta fram en modell för korträntan via kalibrering av en Vasicek modell. Därefter utvecklas, genom multipelregression, modeller för att beskriva bankens inlåningsvolym och inlåningsränta.

Dessa modeller används för att kvantifiera likviditets- och ränterisken för inlåningsvolymen, vilka beräknas och presenteras separat. Likviditetsrisken bestäms genom att en likviditetskvantil tas fram, vilken estimerar den minimala inlåningsvolymen som förväntas kvarstå hos banken över en given tidsperiod. Ränterisken kvantifieras med en arbitragefri värdering av inlåningen och resultatet används för att bestämma känsligheten för hur nuvärdet av inlåningsvolymen påverkas av ett parallellskifte. Utöver detta bestäms en immuniseringsportfölj samt en replikerande portfölj och resultatet av dessa utvärderas mot hur nuvärdet förändras givet att samma parallellskifte i ränteläget som tidigare introduceras.

Slutsatsen av projektet är att det framtagna ramverket för att bestämma likviditetsrisken för inlåningen gav bra resultat och skulle kunna implementeras i dagsläget av banken, förutsatt att volymmodellen estimeras på representativ data samt att en bättre modell för korträntan används. Ramverket för att bestämma ränterisken gav dock inte lika tillförlitliga resultat och är mer utmanande att implementera då en mer avancerad modell för inlåningsräntan krävs.

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Christina Skoglund and Peter Henningsson

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1 Introduction

1.1 Background

A bank is formally defined as a financial institution that is allowed to receive money of its depositors. The main task of a commercial bank is to manage the deposits and withdrawals of its depositors as well as providing loans to clients. A bank's deposit accounts consist of money that has been placed for keeping by the clients, i.e. the depositors, in accounts such as money market accounts, savings accounts etc, see Frauendorfer, K. and Schürle, M. [10]. The deposit itself is seen as a liability that is owed by the bank to the depositor. Even though the bank periodically pays the depositor a deposit rate, it is able to make a profit from the spread between the interest rate received from outstanding loans and the paid deposit rate. A bank is allowed to adjust the customer interest rate at any time and the rate is mainly determined by variations in the market rates with some delay, see Frauendorfer, K. and Schürle, M. [10].

As the depositors are free to withdraw their money from the bank whenever they please, the bank's deposits are therefore characterized as non-maturing liabilities since they lack predetermined maturity dates. This "option" of the depositors is an attribute that generates a risk for the bank as the deposits represent a significant part of the bank's liabilities. This, together with the bank's option to change the deposit rate in response to market rate changes constitute a risk and all these factors make non-maturing liabilities a complex financial instrument to manage, see Bardenhewer, M. [3] and Maes, K. and Timmermans, T. [18].

Due to the complex nature of non-maturing liabilities it is of high interest for financial institutions, such as banks, to develop a framework that models the emerging risk from such instruments. This topic has therefore previously been studied in various articles such as Kalkbrener, M. and Willing, J. [15], Jarrow, R.A. and Van Deventer, D.R. [14] and Maes, K. and Timmermans, T. [18] where different approaches have been presented on how to deal with and model these types of risks. However, the common denominator for these studies is that the risk management is mainly divided into two sectors:

- **Liquidity risk**
  The risk that occurs due to the fact that financial institutions do not know when future withdrawals will occur and how much money is to be withheld.

- **Interest rate risk**
  The risk that arises due to changes in future interest rates and the possible impact that this might have on the value of the institution's assets and liabilities.

In order to quantify these two risk types, there are a few steps that need to be addressed, see Jarrow, R.A. and Van Deventer, D.R. [14]. First, a model which describes the bank's deposit volume in the most accurate way should be developed in order to be able to investigate how future outcomes of the deposit volume will behave. There are various methods for determining the deposit volume model that are presented in previous literature and a common method is to use multiple regression. The deposit volume model is then utilized in order to predict how the forthcoming liquidity of the bank will behave and in that way derive the liquidity constraint, see Kalkbrener, M. and Willing, J. [15].

Regarding the quantification of the interest rate risk, the core is to determine a deposit rate model in order to investigate how the spread between the market rates and the deposit rates is affected by market changes as well as how it directly affects the bank's liabilities and assets, see Frauendorfer, K. and Schürle, M. [10]. Thereafter, there are different strategies that can be used in order to present the risk that is related to interest rate changes, see Bardenhewer, M. [3].

In this report, a framework for determining the bank's liquidity risk and interest rate risk respectively is suggested. Methods and concepts such as multiple regression, Monte Carlo simulation, valuation of demand deposits, immunization, duration and replicating portfolio are all introduced and explained in order to finally present the liquidity risk and the interest rate risk respectively.

1.2 Problem formulation

Skandiabanken, in this report referred to as "the bank", is a Swedish commercial bank which needs to manage the liquidity and interest rate risk of its non-maturing liabilities. The outcome of this project will be used as a pre-study for the implementation of the liquidity and interest risk management within their general risk management framework.

The objective of this thesis is to formulate and present a framework which the bank can use in order to estimate its liquidity risk and interest rate risk respectively. Hence, the targets are to:

1. Determine a stochastic model of the short rate
2. Determine a suitable model of the bank's deposit volume
3. Determine a suitable model of the bank's deposit rate
4. Determine a framework for calculating the bank's liquidity risk
5. Determine a framework for calculating the bank's interest rate risk

1.3 Practical implementation

- All computations have been carried out in R and Python.
- All data that is presented has been scaled. No original data is illustrated.

1.4 Outline

The project is structured in such a way that the first part presents all necessary theoretical background in order to be able to solve the problem and thereafter the methodology section is presented. The report is then concluded with a presentation of the obtained results and an analysis.

1.5 Delimitations

In order to delimit this study, the following delimitations have been made. This leaves room for future development, which is addressed at the end of the report.

To derive the desired deposit volume model, the only macro variables that have been taken into account is the STIBOR market rate and swap rates. This is due to the fact that an extensive in-depth econometric analysis of all possible macro-variables is required in order to derive a model without risking over fitting. As the STIBOR and swap rates are good indicators for market movements and are frequently used in these contexts, see Kalkbrenner, T. and Willing, J. [15] and Jarrow, R.A. and Van Deventer, D.R. [14], it is expected that by using them sufficient market information is provided for this study.

Before implementing these frameworks, a closer analysis should be made concerning the deposit rate model. Modeling of deposit rates can be somewhat complex as they tend to be asymmetric. For instance, at a market drop the bank's may be eager to let the deposit rate drop with the same speed. However, at a market upturn, the deposit rates tend to rise more slowly, see Clausen, V. [7]. Therefore, another delimitation in this study is the fact that it is assumed that there are no asymmetries in the behaviour of the deposit rate and that the deposit rate can be expressed as a linear function of a market rate.

Another delimitation is that all different kinds of deposits of the bank have been merged into one "deposit account". This is simply due to the lack of historical data that has been provided as new deposit accounts have been introduced during the last 10 years. Considering that the data is provided with a monthly frequency, these new product introductions have caused major fluctuations in the data history for each deposit type. Due to this, deriving individual models to each specific deposit account has not been possible.

2 Theoretical framework

This section introduces all necessary theoretical background that is needed in order to be able to answer the research questions.

2.1 Non-maturing liabilities

One of the objectives of commercial banks is to profit from the positive spread between its assets and liabilities. The assets are defined as the investments of the bank, while the liabilities are its costs, see Nyström, K. [21]. The volume that each individual customer deposits is seen as an amount of money that has been "lent" to the bank. Hence, the aggregated volume of the deposited amount is from the bank's perspective seen as a liability.

The liabilities that include deposit accounts are called non-maturing liabilities as the depositors are free to, at any time, deposit or withdraw capital from their deposit account. Therefore, they lack a predetermined maturity date. Every commercial bank or financial institution that manages these types of non-maturing liabilities is eager to be able to model the liability's behaviour over time in order to protect themselves of potential risks. Having a good understanding and a model of the behaviour of the non-maturing liability would allow banks better prepare themselves against rare events such as bank runs, where large amounts of money are withdrawn in a short period of time. These events are difficult to predict but by having a proper risk management it is possible to minimize the impact of such event. A financial risk manager wants to know how long the deposits will stay in the bank, assuming that the deposit rates are adjusted appropriately when market rates change, see Bardenhewer, M. [3].

Demand deposits are *floating rate instruments*, meaning that they do not have a fixed rate of interest. Demand deposits pay the interest demand deposit rate $d(t)$ every period to the depositors and an important note is that banks cannot buy deposits but they can however stop issuing them while individuals cannot issue demand deposits. It is assumed that individuals have no arbitrage opportunities but there are arbitrage opportunities for banks, implying that $d(t) \leq r(t)\ \forall t$, where $r(t)$ is a market rate, see Jarrow, R.A. and Van Deventer, D.R. [14].

2.2 Modeling non-maturing liabilities

The development of models for non-maturing liabilities has previously been studied in, among other, the Jarrow, R.A. and Van Deventer, D.R. [14] article and in the article by Kalkbrener, M. and Willing, J. [15]. Both studies propose a log-normal deposit volume model including a stochastic factor which describes the evolution of the deposit volume of a financial institute.

2.2.1 Log-normal distribution

The random variable X is log-normally distributed if

$$X = \exp\{Y\},$$

where Y is a normally distributed random variable with mean μ and standard deviation σ

$$Y \sim N(\mu, \sigma^2).$$

A log-normal distribution is a continuous probability distribution that will only take positive real values.

2.3 Stochastic Differential Equations

If the local dynamics of a stochastic process $X(t)$ can be approximated by a stochastic difference equation

$$X(t + \Delta t) - X(t) = \mu(t, X(t))\Delta t + \sigma(t, X(t))Z(t), \tag{1}$$

where μ and σ are deterministic functions and $Z(t)$ is a normally distributed disturbance term, then the process $X(t)$ is said to be a diffusion. The functions μ and σ are called the drift and the diffusion term respectively, see Björk, T. [4].

In order to model the Gaussian disturbance term, the concept of a Wiener process is defined. A Wiener process is a stochastic process which has the following properties:

1. $W(0) = 0$.
2. $W(t)$ has independent increments, i.e. $W(s) - W(t)$ is independent of $W(r) - W(u)$ if $u < r < t < s$.
3. $W(t)$ has Gaussian increments, i.e. $W(s) - W(t) \sim N(0, s - t)$.
4. $W(t)$ has continuous paths.

The Wiener process can be used in order to rewrite Equation (1) into

$$X(t + \Delta t) - X(t) = \mu(t, X(t))\Delta t + \sigma(t, X(t))\Delta W(t), \tag{2}$$

where

$$\Delta W(t) = W(t + \Delta t) - W(t). \tag{3}$$

By letting $\Delta t \to 0$ in Equation (2), the stochastic differential equation can be formulated as

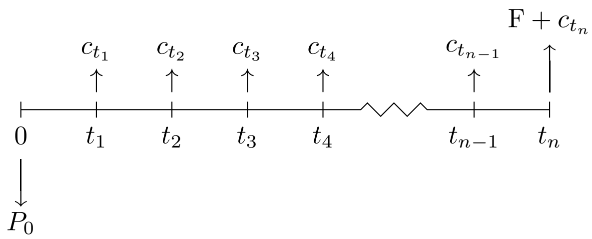
$$\begin{cases} dX(t) = \mu(X(t), t)dt + \sigma(X(t), t)dW(t), \\ X(0) = a, \end{cases} \tag{4}$$

where $X(0) = a$ denotes the initial condition, see Björk, T. [4].

2.4 Market interest rates

2.4.1 Zero rates

A coupon bearing bond is a debt obligation with predetermined periodically paid coupons attached which represent interest payments.



The time-line above illustrates the deterministic cash flows over a time period $[0, t_n]$, where $t_n = T$ is the maturity, that are exchanged in a coupon bond agreement between two parties. At time $t = 0$ the amount P_0 is paid from one party and at the times $t = t_1, \ldots, t_n$ a coupon of c_{t_i} is received up until the maturity date T, where at that point the face value F is received as well. The present value of the coupon bond is

$$P_0 = P(0, T) = \sum_{i=1}^{n} c_{t_i} e^{-y(t_i)t_i} + \mathrm{F}e^{-y(T)T}, \tag{5}$$

where $y(t_i)$is the continuously compounded *zero rate*.

For a bond with no coupon payments, i.e. $c_{t_i} = 0$, the present value is

$$P(0, T) = \mathrm{F}e^{-y(T)T} \tag{6}$$

and this bond is referred to as a *zero coupon bond*.

2.4.2 STIBOR rates

The Stockholm Interbank Offered Rate is a reference rate that depicts an average of the interest rates which a number of Swedish banks, called the STIBOR banks, are willing to borrow each other money without insurance for different maturity dates, see Sveriges Riksbank [**riksbanken**].

The simple spot rate for the interval $[t, T]$ is referred to as the STIBOR spot rate and is a simple interest rate defined as

$$\mathrm{STIBOR}(t, T) = -\frac{P(t, T) - 1}{(T - t)P(t, T)}, \tag{7}$$

where $P(t, T)$ is the price at time t of a zero coupon bond with maturity T, see Björk, T. [4].

2.4.3 Swap rates

An interest rate swap is basically a scheme where a payment stream at a fixed rate of interest is exchanged for a payment stream at a floating rate of interest between two parties over a time period t_n. The fixed rate of interest is called the *swap rate*, denoted as R_n, while the floating rate of interest is commonly a STIBOR rate. The swap rate R_n is determined such that the initial value of the swap, where the total value of the swap is denoted as $\Pi(t)$, equals zero i.e. $\Pi(0) = 0$. The total value of the swap is given by

$$\Pi(t) = \mathrm{F} \sum_{i=1}^{n} [P(t, t_{i-1}) - (1 + \Delta t \cdot R_n \cdot P(t, t_i))], \tag{8}$$

according to Björk, T. [4].

Assume that the fixed payments of the swap occur on a fixed number of dates $t_1, \ldots, t_n$ where T_0 is the emission date of the swap while $t_1, \ldots, t_{n-1}$ are the payment dates and $t_n = T$ is the maturity and let $\Delta t = t_i - t_{i-1}$. The swap rate is determined such that the price of the swap at $t \leq t_0$ equals zero, i.e.

$$\Pi(t) = P^{\text{fixed}}(t) - P^{\text{floating}}(t) = 0. \tag{9}$$

Assuming that the contract has been written at $t = 0$ and that $t_0 = 0$, solving Equation (8) for R_n gives

$$R_n = \frac{1 - P(0, t_n)}{\Delta t \sum_{i=1}^{n} P(0, t_i)}. \tag{10}$$

As the zero coupon bond price $P(0, t_i)$ is expressed as

$$P(0, t_i) = \frac{1}{(1 + z(t_i))^{t_i}}, \tag{11}$$

where $z(t_i)$ is the simple zero rate for maturity t_i, it is possible to derive market zero rates from interest rate swaps by inserting Equation (11) into (10) in order to get

$$R_n \cdot \Delta t \sum_{i=1}^{n} \frac{1}{(1 + z(t_i))^{t_i}} = 1 - \frac{1}{(1 + z(t_n))^{t_n}}, \tag{12}$$

where R_n is the current market listing of a interest rate swap with maturity t_n years and where $z(t_i)$ is the zero rate which is compatible with the market according to Finansinspektionen [9].

The interest rate swap can be interpreted as a par coupon bond, where R_n

is the coupon rate. This can be illustrated in the following equation

$$F = \sum_{i=1}^{n} \frac{R_n \cdot F \cdot \Delta t}{(1 + z(t_i))^{t_i}} + \frac{F}{(1 + z(t_n))^{t_n}}, \tag{13}$$

where F is the face value, see Finansinspektionen [9].

2.4.4 Yield curve

When comparing similar market rate securities of different maturities it is possible to create a curve which illustrates how the yields of the market rate securities vary depending on the maturities. In this way it is possible to understand the relationship between the yield and the maturity, see Waring, D. [23].

There are various ways for how a yield curve can be constructed. It can for example be constructed by using different market rates such as swap rates or zero rates derived from interest rate securities. For instance, zero rates can be derived from swap rates by using Equation (12).

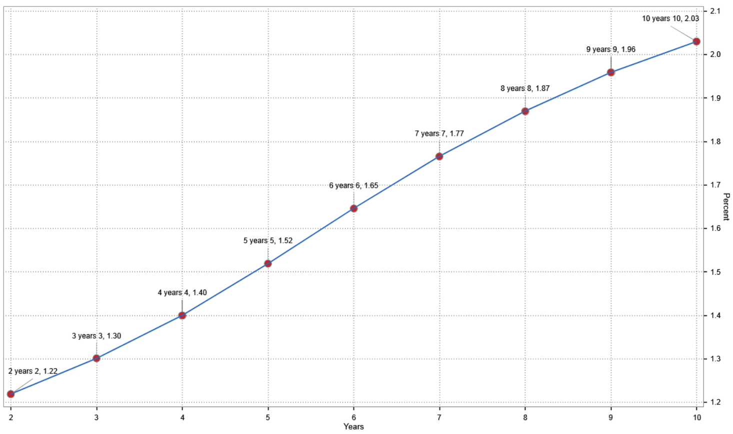


Figure 1: An illustration of a yield curve using Swedish swap rates on a specific date, see Macrobond [17].

An example of a yield curve on a specific date, which has been constructed using Swedish swap rates, is illustrated in Figure 1.

2.5 The short rate

In order to model the market rates one has to derive a model which describes the evolution of the yield curve over time. The short rate is a theoretical rate and can be interpreted as the risk less rate at which one can borrow and lend money over an infinite small time period, see Hull, J.C. [13].

The stochastic short rate $r(t)$ is assumed to have the following dynamics under the martingale measure $\mathbf{Q}$

$$dr(t) = \mu(t, r(t))dt + \sigma(t, r(t))dW(t). \tag{14}$$

Equation (14) is the stochastic differential equation that describes the dynamics of $r(t)$, where μ and σ are the drift and diffusion coefficients and $W(t)$ is a Wiener process, which has previously been introduced in Section 2.3.

Furthermore, the money account process is defined as

$$B(t) = \exp\left\{\int_0^t r(s)ds\right\}, \tag{15}$$

i.e.

$$\begin{cases} dB(t) = r(t)B(t)dt, \\ B(0) = 1, \end{cases}$$

where $B(0) = 1$ is the initial condition. This can be viewed as the value of investing money in a bank account which pays a stochastic rate of interest $r(t)$, see Björk, T. [4].

2.5.1 The Vasicek model

The Vasicek short rate interest model is one of the most commonly used interest rate models in the financial industry. The dynamics of the Vasicek short rate model, specified under the martingale measure $\mathbf{Q}$, are given by

$$\begin{cases} dr(t) = (b - a \cdot r(t))dt + \sigma dW(t), \\ r(0) = r_0, \end{cases} \tag{16}$$

where $r(0) = r_0$ is the initial condition, see Björk, T. [4]. By letting

$$\begin{cases} \mu = \frac{b}{a}, \\ \lambda = a, \end{cases}$$

the dynamics of the short rate are expressed as

$$dr(t) = \lambda(\mu - r(t))dt + \sigma dW(t). \tag{17}$$

From this one can see that the model has the property of being mean reverting, with λ being the speed of the reversion. The solution to the stochastic differential equation in Equation (17) is according to Lesniewski, A. [16]:

$$r(t) = r_0 e^{-\lambda t} + \mu(1 - e^{-\lambda t}) + \sigma \int_0^t e^{-\lambda(t-s)} dW(s). \tag{18}$$

The expected value of $r(t)$ is

$$\mathrm{E}\big[r(t)\big] = r_0 e^{-\lambda t} + \mu\big(1 - e^{-\lambda t}\big) \tag{19}$$

and the variance

$$\mathrm{Var}\big[r(t)\big] = \frac{\sigma^2}{2\lambda}\big(1 - e^{-2\lambda t}\big). \tag{20}$$

2.5.2 Calibration

In order to derive an accurate short rate model, the dynamics of the Vasicek short rate model should be specified under the martingale measure $\mathbf{Q}$. Therefore, the parameters a, b and σ in Equation (16) have to be calibrated to current market data, meaning that they are calibrated to current bond prices implied by the market, see Björk, T. [4].

The implied prices from the market can be determined by using observed market rates. The price of a zero coupon bond, $P(t,T)$ is the price one has to pay at time t in order to receive 1 unit of currency at date T. Hence, for the bond market to be free of arbitrage, the price $P(t,t)$ is always equal to 1, see Björk, T. [4].

One can today ($t = 0$) observe market rates and from them it is possible to determine the the implied observed zero coupon price $P(0,T)$ by using the theory described in Section 2.4.2. These implied observed bond prices are denoted as $\{P^*(0,T); T \geq 0\}$.

The goal is to choose the parameters (a, b, σ) from Equation (16) such that the theoretical price curve fits the empirical curve, i.e.

$$\{P(0,T; a, b, \sigma); T \geq 0\} = \{P^*(0,T); T \geq 0\}.$$

This will give the calibrated parameters

$$(a^*, b^*, \sigma^*).$$

The Vasicek short rate model has the property that it has an affine term structure, which means that the theoretical price of a zero coupon bond can be written on the following form

$$P(t,T) = e^{A(t,T) - B(t,T) r(t)}, \tag{21}$$

where $r(t)$ is the short rate at time t and where $A(t,T)$ and $B(t,T)$ are given by

$$B(t,T) = \frac{1}{a}\{1 - e^{a(T-t)}\} \tag{22}$$

and

$$A(t,T) = \frac{\{B(t,T) - T + t\}(ab - \frac{1}{2}\sigma^2)}{a^2} - \frac{\sigma^2 B^2(t,T)}{4a} \tag{23}$$

respectively, for the Vasicek short rate model, see Björk, T. [4].

The implied observed market prices $P^*(0,T)$ should match as closely as possible the theoretical price obtained from the term structure of the Vasicek model in Equation (21). This gives a system of equations that is to be solved.

As the one-factor Vasicek model only has three free parameters, there is a small chance of obtaining a perfect fit of the observed prices and the theoretical prices.

2.6 Statistical analysis

2.6.1 Multiple regression

Multiple regression is a statistical tool that allows for the investigation of how numerous independent variables are related to a dependent variable. When having identified these independent variables it is thereby possible to excerpt the information that they provide in order to carry out predictions about future outcomes of the dependent variable. An example follows:

$$Y' = b_0 + b_1 X_1 + b_2 X_2. \tag{24}$$

Equation (24) is an example of a multiple regression model where Y' is the predicted value of the dependent variable Y and where X_1 and X_2 are the independent variables. b_0, b_1 and b_2 are estimated from historical values of Y, X_1 and X_2, see Higgins, J. [12]

2.6.2 Maximum likelihood

Maximum likelihood estimation (MLE) is a statistical method which is used for the estimation of parameters of statistical models given observed historical data.

Assume there is a data sample of n observations $x_1, ..., x_n$ from a random variable with probability density function $f(x_i|\theta)$, where θ is a vector of parameters. The goal is to find an estimator $\hat{\theta}$ which will assume the closest value to θ as possible. To do this, the likelihood function is formulated where all observations x_k for $k = 1, ..., n$ are treated as fixed values whilst all parameters θ are let to vary and due to independence, the following expression can be formulated

$$\mathcal{L}(\theta; \mathbf{x}) = f(x_1, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta). \tag{25}$$

Thereafter, the average log-likelihood function $\hat{l}$ is defined as the logarithm of $\mathcal{L}$:

$$\hat{l}(\theta; \mathbf{x}) = \frac{1}{n} \ln \mathcal{L}(\theta; x_1, \ldots, x_n) = \frac{1}{n} \sum_{i=1}^{n} \ln f(x_i|\theta). \tag{26}$$

By maximizing the average log-likelihood function $\hat{l}$ it is possible to find the estimates $\hat{\theta}$ that maximize $\hat{l}(\theta; x)$, see Rodriguez, G. [22].

2.6.3 Aikake Information Criterion

When dealing with model constructions it is important to keep in mind that there are no "true" models, only models that approximate the reality. Therefore, one is always eager to find the model that best approximates reality, see Mazerolle, M.J. [19]. The Aikake Information Criterion is a method for model selection and is known as the AIC. The AIC statistic is defined as

$$AIC = -2ln(\mathcal{L}) + 2k \tag{27}$$

where $\mathcal{L}$ is the maximum value of the likelihood function for the model that is tested and k is the number of parameters used in the model. The model that is preferred is the one with the smallest AIC value, see Brockwell, P.J. [5].

However, it has been shown that the AIC has a tendency to over fit the models. Therefore, the AICC statistic is often used instead and it translates to the AIC but with a correction, decreasing the probability of selecting models that are over fitted.

$$AICC = AIC + \frac{2k(k+1)}{n-k-1} \tag{28}$$

where n denotes the sample size, see Brockwell, P.J. [5].

2.7 Asset Liability Management

2.7.1 Liquidity risk management

The definition of liquidity is the unhindered flows among the agents of the financial system, with a particular focus on the flows among the central bank, commercial banks and market, see Jarrow, R.A. and Van Deventer, D.R. [14]. Liquidity is the ability to execute these flows. If a bank or a financial institution would not be able to execute these flows, it would be considered to be illiquid, see Nikolaou, K. [20]. Hence, for a commercial bank to be liquid it means that it has the ability to realise its liability cash flows at maturity. For this to be possible, it is vital that the bank has deposit volumes that are sufficient enough.

Liquidity risk management involves managing the risks that origin from non-maturity liabilities and their characteristics, see Kalkbrener, M. and Willing, J. [15], i.e. the fact that the future cash flows of the deposits are unknown which makes the bank unaware of the fund volume they need to have in readiness at any given time.

A framework for quantifying the liquidity risk, i.e. the liquidity needed to execute the future liability cash flows, of a financial institution has been developed by Kalkbrener, M. and Willing, J. [15]:

Assume that $V(t)$ is the **deposit volume process** of a bank. The target is to specify the *minimal volume*, $M(t)$, of the demand deposit over a given time period $[0, t]$, i.e.

$$M(t) := \min_{0 \leq s \leq t} V(s), \tag{29}$$

where $M(t)$ is computed by Monte Carlo simulation by generating several paths of $V(t)$. The p-quantile of $M(t)$ for $0 < p < 1$ is denoted $m(t, p)$. Therefore, $m(t, p)$ is with the probability p the exact amount which will be available for investment over the time interval $[0, t]$, see Kalkbrener, M. and Willing, J. [15].

This can be used in a liquidity risk context to derive an estimate of the bank's core volume, i.e. the volume that will remain in a bank over the time period $[0, t]$ with a certainty of $(1 - p)$. Furthermore, it is also a good way of visualizing the behaviour of a bank's core deposit volume over time.

2.7.2 Interest rate risk management

Interest rate risk reflects a financial institution's sensitivity to changes in interest rates, i.e. it is the risk that the value of the institution's assets and liabilities are negatively affected by an interest rate change, see Basel Committee on Banking Supervision [2]. For instance, interest rate changes expose a major risk to for instance bondholders as when the interest rates increase, the bond prices fall and vice versa. Therefore, the value of the bank's assets is directly correlated with the interest rates. Another factor that is important to recognize is how each individual bank customer reacts to interest rate changes. The customer's option to withdraw money has been shown to be deeper if interest rates are expected to rise than if they are expected to fall. Interest risk management involves managing the risks that origin from the changes in interest rates, see Bardenhewer, M. [3].

Bank's profit from the fact the market rates are, in most cases, higher than the deposit rate given to customers. In periods of low market rates the banks margin for profit decreases. This is the type of interest rate risk that a bank issuing demand deposits can face, see Jarrow, R.A. and Van Deventer, D.R. [14].

Interest rate risk is the risk for the net present value of the demand deposits to decrease due to changes in the market rates. For this purpose a methodology for valuing and pricing of the non-maturing demand deposits is needed. The liability of the demand deposits includes both interest payments to customers and withdrawals of money by the customers. These interest rate payments occur every time period, usually every month, and may vary in size depending on the current market rates.

2.8 Valuation of demand deposits

Jarrow, R.A. and Van Deventer, D.R. [14] propose a method for the arbitrage-free valuation of demand deposits for a financial institution over a time period T. The following relationship holds

$$V(0) = PV + PV_L, \tag{30}$$

where $V(0)$ is the current demand deposit volume of the bank, PV is the **net present value of the demand deposit** and PV_L is the **present value of the bank's total liability**.

They state that the **present value of the liability** can be formulated as

$$\begin{aligned} PV_L = {} & E^{\mathbf{Q}}\left[\sum_{t=0}^{T-2} \frac{V(t) - V(t+1)}{B(t+1)}\right] + E^{\mathbf{Q}}\left[\sum_{t=0}^{T-1} \frac{d(t) \cdot V(t)}{B(t+1)}\right] \\ & + E^{\mathbf{Q}}\left[\frac{V(T-1)}{B(T)}\right], \end{aligned} \tag{31}$$

where $V(t)$ is the deposit volume at time t and $d(t)$ is the deposit rate at time t and where $E^{\mathbf{Q}}[\,\cdot\,]$ denotes the expectation with respect to the martingale measure, $\mathbf{Q}$, and $B(t)$ is the numeraire. The first term is the difference in volume between two time steps and the second term are interest rate payments. The third term origins from the assumption that the remaining deposit volume is payed back to the depositors after time T.

The **net present value of the demand deposit** today can therefore, by following Equation (30), be written as

$$\begin{aligned} PV = V(0) & - E^{\mathbf{Q}}\left[\sum_{t=0}^{T-2} \frac{V(t) - V(t+1)}{B(t+1)}\right] - E^{\mathbf{Q}}\left[\sum_{t=0}^{T-1} \frac{d(t) \cdot V(t)}{B(t+1)}\right] \\ & - E^{\mathbf{Q}}\left[\frac{V(T-1)}{B(T)}\right], \end{aligned} \tag{32}$$

where the present value of the liability cash flows, stated in Equation (31), have been subtracted from the current volume $V(0)$. The study shows that Equation (32) can be re-written as

$$PV = E^{\mathbf{Q}}\left[\sum_{t=0}^{T-1} \frac{V(t)[r(t) - d(t)]}{B(t+1)}\right]. \tag{33}$$

Equation (33) is therefore the formula for valuing the demand deposits and Equation (31) is the expression for the value of the liability cash flows.

The main idea for banks to issue demand deposits is for them to have a positive net present value for the bank. The demand deposit can in fact be viewed as an exotic interest rate swap. Banks pay a floating rate $d(t)$ and receive a floating rate $r(t)$ on the principal of $V(t)$, where $r(t) > d(t)$, see Kalkbrener, M. and Willing, J. [15].

2.9 Duration

Duration is a normalized measure of an asset's sensitivity to a small instantaneous parallel shift in zero rates. It can be interpreted as a time weighted average of the present value of an asset's cash flows. It is often used for interest risk management purpose when comparing and hedging liability cash flows.

The formula for the duration D is

$$D = \sum_{k=1}^{n} t_k \frac{c_{t_k} e^{-z(t_k)t_k}}{\sum_{j=1}^{n} c_{t_j} e^{-z(t_j)t_j}}, \tag{34}$$

where c_{t_k} is the cash flow occurring at time t_k and $z(t_k)$ is the zero rate for maturity t_k, see Hammarlid, O. et al. [11].

2.10 Immunization

Immunization is used for hedging asset or liability cash flows against a parallel shift in the zero rates. It matches the duration and present value of a liability to the portfolio of for instance bonds with fixed cash flows and maturities. Immunization of a single liability requires only investing in two bonds with different durations, see Hammarlid, O. et al. [11]. However, the duration of the two bonds must satisfy the condition

$$D_j < D < D_k, \tag{35}$$

where D_j and D_k are the durations of bond j and bond k respectively and D is the duration of the liability. This condition guarantees that the solution of the immunization portfolio only consists of long position in the two bonds.

The solution for the immunization portfolio is given by the following system of equations

$$\begin{bmatrix} P_j & P_k \\ P_j D_j & P_k D_k \end{bmatrix} \begin{bmatrix} h_j \\ h_k \end{bmatrix} = \begin{bmatrix} P_L \\ P_L D_L \end{bmatrix}.$$

Here, P and D denote the present value and the duration respectively and the subscript L denotes the liability. The portfolio weights h_j and h_k are the solution for the immunization portfolio, see Hammarlid, O. et al [11].

2.11 Replicating portfolio

The replicating portfolio approach is used in order to find a portfolio of assets that matches the cash flows of a liability or of another asset. The cash flows should match in size and also time at which the cash flows occurs, see Devineau, L. and Chauvigny, M. [8].

In some cases the cash flows of a liability are not deterministic but instead stochastic and are therefore impossible to determine exactly beforehand. However, by generating many scenarios for random outcomes of the liability cash flows, one can find a solution that on average matches these scenarios best. By using the replicating portfolio framework one can translate a liability that produces stochastic cash flows into portfolio of fixed income assets, for example a bond portfolio. This is also a common way to hedge these kinds of liabilities, see Mausser, H. et al. [6]. The replicating portfolio weights can be found by solving the following optimization problem

$$\begin{aligned}
\underset{w_1,\ldots,w_n}{\operatorname{argmin}} \quad & \sum_{s=1}^{S}\sum_{t=1}^{T}\left[CF_L(s,t) - \sum_{k=1}^{n} w_k CF_k(s,t)\right]^2, \\
\text{s.t.} \quad & w_i \geq 0, \\
& \sum_{i=1}^{n} w_i \leq V(0),
\end{aligned} \tag{36}$$

where $CF(s,t)$ denotes the cash flow at time t for scenario s, the subscript L denotes the liability cash flow and k denotes the replicating assets. Here, $w_1, \ldots, w_n$ are the portfolio weights for each asset, see Devineau, L. and Chauvigny, M. [8]. The constraints guarantee that the portfolio only consists of long positions and $V(0)$ is the maximum amount available for investment.

3 Data

3.1 Data deposit volume

The data set for the monthly observations of the bank's deposit volumes is a time series that consists of 129 data points between January 2004 and September 2014 and is illustrated in Figure 2.

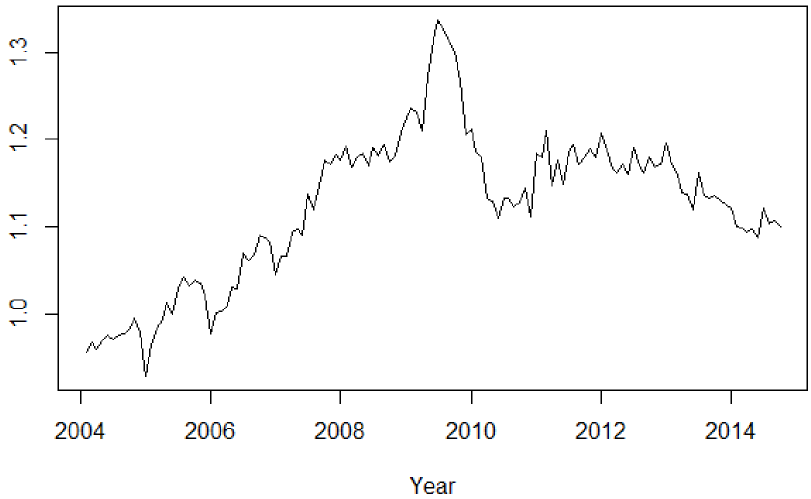


Figure 2: Historic volume of the bank.

Figure 2 illustrates the aggregated volume of all deposit accounts of the bank on the Swedish market. The data has been scaled for business reasons. What is observed from Figure 2 is that there is a substantial increase in deposit volume during the year 2009, which decreases within a year. This substantial increase is due to the fact that the management of the bank raised the rates for the fixed income deposits sharply as a reaction to the falling market rates, which in turn are due to the financial crisis, see Skandiabanken [1]. A description of the data set is provided in Table 1.

Table 1: Detailed time line of historic volume

2004	Data set starts at 2004-01-30.
2007	The volume has a heavy increasing trend during 2004-2007.
2008	Financial crisis, the market drops.
2009	Heavy peak in data set caused by internal strategy.
2011	The volume heavily decreases until present time.
2014	End of data set at 2014-09-30.

In this study, the aggregated volume of all different deposit accounts is seen as one demand deposit.

3.2 Data deposit rates

The bank offers different types of rates for different types of deposit accounts to its customers, the most important ones are listed below.

- **Savings account**
  The savings account was introduced during year 2008 and is an account without maturity that offers free withdrawals. The interest rate paid for the savings account depends on the amount deposited.

- **Deposit account**
  The deposit account allows for immediate withdrawals and offers the lowest level of interest rate.

- **Fixed interest account**
  The fixed interest account offers the highest interest rates which is dependent on the maturity dates. A fee is required in order to withdraw money before maturity and the interest rate level depends on the maturity and the amount of money that is deposited.

Similarly to the data of the deposit volumes provided by the bank, the data of the deposit rates consists of 129 monthly observations between January 2004 and September 2014. The interest rates represent the amount of interest paid to the specific deposit holder by the bank.

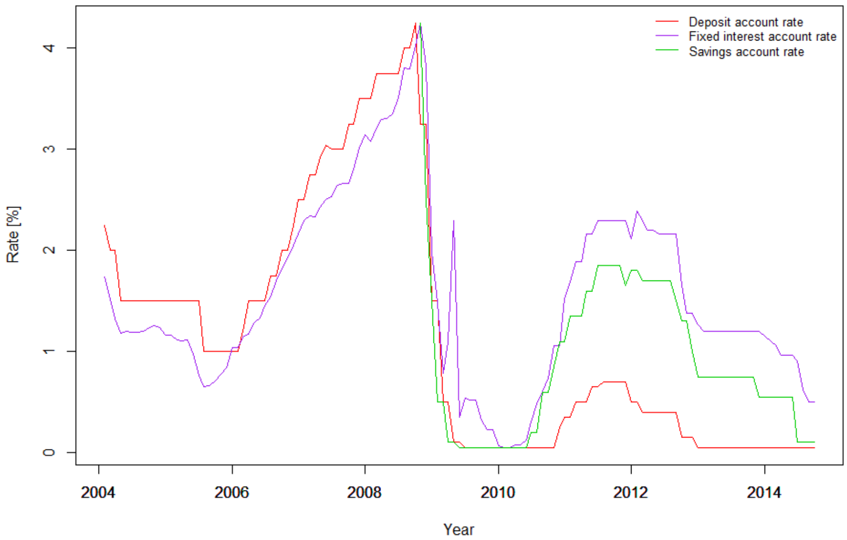


Figure 3: Historic deposit account rates.

What can be observed from Figure 3 is that all different kinds of deposit rates seem to be following a similar behaviour. As expected, the fixed rate deposit rate is always higher than the deposit and the savings deposit rates. Furthermore it is confirmed that the fixed interest rate has a clear spike at year 2009, confirming the decision of the bank's management to raise the fixed interest rate during that period, as previously mentioned.

Due to the fact that the rates have a similar behaviour, although at different levels, in this report it is assumed that all deposit accounts have the same rate, the demand deposit rate which is denoted as the deposit account rate in Figure 3.

3.3 Data market rates

The market rate securities that are used for this study are the STIBOR rates, explained in Section 2.4.2, and swap rates for interest rate swaps, explained in Section 2.4.3.

Table 2: Market rates included in this study.

STIBOR	**Interest rate swap**
1-month	1-year
3-month	2-year
6-month	3-year
	4-year
	5-year
	6-year
	7-year
	8-year
	9-year
	10-year

The rates that are included in this study are presented in Table 2.

The chosen date from which the market data is taken is 2014-09-30, as previously mentioned. These swap rates, R_n, and STIBOR rates, r^S, are presented in Table 3.

Table 3: Market rates from date 2014-09-30.

	$\mathbf{r^S}$	$\mathbf{R}_n$
1-month	0.00396	-
3-month	0.00473	-
6-month	0.00538	-
1-year	-	0.00453
2-year	-	0.00553
3-year	-	0.00712
4-year	-	0.00895
5-year	-	0.01750
6-year	-	0.01247
7-year	-	0.01405
8-year	-	0.01543
9-year	-	0.01662
10-year	-	0.01764

The data consists of 129 monthly observations for each security and is retrieved from the Swedish Central Bank and Nasdaq.

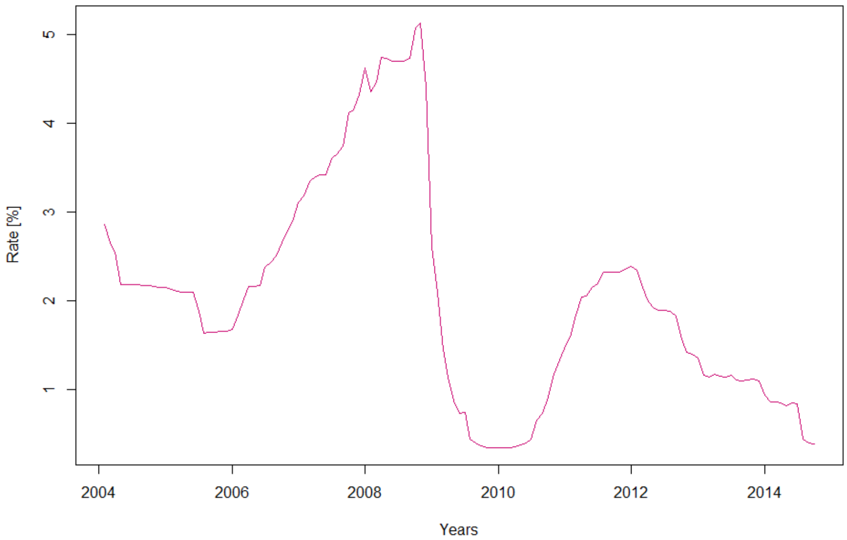


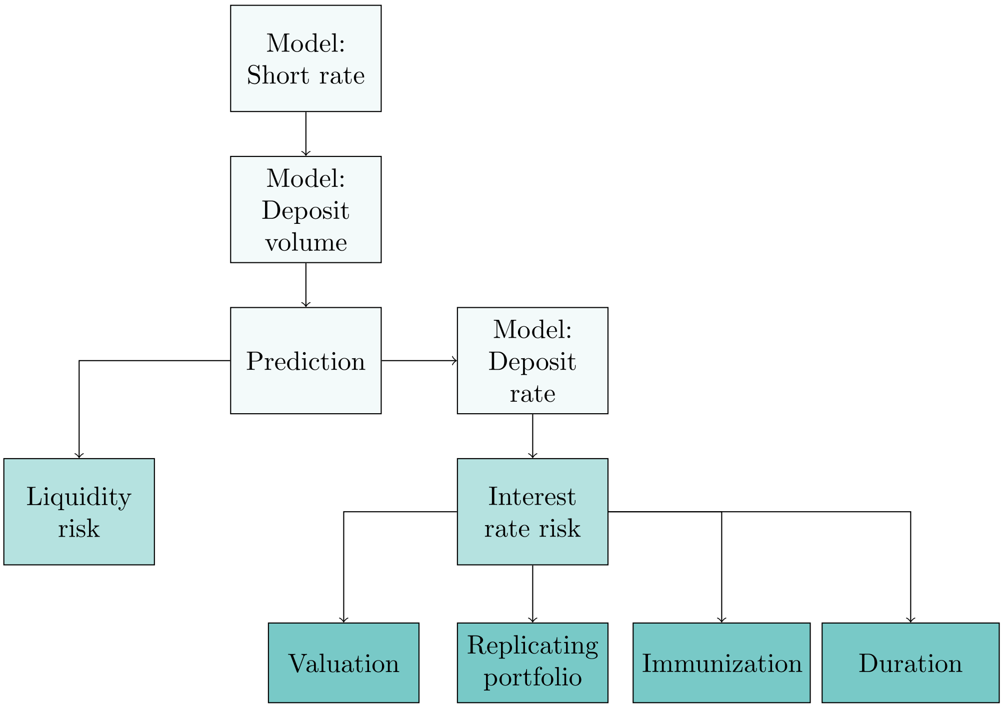
Figure 4: Historic STIBOR 1M.

Figure 4 presents the 129 monthly observations for the STIBOR 1M which used as a proxy for the short rate in this report.

4 Methodology

This section introduces the different methods used in order to answer the research questions of the study.

4.1 General approach



Flow chart 1: Flow chart of study approach.

The study is broken down into several sub-problems and the solution route for it is presented above in Flow Chart 1. There are three main sub-problems that have to be addressed before reaching the target of determining the liquidity and interest rate risk. The first sub-problem is the modelling of the short rate which will allow for simulations of future evolution of the short rate. The second sub-problem is to find a model that is accurate enough and that describes the evolution of the volume data of the bank. The third sub-problem is to determine a model that describes the bank's deposit rate well enough.

When the chosen methods for how to address these three main sub-problems have been described, the frameworks for determining the liquidity risk and the interest rate risk of the bank are presented respectively.

Note that the short rate model and the deposit volume model are utilized in order to determine the liquidity risk while the short rate model, the deposit volume model and the deposit rate model are utilized in order to determine the interest rate risk.

4.2 Short rate model

To be able to value the demand deposit and calculate interest rate risk associated with the non-maturing liability in an accurate way, one has to develop a stochastic model for the evolution of future interest rates, namely the short rate.

4.2.1 Estimating the yield curve

The yield curve is referred to as the zero rate curve for the specific date 2014-09-30. As the STIBOR rates are interpreted as zero rates, they are used directly as the first three data points in the yield curve. The rest of the zero rates $z(t_n)$ for maturity t_n are derived from the interest rate swaps by using Equation (12). Hence, the zero rates for the corresponding swap rates are determined by

$$z(t_n) = \left[\frac{R_n + 1}{1 - \sum_{i=1}^{n-1} \frac{R_n}{(1+z(t_i))^{t_i}}} \right]^{1/t_n} - 1. \tag{37}$$

This is done for all interest swap rates used in this study, see Table 2.

Three different cases are used in order to derive the interest rate risk results, one unstressed case $\boxed{1}$ and two stressed cases $\boxed{2}$ and $\boxed{3}$. For each of the stressed cases, a parallel shift in the zero rate curve is introduced where a value δ is added to all current zero rates, thus generating a new yield curves consisting of the data points

$$z_{\text{shift}}(t) = z(t) + \delta_i \quad \text{for} \quad i = 1, 2, 3. \tag{38}$$

The stress cases in this study are the following

Table 4: Stress cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$.

δ_1	δ_2	δ_3
0	0.01	-0.005

Note in Table 4 that $\delta_1 = 0$ is added to the unstressed case $\boxed{1}$, i.e. the original zero curve is utilized. The reason to why the stress case $\boxed{3}$ is chosen in this manner is because a larger downshift in the yield curve would produce very low market rates, given the current yield curve.

4.2.2 Price curve

As the zero yield curve has been determined it is now possible to continue and determine the implied observed markets price of a zero coupon bond $P^*(0, T)$ with maturity T, using Equation 6. However, as the prices are obtained by using the zero rates, all prices that are given from today to maturity date T are denoted $P^*(0, T)$.

The observed price curve is constructed for each of the yield cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ presented in Section 4.2.1 above.

4.2.3 Calibration

Calibrating the Vasicek model is done according to the calibration procedure described in Section 2.5.2. The observed market prices $P^*(0,T)$ for the current market data have been derived in previous Section 4.2.2 and are used for the calibration.

The parameters (a, b, σ) of the Vasicek model are now derived by matching the theoretical price of a zero coupon bond, given by Equation (21), to the observed market prices. This results in the following optimization problem

$$\underset{(a,b,\sigma)}{\text{argmin}} \quad \sum_{i=1}^{n} \left[P^*(0,t_i) - e^{A(0,t_i)-B(0,t_i)r(0)} \right]^2, \tag{39}$$

where $A(t,T)$ and $B(t,T)$ are given by Equations (23) and (22) respectively and it is in these equations that the parameters a, b and σ appear.

Furthermore, the short rate today, i.e. $r(0)$, is approximated through linear extrapolation using the zero rates, see below.

$$r(0) \approx z(t_1) - \frac{z(t_2) - z(t_1)}{t_2 - t_1} t_1. \tag{40}$$

By solving the optimization problem in Equation (39) one obtains the parameters (a^*, b^*, σ^*) for the Vasicek model that fits the current observed market data.

After calibrating the parameters of the Vasicek model, a Monte Carlo simulation of future outcomes of the short rate is carried out, with a number of simulations of $N = 10000$. This is done in order to use these results when simulating future deposit volume and deposit rate outcomes.

This calibration procedure is carried out for each of the cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ presented in Section 4.2.1.

4.3 Deposit volume model

It is assumed that the deposit volume follows a log-normal distribution as previously studied in Jarrow, R.A. and Van Deventer, D.R. [14]. Therefore, the deposit volume $V(t)$ is modelled as a model with a relationship between the risk factors and its log-variations through multiple regression analysis, as described in Sections 2.2 and 2.6.1.

The model of the deposit volume can in general terms be expressed as

$$\log V(t) = \gamma_0 + \gamma_1 X_1 + ... + \gamma_k X_k + \gamma_{k+1} \log V(t-1) + \sigma Z, \quad Z \sim N(0,1) \tag{41}$$

where $[X_1, ..., X_k]$ are the independent variables of the model, γ_i for $i \in [0, k+1]$ are the parameters that are to be estimated and $k+3$ is the total amount of parameters in the model. The goal is to determine which of the variables $[X_1, ..., X_k]$ that are going to be included in the final model for this study.

The historic deposit volume illustrated in Figure 2 is the dependent variable $V(t)$ and the risk factors are considered to be the deposit rate and the market interest rate.

Possible variables

The variables and the model alternatives that are tested are chosen based on the fact that they are either expected to have an affect on the dependent variable $V(t)$ or that they have been tested in similar cases from scientific papers as in Kalkbrenner, M. and Willing, J. [15] and Jarrow, R.A. and Van Deventer, D.R. [14].

The model will have the following appearance

$$\begin{aligned}\log V(t) = \gamma_0 + \gamma_1 r(t) + \gamma_2 d(t) + \gamma_3 (r(t) - r(t-1)) \\ + \gamma_4 (r(t) - d(t)) + \gamma_5 \log V(t) + \sigma Z, \quad Z \sim N(0,1)\end{aligned} \tag{42}$$

where the variables that are going to be analyzed are specified in Table 5 below.

Table 5: List of variables that are tested.

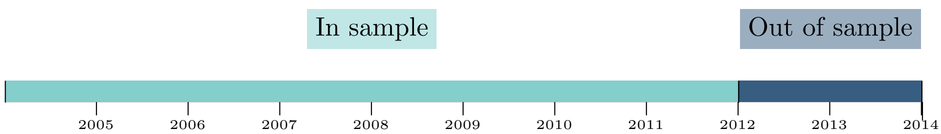
Variable notation	Type	Unit	Comment
γ_0	Constant	-	Time constant
r_t^S	Rate	-	Short rate
d_t	Rate	-	Deposit rate
$(r_t^S - r_{t-1}^S)$	Rate difference	-	(Short rate - lag short rate)
$(r_t^S - d_t)$	Rate difference	-	(Short rate - deposit rate)
$log(V_{t-1})$	Lagged volume	SEK	Lagged deposit volume

Note: STIBOR 1M, $r^S(t)$, is used as proxy for the short rate $r(t)$.

Model validation

As the aim is to develop a multiple regression model to the historical deposit data, it is beneficial to reserve a time period of this historical data for testing purposes and which allows to verify the model on data that has not been a component in the optimization procedure.

The *in sample* interval of the historical deposit volume data set, in which the model is estimated, stretches from 2004/01/31 - 2012/09/30. Hence, the *out of sample* interval is consequently 2012/09/30 - 2014/09/30, see the time line illustrated below.



Since the data set consists of the bank's deposit volume on a monthly basis, the in sample data includes 105 data points while the interval for the out of sample only includes 24 data points.

Model selection
The parameter estimation analysis presented above is carried out for all model alternatives that are presented in Table 6.

Table 6: Different deposit models that are analyzed.

Variable:	Constant	$r^S(t)$	$d(t)$	$r^S(t) - r^S(t-1)$	$r^S(t) - d(t)$	$\log V(t)$
Parameter:	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
Model 1	x	x	x	x	x	x
Model 2	x	x	x	x		x
Model 3	x	x	x		x	x
Model 4	x	x			x	x
Model 5	x	x		x		x
Model 6	x		x	x	x	x
Model 7	x		x		x	x
Model 8	x		x	x		x
Model 9	x	x	x			x
Model 10	x		x			x
Model 11	x	x				x

As it is assumed that the deposit volume $V(t)$ has a log-normal distribution, the parameters can be estimated using the maximum likelihood method described in Section 2.6.2 where f represents the density of the normal distribution.

The likelihood function is determined by

$$\mathcal{L}(\boldsymbol{\gamma}; \log V_1, \log V_2, ..., \log V_{105}) = f(\log V_1, \log V_2, ..., \log V_{105}|\boldsymbol{\gamma}) = \prod_{i=1}^{105} f(\log V_i|\boldsymbol{\gamma}) \tag{43}$$

and the log-likelihood function is determined by

$$\ln \mathcal{L}(\boldsymbol{\gamma}; \log V_1, \log V_2, ..., \log V_{105}) = \sum_{i=1}^{105} \ln f(\log V_i|\boldsymbol{\gamma}) \tag{44}$$

where the average log-likelihood is

$$\widehat{l}(\boldsymbol{\gamma}) = \frac{1}{105} \ln \mathcal{L} \tag{45}$$

The parameters $\boldsymbol{\gamma}$ are estimated such that they maximize the log-likelihood function in Equation (45).

The model that is of highest quality is the one that will provide the lowest AICC value, see Section 2.6.3, and is therefore the model that is going to be used in this study. Furthermore, the significance of the parameters is analyzed.

4.4 Deposit rates

The definition of the deposit rate is the interest rate paid to the depositors in return for them depositing money in the bank. The model for the deposit rate $d(t)$ is derived using the same procedure as when modeling the deposit volume in Section 4.3, however, the following assumptions have been made:

- **Deterministic function of short rate**
  The deposit rate is assumed to be a deterministic linear function of the short rate and will thereby not include an additional stochastic term.

- **All accounts pay same deposit rate**
  For simplicity it is assumed that all different deposit accounts are merged to one single deposit, as earlier mentioned, and pay the same deposit rate $d(t)$.

- **Not assume negative values** $d(t) \geq 0$
  As it is unlikely that the bank will issue negative interest to their depositors, the deposit rate is not allowed to assume negative values.

Similar to the deposit volume model derived in Section 4.3, the deposit rate model is a multiple regression model where the deposit rate is expressed as

$$d(t) = \big(\beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k\big)\mathbb{1}\{d(t) \geq 0\}, \tag{46}$$

where X_i for $i \in [1, k]$ are the independent variables of the model, β_i for $i \in [0, k]$ are the parameters that are to be estimated and $k + 1$ is the total amount of parameters in the model.

Possible variables
The variables that are tested are variables that are either expected to have an affect on our dependent variable $d(t)$ or variables that have been tested in similar cases from scientific papers, such as in Jarrow, R.A. and Van Deventer, D.R. [14].

The model will have the following appearance

$$\begin{aligned} d(t) = (&\beta_0 + \beta_1 r(t) + \beta_2 d(t-1) \\ &+ \beta_3(r(t) - d(t-1)) + \beta_4(r(t) - r(t-1)) \\ &+ \beta_5(r(t-1) - d(t-1)))\mathbb{1}\{d(t) \geq 0\}, \end{aligned} \tag{47}$$

where the variables that are going to be analyzed are specified in Table 7 below.

Table 7: List of independent variables that are tested for model of deposit rate

Variable notation	Type	Unit	Comment
β_0	Constant	-	-
r_t^S	Rate	-	Short rate
d_{t-1}	Rate	-	Lag deposit rate
$(r_t^S - d_{t-1})$	Rate difference	-	(Short rate - lag deposit rate)
$(r_t^S - r_{t-1}^S)$	Rate difference	-	(Short rate - lag short rate)
$(r_{t-1}^S - d_{t-1})$	Rate difference	-	(Lag short rate - lag deposit rate)

Note: STIBOR 1M, $r^S(t)$, is used as proxy for the short rate $r(t)$.

Model selection
The parameter estimations analysis presented above is carried out for all model alternatives that are presented in Table 8.

Table 8: Different deposit rate models that are analyzed.

Variable:	Constant	$r^S(t)$	$d(t-1)$	$r^S(t) - d(t-1)$	$r^S(t) - r^S(t-1)$	$r^S(t-1) - d(t-1)$
Parameter:	β_0	β_1	β_2	β_3	β_4	β_5
Model 1	x	x	x	x	x	x
Model 2	x	x	x	x	x	
Model 3	x	x	x	x		
Model 4	x	x	x			
Model 5	x	x				
Model 6	x	x		x		
Model 7	x	x			x	
Model 8	x	x				x

The same model selection and validation procedure as in Section 4.3 is used.

4.5 Risk Management

4.5.1 Liquidity risk management

The method chosen to model the liquidity risk of the bank is the same method used in Kalkbrener, M. and Willing, J. [15]. A Monte Carlo simulation, with a number of simulations of $N = 10000$, of future volume levels is carried out, where $V^i(t)$ stands for the total volume of the deposits at time t for simulation i for $i = 1, \dots, N$. $V^i(t)$ includes both the decreases in volume due to withdrawals as well as increases in volume due to new deposits within the period $[0, t]$. However, in order to determine the liquidity risk of the bank, only the decreases in volume are interesting to consider. Therefore, it is only of interest to examine the minimal volume which is available for investment over the period $[0, t]$, see Section 2.7.1. This is done by looking at

$$M^{(i)}(t) := \min_{0 \le s \le t} V^{(i)}(s), \quad i = 1, 2, ..., N \tag{48}$$

where $M(t)$ is

$$M(t) = \left[M^{(1)}(t), \ldots, M^{(N)}(t)\right]. \tag{49}$$

This means that $M(t)$ is a vector containing the sample of minimal volumes in $[0, t]$.

Now, the obtained sample $M(t)$ is ordered such that $M^{(1)}(1) \leq ... \leq M^{(N)}(t)$ so that a liquidity quantile can be derived.

The liquidity quantile $m^p(t)$ is expressed as

$$m^p(t) := M_{[Np]}(t), \tag{50}$$

where $m_{[Np]}(t)$ is the $[Np]$:th element in the ordered vector $M(t)$. This denotes the empirical $(1-p)$-quantile of $M(t)$. $m^p(t)$ is the amount that is available for investment over the period $[0, t]$ with probability $(1-p)$. It will be referred to as the liquidity quantile. The reader might notice that the expression of the liquidity quantile $m^p(t)$ does not include an index i. This is due to the fact that $m^p(t)$ is derived using Monte Carlo simulation of the volume process $V(t)$, i.e. the liquidity quantile does not depend on the simulation i and can instead be interpreted as a worst case outcome of the volume process $V(t)$.

The liquidity quantile is carried out for the 95:th and the 99:th percentile respectively, i.e. for $p = 0.05$ and $p = 0.01$.

From Equation (48) one can see that $m^p(t)$ is non-increasing. Hence, it will only consider the decreasing effects that the risk factors may generate.

The objective of deriving a liquidity quantile is to, from a risk management perspective, estimate how much money that could be withdrawn in a worst case scenario over a given time period. To illustrate this, the liquidity quantile $m^p(t)$ can then be plotted as a continuation of the current volume. Another way of illustrating the result is to use $m^p(t)$ in order to derive the withdrawals that occur over different time periods. Since $m^p(t)$ is non-increasing, these cash flows are the difference in the liquidity quantile $m^p(t)$ between two given times. These cash flows are maximum amounts that will be withdrawn over the given time period and are referred to as *liquidity buckets*, i.e. the maximum amount that is expected to be withdrawn over a time period is presented as a bucket.

4.5.2 Interest rate risk management

Valuation of demand deposit
In order to be able to derive interest risk measurements the expected value of net present value of the demand deposit must be computed, following the theory presented in Section 2.8. The valuations are done using Monte Carlo simulation, as described in the previous section, where the number of simulations is $N = 10000$ in order to get a reliable estimate. The valuation formula expressed in Equation (33) can be derived using Monte Carlo simulation, for a time period T, as

$$PV = E^{\mathbf{Q}}\left[\sum_{t=0}^{T-1} \frac{V(t)[r(t) - d(t)]}{B(t+1)}\right] \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=0}^{T-1} \frac{V^i(t)[r^i(t) - d^i(t)]}{B^i(t+1)}\right], \quad (51)$$

where $r^i(t)$ is the Vasicek short rate time t for simulation i, $d^i(t)$ is the corresponding deposit rate at time t for simulation i, $V^i(t)$ is the corresponding deposit volume at time t for simulation i and $B^i(t+1)$ is the money account process presented in Equation (15) for simulation i, where $i = 1, \ldots, N$. The deposit volume process, the Vasicek short rate process and the deposit rate process are developed in Section 4.3, Section 4.2 and Section 4.4 respectively.

From Equation (32) it follows that the present value of the liability cash flows PV_L can expressed as

$$PV_L = V(0) - PV. \quad (52)$$

The time horizons for which this is computed is set to five and ten years respectively, where the time step for these simulations is one month, i.e. $\frac{1}{12}$ years.

Furthermore, the Monte Carlo value that is determined by Equation (51) is also computed when using the liquidity quantile $m^p(t)$ as the volume process in order to get a more conservative measurement of the net present value of the demand deposit. Hence, the formula for the conservative case is expressed as

$$PV^m = E^{\mathbf{Q}}\left[\sum_{t=0}^{T-1} \frac{m^p(t)[r(t) - d(t)]}{B(t+1)}\right] \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{t=0}^{T-1} \frac{m^p(t)[r^i(t) - d^i(t)]}{B^i(t+1)}\right] \quad (53)$$

where the framework for the liquidity quantile $m^p(t)$ is developed in Section 4.5.1. The present value of the liability cash flows when using the liquidity quantile $m^p(t)$ can be stated as

$$PV_L^m = V(0) - PV^m. \quad (54)$$

In order to analyze the interest rate risk one has to compare how the net present value of the demand deposits reacts to a change in the market. For this purpose, the parallel shifts in the zero rates is used.

The short rate model is now re-calibrated to the new market rates for each stress case, which implies calibrating new parameters for the Vasicek model for each stress case $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ respectively. These new versions of the Vasicek model are then used to derive new values of the net present values expressed in Equations (51) - (54). This will therefore allow one to observe in what way a shift in the market rates affects the net present value by comparison to the original value.

Duration

The duration of the liability cash flows is determined by using Equation (34). As the liability cash flows are stochastic and not deterministic, the expected value of the duration is estimated using the Monte Carlo simulation technique mentioned above, with the same number of simulations $N = 10000$. Rewriting the liability cash flows stated in Equation (31) the cash flows for scenario i at time t can now be written as

$$CF^i(t) = \begin{cases} V^i(t-1) - V^i(t) + d^i(t-1) \cdot V^i(t-1) & \text{if } t = 1, 2, ..., T-1 \\ V^i(T-1) + d^i(T-1) \cdot V^i(T-1) & \text{if } t = T \end{cases}$$

By replacing the volume process $V(t)$ by $m^p(t)$ it is possible to state the cash flows as

$$CF^i(t) = \begin{cases} m^p(t-1) - m^p(t) + d^i(t-1) \cdot m^p(t-1) & \text{if } t = 1, 2, ..., T-1 \\ m^p(T-1) + d^i(T-1) \cdot m^p(T-1) & \text{if } t = T \end{cases}$$

And the present value of these liability cash flows is

$$PVCF^i(t) = \frac{CF^i(t)}{B^i(t)}. \tag{55}$$

The duration of the liability cash flows is defined as

$$D_L = \frac{1}{N} \sum_{i=1}^{N} \left[\sum_{t=1}^{T} \frac{t}{12} \frac{PVCF^i(t)}{\sum_{s=1}^{T} PVCF^i(s)} \right], \tag{56}$$

where i is the simulation and N is the total number of simulations. The duration measurement is done for both the non-stressed case as well for the two stressed cases and for a five respectively ten year time period.

Furthermore, the duration measurement is derived for when the volume process $V(t)$ is replaced with the liquidity quantile $m^p(t)$, thus allowing the cash flows to be positive for all t. In this way, a more conservative duration measure is derived.

Immunization

The securities for which the immunization portfolio can consist of are presented in Table 2. As previously described in Section 2.10, in order to determine the portfolio weights (h_1, h_2) that will immunize the present value of the liability PV_L, derived above, one has solve the equation system presented in Section 2.10

$$\begin{bmatrix} P_1 & P_2 \\ P_1 D_1 & P_2 D_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} PV_L \\ PV_L D_L \end{bmatrix},$$

where P_1, P_2, D_1 and D_2 are the present values and the durations for the two assets while PV_L and D_L are the present value and the duration of the liability. The duration of the liability D_L have been derived in the previous section.

The immunization methodology is applied for when the present value of the liability PV_L is derived using the liquidity quantile $m^p(t)$ as the volume process.

By deriving an immunization portfolio it can be examined how well the derived portfolio matches the present value of the liability when the market rates are shifted. The present value of the immunization portfolio is then re-calculated using the shifted market rates and can then be compared to the net present value of the liability PV_L, derived using the shifted market rates.

Replicating portfolio

In this report the replicating portfolio framework described in Section 2.11 is used to find a portfolio of market securities that replicate the present value of the liability cash flows. Since the liability cash flows from the demand deposit occur on a monthly basis while the cash flows from the market securities occur on different maturity dates, this report proposes an approach where the present value of the liability cash flows $PVCF^i(t)$, see Equation (55), are summed into different time buckets TB^i_k, which are described further down.

The replicating portfolio is derived using only the liquidity quantile $m^p(t)$ as the volume process in order to meet the liquidity constraint. The present value of the liability cash flows are summed into different time buckets TB^i_k matching the maturities t_k of the assets used

$$\begin{aligned} TB^i_1 &= PVCF^i(1), \\ TB^i_2 &= PVCF^i(t_1+1) + \cdots + PVCF^i(t_2), \\ &\vdots \\ TB^i_k &= PVCF^i(t_{k-1}+1) + \cdots + PVCF^i(t_k), \end{aligned}$$

where t_k are the maturities, in months, of the corresponding assets. The optimization problem stated in Equation (36) is now translated to

$$\begin{aligned} \underset{w_1,\ldots,w_n}{\text{argmin}} \quad & \sum_{i=1}^{N}\sum_{k=1}^{n}\left[TB^i_k - \sum_{j=1}^{n} w_j \frac{CF^i_j(t_k)}{B^i(t_k)}\right]^2, \\ \text{s.t.} \quad & w_j \geq 0, \\ & \sum_{i=j}^{n} w_j \leq V(0), \end{aligned} \tag{57}$$

where $CF^i_j(t_k)$ is the cash flow of the replicating asset j at time t_k in simulation i and n is the number of replicating assets used. $V(0)$ is the deposit volume at time 0, i.e. the amount available for investment at time 0. N is the number of simulations, in this case $N = 10000$ as in previous sections. The replicating portfolio is constructed for both a five and a ten year time period.

5 Results

In this section the results that have been obtained in this study are presented. The methodology procedure for how these results are derived is described in Section 4.

5.1 Short rate

By incorporating the methodology presented in Section 4.2 a short rate model is obtained.

By then utilizing Equation (37) the zero rates are derived from the swap interest rates that are given in Table 3. By combining the derived zero rates with the STIBOR rates, from the same date, the yield curve is derived as described in Section 4.2.1.

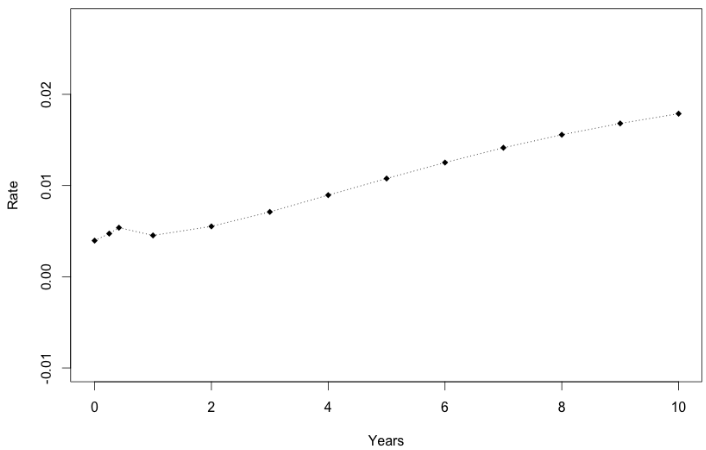


Figure 5: Yield curve for case $\boxed{1}$.

Figure 5 illustrates the non-stressed yield curve, i.e. case $\boxed{1}$ where $\delta_1 = 0$. The three first data points represent the 1M STIBOR, 3M STIBOR and 6M STIBOR market rates respectively. The rest of the data points are the zero rates derived from the swap rates.

By following the methodology given in Section 4.2, by using the Equation (38) the yield curves are derived for the stressed cases $\boxed{2}$ and $\boxed{3}$ and they are illustrated in Figure 6 below. The cases $\boxed{2}$ and $\boxed{3}$ can be denoted as the "stressed cases" while case $\boxed{1}$ is denoted as the unstressed case.

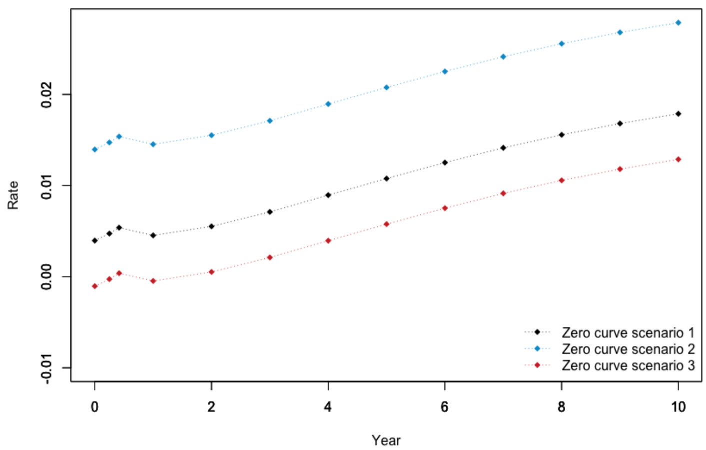


Figure 6: Yield curves for the cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$.

In Figure 6 it is clear that case $\boxed{2}$ can be interpreted as a market increase by $+0.01$ in relation to case $\boxed{1}$ while case $\boxed{3}$ then represents a market drop by -0.005. This is good to keep in mind when deriving and analyzing the results.

The zero rates are, as described in Section 4.2.2, used to determine the implied observed market prices of the zero coupon bonds $P^*(0, T)$ for the maturities

$$T = \left[\tfrac{1}{12}, \tfrac{3}{12}, \tfrac{6}{12}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\right],$$

where T denotes maturity year. The observed market prices are calculated using Equation (11) and the derived yield curves presented in Figure 6 and the result for case $\boxed{1}$ is illustrated in Figure 7 below.

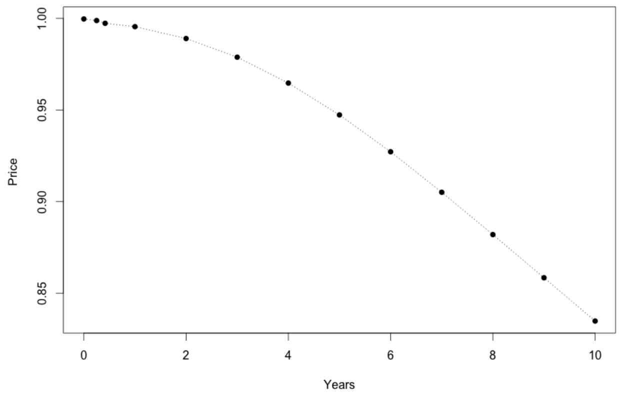


Figure 7: Price curve $P^*(0,T)$ for a zero coupon bond according to case $\boxed{1}$.

The curve in Figure 7 is then used to calibrate the Vasicek model, meaning that the Vasicek model is calibrated with respect to the implied observed market prices for the different zero rate cases by using the optimization formula which is stated in Equation (39).

The result for case $\boxed{1}$ is illustrated in Figure 8 which illustrates the implied observed market prices and the theoretical prices given by the Vasicek model.

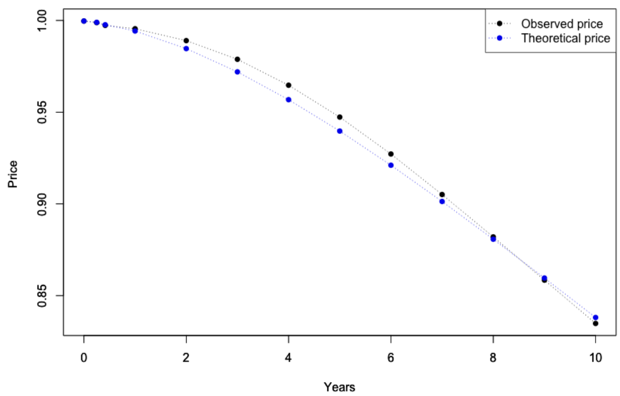


Figure 8: Implied observed market prices $P^*(0,T)$ and the theoretical prices $P(0,T)$.

It can be observed from Figure 8 that the theoretical prices do not completely match with the implied observed prices for the maturity years two, three, four, five, six, seven and ten.

Table 9: Results of calibrated parameters for the three cases.

Stress case	a^*	b^*	σ^*
1	0.1454	0.0046	0.0131
2	0.1268	0.0055	0.0115
3	0.1579	0.0040	0.0128

The results of the calibrated Vasicek parameters for the different cases are presented in Table 9 above. Thus, these are the parameters that are produced under the martingale measure $\mathbf{Q}$ and that will be used in the short rate model.

These parameters, that are stated in Table 9, are therefore used to carry out a Monte Carlo simulation for $N = 10000$ number future possible outcomes of the short rate for a ten year period ahead for all three cases. The result for 20 simulations of the unstressed case 1 is illustrated in Figure 9 below.

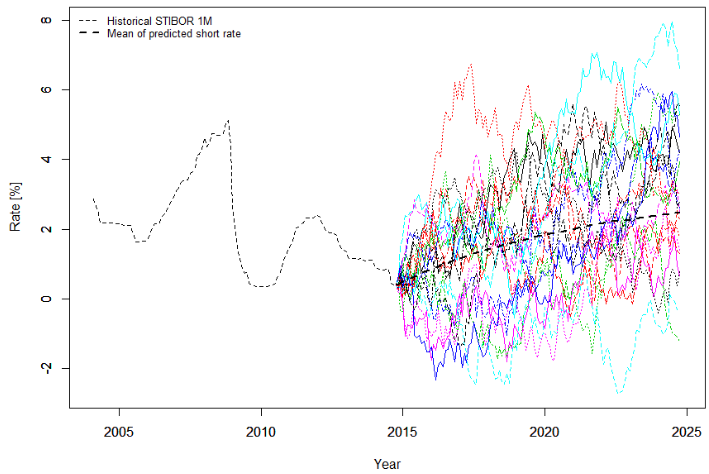


Figure 9: 20 predicted outcomes of the Vasicek short rate for case 1 over a ten year period.

An observation of Figure 9 is that the mean value, see Equation (19), of the simulated short rate is expected to increase within the coming ten years.

5.2 Deposit volume

By following the model selection methodology presented in Section 4.3, the following results are obtained.

The result of the model selection is presented in Table 10 where the AICC values are obtained for all 11 of the different model alternatives that are presented in Table 6.

Table 10: AICC results of the different model alternatives.

Model 1-6	1	2	3	4	5	6
AICC	536.1	536.0	534.7	532.5	532.6	536.0
Model 7-11	7	8	9	10	11	-
AICC	532.5	531.7	532.5	529.3	529.1	-

In Table 10 it is observed that the model which produces the smallest AICC value is Model 11. This means that the model which describes the deposit volume data most accurately is formulated as

$$\log V(t) = \gamma_0 + \gamma_1 r(t) + \gamma_5 \log V(t-1) + \sigma Z(t), \quad Z(t) \sim N(0,1) \tag{58}$$

meaning that the model can be expressed as

$$V(t) = V(t-1)^{\gamma_5} \exp\left\{\gamma_0 + \gamma_1 r(t) + \sigma Z(t),\right\} \tag{59}$$

where $Z(t) \sim N(0,1)$.

The parameters in this model are estimated based on the historical in sample data, as described in Section 4.3, by using the maximum likelihood method. The results are presented in Table 11 below.

Table 11: Results of MLE-estimated parameters for the deposit volume model.

Parameter	Parameter value	p-value
γ_0^*	0.9610	0.0492
γ_1^*	0.1932	0.1186
γ_5^*	0.9599	<2.00e-16
σ^*	0.0201	-

An observation from Table 11 is that the p-value for parameter γ_1^* indicates that the short rate variable is not significant at a 90% level, however it is important to note that it is not remarkably high either, but suggesting that it is plausible that the model can instead be expressed as

$$V(t) = V(t-1)^{\gamma_5} \exp\left\{\gamma_0 + \sigma Z(t)\right\} \quad (60)$$

which will henceforth be referred to as **Model 12**.

The AICC value and the parameters for Model 12 are estimated in the same manner as described above and the results are presented in Tables 12 and 13 respectively.

Table 12: The AICC result of Model 12.

	Model 12
AICC	528.4

Table 13: Results of MLE-estimated parameters for Model 12.

Parameter	Parameter value	p-value
γ_0^*	1.1298	0.0343
γ_5^*	0.9531	<2.00e-16
σ^*	0.0193	-

What can be seen from Table 13 is that all parameters are significant at a 95% level.

What can be observed is that the AICC value for Model 12 is not notably lower than the AICC value of Model 11. Also, since the p-value of the short rate parameter in Model 11 is not remarkably high, it is worth to compare the performance of these two models in order to observe which one that produces the most reasonable results.

An in sample analysis is done for both models where a 105-step prediction is carried out and compared to the in sample deposit volume data in order to confirm that the expected value of the predictions behaves similarly to the historical deposit data set. The start value for these predictions is the first value of the original data set an the results are illustrated in Figures 10 and 11 below.

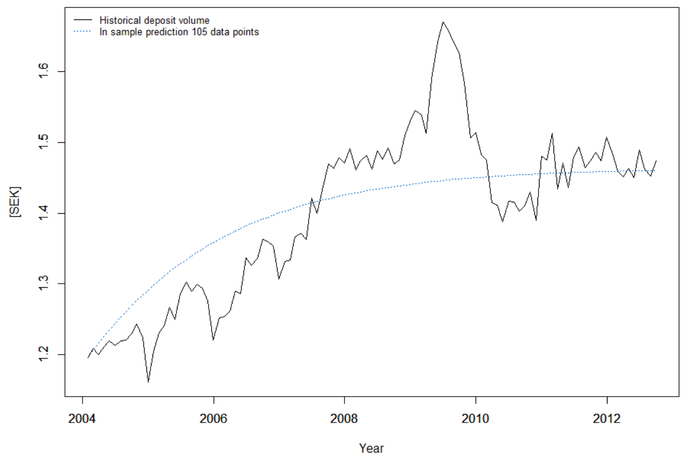


Figure 10: In sample prediction of deposit volume for Model 12.

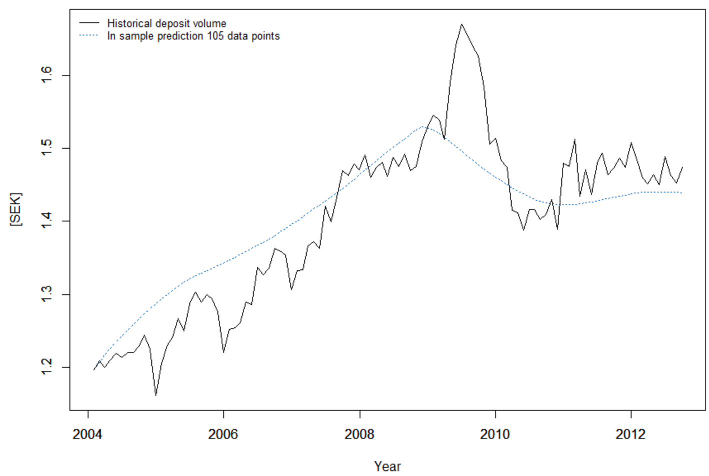


Figure 11: In sample prediction of deposit volume for Model 11.

From Figure 10 it can be observed that Model 12 does not fully capture the behaviour of the historical data set, while one can see in Figure 11 that Model 11 manages to capture the behaviour fairly good. However, it can clearly be observed that none of the models capture the appearance of the original data set during the years 2009-2010 due to the heavy peak in the data set.

Thereafter, the models are used to predict two years, i.e. 24 data points, in order to compare the expected value with the out of sample data set. The starting value for these predictions is the value of the original data set at date 2012-09-30 and the predictions are until date 2014-09-30. The results of the out of sample prediction of the two models are illustrated in Figures 12 and 13 below.

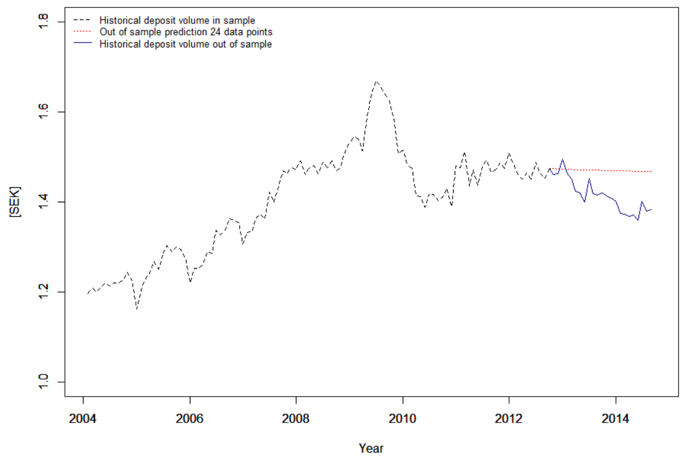


Figure 12: Out of sample prediction of deposit volume compared to actual volume using Model 12.

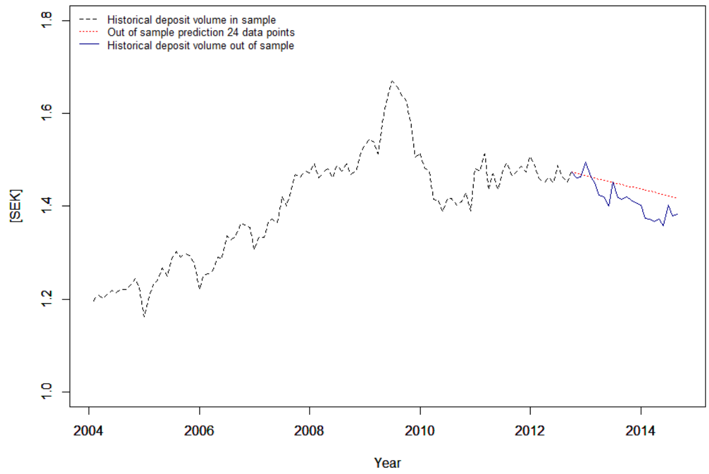


Figure 13: Out of sample prediction of deposit volume compared to actual volume using Model 11.

From Figure 12 one can clearly see that the prediction of Model 12 does not follow the original data set. While the original data set declines in volume after year 2012, the prediction of Model 12 stay almost on a constant level over this two year period. However, it is observed in Figure 13 that Model 11 does a better job of predicting the out of sample data, compared to Model 12.

Based on these results, the model that is chosen for this study is Model 11. Even though the statistical analysis suggests that Model 12 is a better model, it has not remarkably lower AICC value compared to Model 11 and the p-value for the short rate coefficient in Model 11 is not extremely high. It has also been shown that Model 11 captures the behaviour of the historical data set and that it also produces good results when predicting the out of sample data.

Apart from this, another reason to use Model 11 is due to the assumption that the model should have a dependence on the current market rate. This is reasonable from an economical perspective since the behaviour of the deposit volume should have some relationship to the current market situation. This dependence has been included in similar models that are analyzed in previous studies, such as in Kalkbrener, M. and Willing, J. [15].

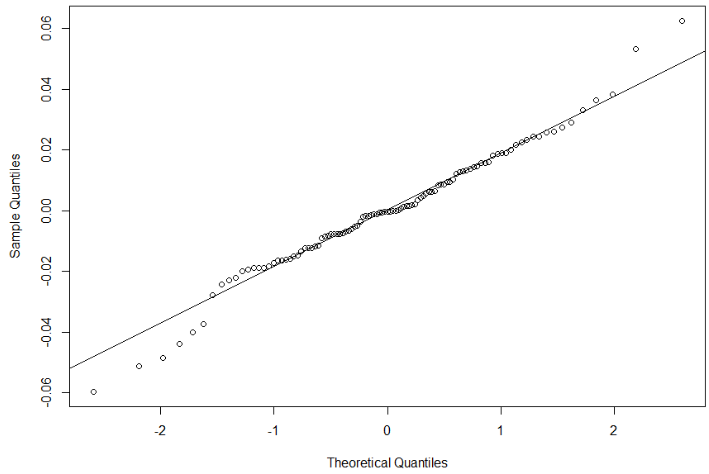


Figure 14: QQ-plot of the model's residuals.

The residuals of the estimation of Model 11 are plotted in a QQ-plot against the normal distribution, see Figure 11. It can be seen that the residuals overall follow a normal distribution but that there exist some outlier data points at the tails. They probably origin from the the heavy peak in the original data set.

By carrying out a 24-step out of sample prediction for Model 11, but this time incorporating the noise term in the simulations, one can obtain the confidence level of the prediction. The result is illustrated in Figure 15 where the confidence levels of 95% and 99% are presented, see Figure 15 below.

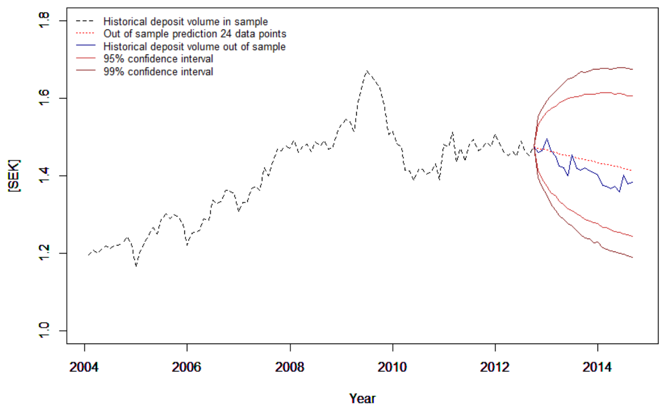


Figure 15: Out of sample prediction of deposit volume with 95% and 99 % confidence level.

The next step is to use the result of the short rate Monte Carlo simulation obtained from Section 5.1, i.e. to use the results of the prediction of the short rate $r(t)$ in order to predict the future possible outcomes of the deposit volume ten years ahead. 20 results for case $\boxed{1}$ are illustrated in Figure 16 below.

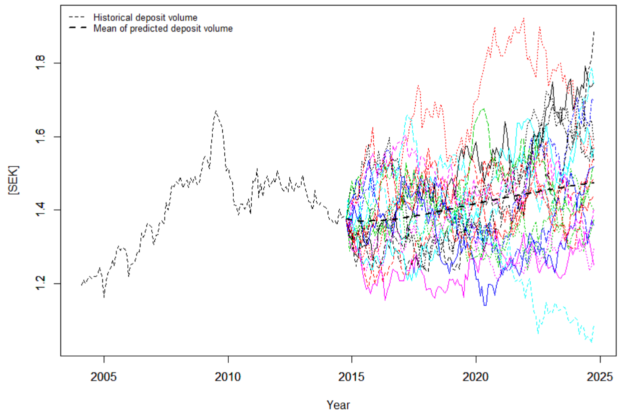


Figure 16: 20 predicted outcomes of the deposit volume model for case $\boxed{1}$ over a ten year period.

It can be observed that for a ten year period, the mean value of the deposit volume is expected to increase, based on the information provided by the market for stress case $\boxed{1}$.

5.3 Deposit rate

By following the model selection methodology presented in Section 4.4, the following results are obtained.

The result of the model selection is presented in Table 14 where the AICC are obtained for all eight of the different model alternatives that are presented in Table 8.

Table 14: AICC analysis result.

Model	1	2	3	4	5	6	7	8
AICc	1058.1	1055.8	984.6	982.5	863.1	982.5	865.3	1042.2

In Table 14 it is observed that the model which produces the smallest AICC value is Model 5. This means that the deposit rate model which describes the deposit rate data most accurately is formulated as

$$\begin{cases} d(t) = \beta_0 + \beta_1 r(t) & r(t) \geq 0, \\ d(t) = 0 & r(t) < 0, \end{cases} \tag{61}$$

This means that the deposit rate is best modeled by a linear model where a constant β_0 and a dependence on the short rate $r(t)$ are incorporated. Furthermore, the model is constructed in such way so that the deposit rate never assumes negative values, i.e. negative rates will never be issued to the depositors.

The parameters to this model are estimated based on the historical in sample data, similarly to the previous section, by using the maximum likelihood method. The results are presented in Table 15 below.

Table 15: The MLE-estimated parameters for the deposit rate model.

Parameter	Parameter value	p-value
β_0^*	-0.0068	8.18e-13
β_1^*	0.9190	<2.00e-16

One can in Table 15 see that both parameters are significant at a 99% level.

As in the previous section, in order to validate this choice of model it is of interest to use Equation (61) in order to carry out an in sample analysis. A 105-step prediction is carried out against the in sample deposit rate data in order to confirm that the the prediction behaves similarly to the historical deposit rate data set. The starting value for this prediction is the first value of the original data set and the result is illustrated in Figure 17 below.

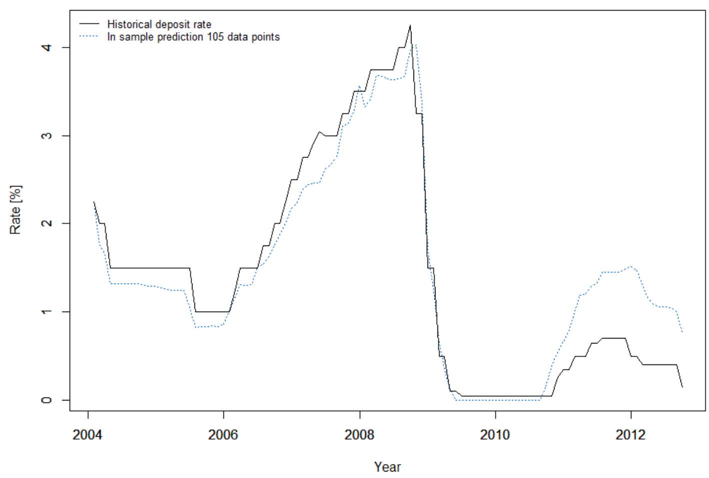


Figure 17: In sample prediction of deposit rate compared to actual volume.

An observation in Figure 17 is that the rate follows the market rate closely until the year 2011. Thereafter, the predicted deposit rate takes on higher values although it keeps the same appearance.

Thereafter, the chosen model is used to predict two years, i.e. 24 data points, in order to compare the prediction with the out of sample data set. The starting value for this prediction is as for the deposit volume, the value of the original data set at date 2012-09-30 and the prediction is until date 2014-09-30. The result of the out of sample prediction is illustrated in Figure 18 below.

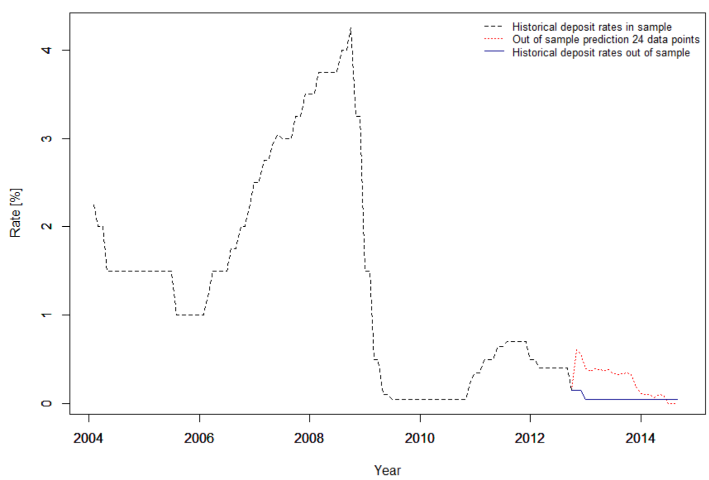


Figure 18: Out of sample prediction of deposit rate compared to actual volume.

It can clearly be seen that the prediction of the 24-step predicted deposit rate does not completely follow the original deposit rate data, although it still keeps on low levels.

5.4 Liquidity risk management

By following the methodology for calculating liquidity risk, described in Section 4.5.1, it is possible to visualize the liquidity risk of the bank. The result will be presented in two different ways; by plotting the liquidity quantile $m^p(t)$ and by displaying the buckets that have earlier been explained in Section 4.5.1.

The liquidity quantile $m^p(t)$ is determined by the framework in Section 4.5.1 and by using the derived deposit volume and short rate model that are obtained in Section 5.2 and Section 5.1 respectively, i.e. the deposit rate model is **not** utilized in this section since the withdrawals are only considered and not interest rate payments to the depositors. This is done for both $p = 0.05$ and $p = 0.01$ for a time period of ten years.

The obtained results of $m^p(t)$ are presented graphically as a continuation of the volume today and is presented for all stress cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ in Figures 19, 20 and 21 below.

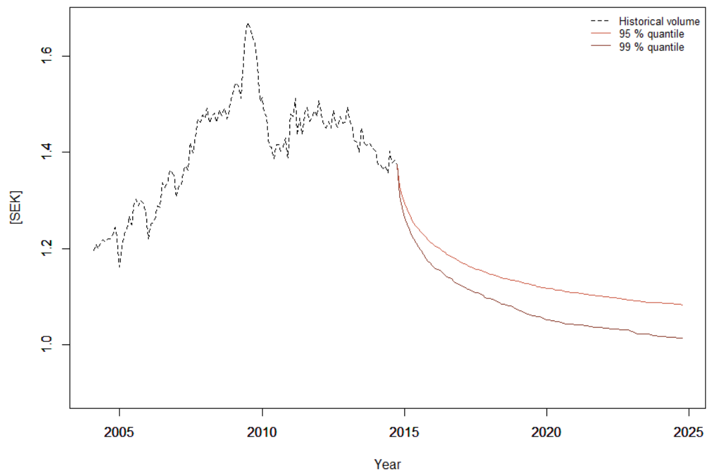


Figure 19: Illustration of the liquidity quantiles $m^p(t)$ for $p = 0.05$ and $p = 0.01$ for case $\boxed{1}$.

Figure 19 illustrates the result of $m^p(t)$ with $p = 0.05$ and $p = 0.01$ for interest rate case $\boxed{1}$. What is presented is the amount of deposit volume that will stay at the bank with a 95% respectively 99% confidence within the next ten years based on the market information from 2014-09-30.

The same result is presented for the shifted market cases 2 and 3 below.

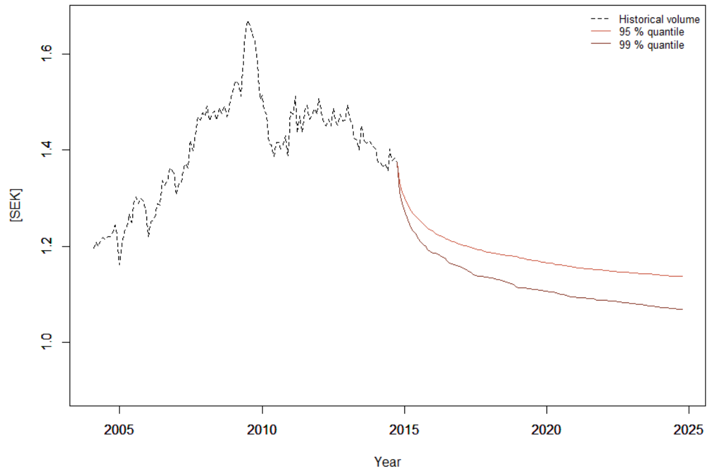


Figure 20: Illustration of the liquidity quantiles $m^p(t)$ for $p = 0.05$ and $p = 0.01$ for case 2.

The liquidity quantile $m^p(t)$ for the shifted case 2 is presented in Figure 20. An observation is that the quantile stays on a bit higher level than it did for case 1, meaning that a larger amount of deposit volume stays at the bank within the next ten years if the yield curve is shifted upwards.

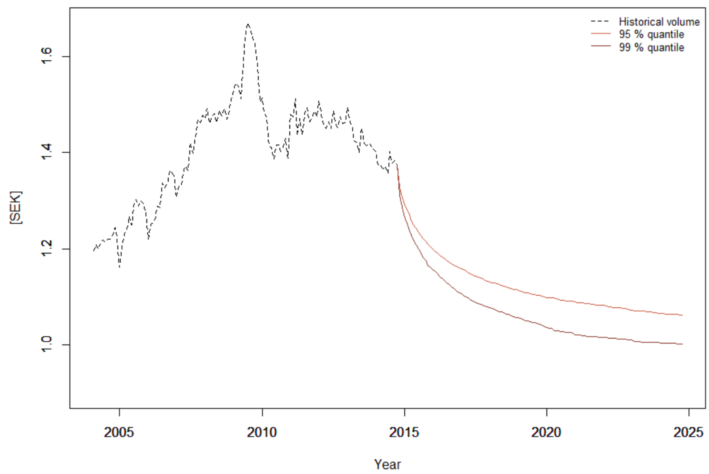
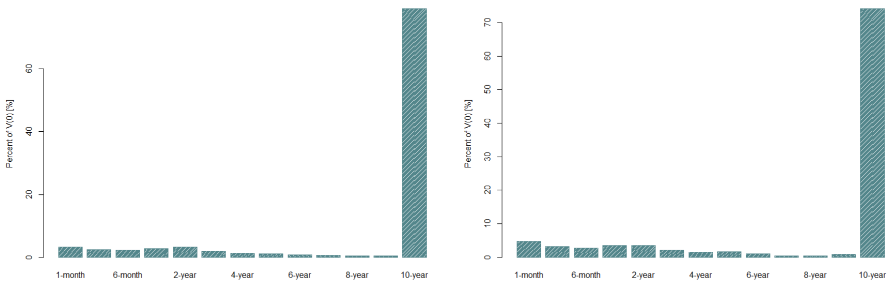


Figure 21: Illustration of the liquidity quantiles $m^p(t)$ for $p = 0.05$ and $p = 0.01$ for case 3.

Figure 21 presents the result from the final shift case $\boxed{3}$. An observation is that a lower amount of the deposit volume is expected to stay in the bank over the next ten years with a confidence of 95% and 99% respectively, compared to the cases $\boxed{1}$ and $\boxed{2}$.

In general, it can be observed from the plotted results of the three cases that the liquidity quantile $m^p(t)$ can be seen to rapidly decline for lower values of t but the decline effect decreases as t increases.

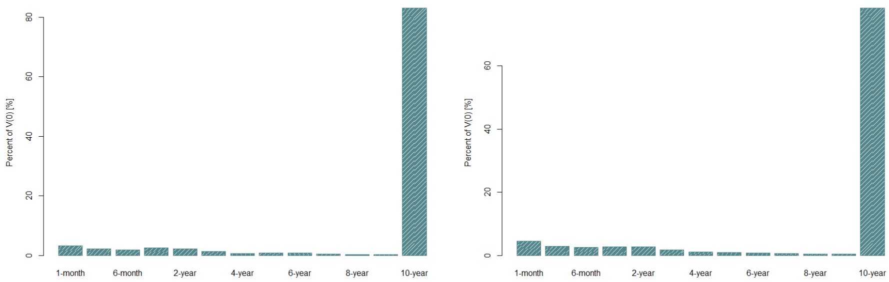
As the liquidity quantile has now been visualized in graphs, one can display the liquidity quantile in buckets of different maturities as well. The liquidity buckets, described in Section 4.5.1, are determined for each of the three cases $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ and can be interpreted as the maximum amount that can be withdrawn from the demand deposit over given time periods as a percentage of the current volume $V(0)$. This is done for a time period of 10 years for both $p = 0.05$ and $p = 0.01$. The result of this is presented in Figures 22, 23 and 24 below.



(a) Liquidity bucketing with $p = 0.05$ (b) Liquidity bucketing with $p = 0.01$

Figure 22: Illustrations of the liquidity bucketing for a ten year period for case $\boxed{1}$.

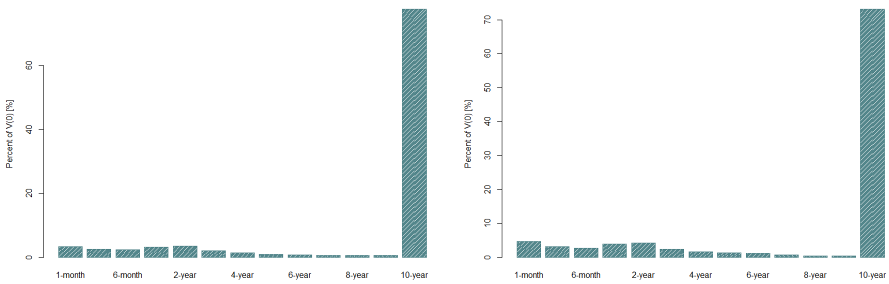
Figure 22a and Figure 22b display the liquidity bucketing for $p = 0.05$ and $p = 0.01$ for interest rate case $\boxed{1}$ over a ten year time period.



(a) Liquidity bucketing with $p = 0.05$ (b) Liquidity bucketing with $p = 0.01$

Figure 23: Illustrations of the liquidity bucketing for a ten year period for case 2.

Figure 23a and Figure 23b displays the liquidity bucketing for p = 0.05 and p = 0.01 for interest rate shift case 2 over a ten year time period.



(a) Liquidity bucketing with $p = 0.05$ (b) Liquidity bucketing with $p = 0.01$

Figure 24: Illustrations of the liquidity bucketing for a ten year period for case 3.

Figure 24a and Figure 24b displays the liquidity bucketing for p = 0.05 and p = 0.01 for interest rate shift case 3 over a ten year time period.

What can be depicted in the illustrations of the liquidity buckets is that a larger amount of the current deposit volume must be allocated in buckets of the shorter maturities for the confidence 99%. Another way of visualizing the results in Figures 22, 23 and 24, is to present the percentage results in a table, see Table 16 below.

Table 16: Liquidity buckets for each case and confidence level.

	Case 1		Case 2		Case 3	
Bucket	$p = 0.05$	$p = 0.01$	$p = 0.05$	$p = 0.01$	$p = 0.05$	$p = 0.01$
1M	3.34%	4.77%	3.18%	4.50%	3.38%	4.68%
3M	2.50%	3.19%	2.26%	2.93%	2.61%	3.24%
6M	2.34%	2.71%	1.88%	2.54%	2.47%	2.75%
1Y	2.79%	3.55%	2.49%	2.79%	3.17%	3.86%
2Y	3.21%	3.47%	2.25%	2.77%	3.60%	4.27%
3Y	1.99%	2.10%	1.37%	1.76%	2.05%	2.42%
4Y	1.27%	1.58%	0.74%	1.07%	1.48%	1.64%
5Y	1.09%	1.65%	0.83%	1.01%	1.01%	1.30%
6Y	0.74%	1.08%	0.75%	0.84%	0.81%	1.22%
7Y	0.54%	0.47%	0.51%	0.59%	0.61%	0.67%
8Y	0.53%	0.43%	0.39%	0.44%	0.60%	0.36%
9Y	0.52%	0.83%	0.33%	0.47%	0.55%	0.46%
10Y	79.14%	74.17%	83.03%	78.29%	77.67%	73.13%

The reason for the dominance of the 10-year liquidity bucket, as can be seen in Table 16, is due to the assumption that all of the remaining deposit volume is paid back (to the depositors) at the end of the considered time period.

5.5 Interest rate risk management

The calculation of the interest rate risk has been carried out according to the methodology described in Section 4.5.2. First, the demand deposits are valuated for the different cases, where the results of case 2 and 3 are compared to the result of case 1. Thereafter, the duration of the liability, an immunization portfolio and a replicating portfolio are derived.

All results in this section are presented as a percentage of the current volume $V(0)$. The duration is expressed in years.

Valuation of demand deposit
As presented in Section 4.5.2, a method for measuring the interest rate risk is the valuation of the demand deposit of the bank for different stress cases in order to observe the changes that occur due to interest rate shifts.

The result of the present values of the demand deposit and consequently the present values of the liability of the bank are computed by using Equations (51) - (54) and are presented in Table 17 below.

Table 17: Valuation of demand deposit for case 1.

	5 years		10 years	
	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$
PV	1.78%	1.42%	4.25%	3.11%
PV_L	98.22%	98.58%	95.75%	96.89%

As presented in Table 17, this is done for both the volume process $V(t)$ and the liquidity quantile $m^p(t)$ for two different time periods, five and ten years respectively. Furthermore, the results for $m^p(t)$ are only computed for the 95% confidence level, p=0.05.

The results in Table 17 can be interpreted as the value of issuing a demand deposit for the bank. All results are presented as a percentage of the current deposit volume, which is the deposit volume that the bank has at date 2014-09-30. Also, the reader might notice that the relationship stated in Equation (52) is fulfilled in Table 17.

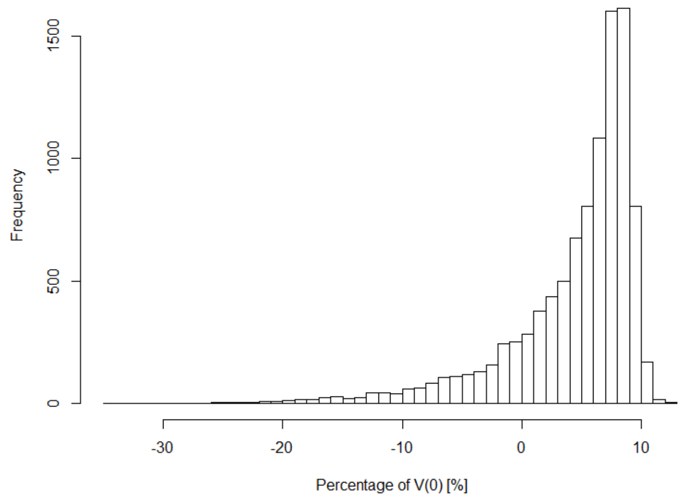


Figure 25: Histogram of the net present value of the demand deposit, PV, for a ten year period when using $V(t)$.

Figure 25 illustrates the distribution of the net present value of the demand deposit, PV, when using $V(t)$ as the volume process and for a ten year time period.

This procedure is carried out for the stressed cases $\boxed{2}$ and $\boxed{3}$ as well, i.e. the measurements for the net present value of the demand deposit, PV, and the present value of the liability, PV_L. The new valuations after the interest rate shifts are presented in Table 18 and Table 19 presents a comparison to the present value of the liability stated in Table 17. The comparison is the percentage increase or decrease of the net present value of the liability after a parallel shift has affected the yield curve.

Table 18: Valuation of demand deposit for stress cases $\boxed{2}$ and $\boxed{3}$.

	Case $\boxed{2}$				Case $\boxed{3}$			
	5 years		10 years		5 years		10 years	
	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$
PV	3.74%	3.16%	7.50%	5.95%	0.65%	0.42%	2.46%	1.58%
PV_L	96.26%	96.84%	92.50%	94.05%	99.35%	99.58%	97.54%	98.42%

Table 19: Comparing the results from stress cases [2] and [3] with case [1].

	Case [2]				Case [3]			
	5 years		10 years		5 years		10 years	
	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$	$V(t)$	$m^{0.05}(t)$
ΔPV_L	-1.20%	-1.77%	-3.39%	-2.93%	+1.15%	+1.01%	+1.87%	+1.58%

As seen in Table 18 the present value of the demand deposit increases if the zero rates rises in case [2] and decreases if the market rates fall in case [3]. The time period over which the valuation is carried out also affects the present value of the demand deposit. When using the liquidity quantile $m^p(t)$ as the volume process one obtains smaller net values of the demand deposit, as mentioned in Section 4.5.2. Furthermore, a comparison has been carried out where the outcome of the outcome of the net present values of the liability for the stressed cases [2] and [3] are compared to the unstressed case [1].

The result seen in Table 19 is that the net present value of the liability decreases for case [2] and increase for case [3]. This means that when there is an upturn in the market rates, the bank's liability decreases while at an eventual downturn the bank's liability increases. Also, the increasing/decreasing difference in the value of the liability depends on the length of the time period for which the valuation is carried out.

Duration

The duration of the liability is derived following the methodology described in Section 4.5.2, Table 20 displays the obtained duration results for all interest rate scenarios.

Table 20: Duration in years of the liability for the different cases [1], [2] and [3].

	5 years	10 years
	$m^{0.05}(t)$	$m^{0.05}(t)$
Case [1] D_L	4.44	7.89
Case [2] D_L	4.29	7.94
Case [3] D_L	4.21	7.89

The results presented in Table 20 show that all duration measurements are skewed towards the end of the considered time period. Furthermore, it is observed that the results of the duration do not vary substantially between the different cases.

Immunization

From the obtained duration of the liability D_L, which has been derived above, it is possible to determine which of the market rate securities to use in the immunization portfolio. The portfolio weights are determined by solving the matrix equation stated in Section 4.5.2. The optimal weight results of the five year and ten year time period are presented in Tables 21 and 22 below.

Table 21: Solution of the immunization portfolio for a five year time period.

Security	1M	3M	6M	1Y	2Y	3Y	4Y	5Y
$m^{0.05}(t)$	-	-	-	-	-	-	47%	51%

Table 22: Solution of the immunization portfolio for a ten year time period.

Security	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
$m^{0.05}(t)$	-	-	-	-	-	-	-	-	-	-	61%	36%	-

Using the optimal weights from the immunization portfolios derived above, it is possible to compute the present value of the portfolios for the stressed cases 2 and 3. The results and a comparison to the values in Table 18 are presented in Table 23 below.

Table 23: Results of the immunization.

	Case 2	ΔPV_L	Case 3	ΔPV_L
5 years	93.73%	-3.21%	100,10%	+0.52%
10 years	89.56%	-4.77%	100.81%	+2.37%

The comparison in Table 23 is the percentage increase or decrease in the liability compared to the values in Table 18.

Replicating portfolio
The results for the replicating portfolio are derived by solving the optimization problem stated in Equation (57). This is done for a five year period as well as for a ten year period.

The optimal portfolio weights are presented in Tables 24 and 25 below for a five and a ten year period.

Table 24: Solution of the replicating portfolio for a five year period in percent.

Security	1M	3M	6M	1Y	2Y	3Y	4Y	5Y
$m^{0.05}(t)$	3.27	2.54	2.42	2.23	2.58	1.98	1.42	82.39

Table 25: Solution of the replicating portfolio for a ten year period in percent.

Security	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
$m^{0.05}(t)$	3.27	2.54	2.42	1.71	2.05	1.44	0.88	0.58	0.46	0.46	0.40	0.38	79.18

The present value of the replicating portfolios are computed for each of the stressed case $\boxed{2}$ and $\boxed{3}$ and the results are presented in Table 26. Similarly to the immunization results, the replicating portfolio results are compared to the values in Table 18.

Table 26: Results of the replicating portfolio.

	Case $\boxed{2}$	ΔPV_L	Case $\boxed{3}$	ΔPV_L
5 years	94.66	-2.25%	100.94	+1.37%
10 years	88.39	-6.02%	99.54	+1.14%

What can be observed in Table 26 is that the percentage increase or decrease of the difference of the present value of the replicating portfolio and the values of the liability presented in Table 18 are higher for a greater shift in the zero rates and for a longer time period.

6 Analysis

The calibration of the Vasicek parameters for the short rate model to current market data is a challenge as the Vasicek model only has three parameters. This in combination with the fact that the yield curve of the chosen date has an odd shape makes it difficult to entirely fit the Vasicek model to the observed market data. The optimization procedure was therefore challenging as several local minimum were obtained. However, even though the calibration produced poor results, it was satisfying enough to derive a liquidity quantile for the deposit volume and determine an estimate of the net present value of the demand deposit. Nonetheless, it would have been interesting to use a more advanced short rate model.

In this study, the final outcome of the deposit volume model is a simple model consisting of a constant, a dependence on the market short rate, a dependence on its previous value and a stochastic component and this model is referred to as Model 11. However, it is shown in Section 5.2 that the coefficient of the short rate is not significant for the model. This might be because of the heavy peak in the historical data set, discussed in Section 3, which affects the outcome of the model parameter estimation. This peak is somewhat interesting to highlight since it is not a deviation caused by the market but is rather a result from a decision that was taken by the management group of the bank and is therefore not correlated to the market situation. However, even though the derived model has been affected by this peak, from Figure 11 and Figure 13 it is possible to see that the model yielded satisfying results both for the in and out of sample data. The out of sample data lies well within the predicted 95% confidence interval of the model prediction. Another reason that justifies this choice of model is the fact that a dependence of a market rate seems reasonable from an economic point of view, as stated in Section 5.2. Although, it is important to emphasize that if the bank would like to use this type of model, the data set on which the model is estimated would have to be carefully selected in order to ensure representative future outcomes of the deposit volume.

By studying Figure 17 and Figure 18 one can conclude that the proposed model for the deposit rate is unable to capture the full behaviour of the historical deposit rate. This might be from the fact that the deposit rate have an asymmetric response to changes in the market rates, where the deposit rate reacts faster to a rise in the market rates than to a fall. However, even though the proposed model could not describe the entire deposit rate data, the result is still acceptable but it is important to bring attention to the fact that the final results will probably improve with a more accurate deposit rate model.

The liquidity quantile, $m^p(t)$, for both $p = 0.05$ and $p = 0.01$ has been derived for all cases of the zero rates and the results are illustrated in Figure 19 - Figure 21. This gives an estimate and an insight of the minimum volume available at a given time t. Given the results in Table 16, it is clear that an upturn in the market implies that a greater amount of the deposit volume is expected to remain in the bank deposits and the other way around if there is a drop in the market rates.

From Table 16 it is also possible to see that 79.14% and 74.17% of the initial volume remain after ten years for case $\boxed{1}$ at a 95%- respectively 99% confidence, indicating that a large portion of the volume would still remain even in a worst case scenario. One can also conclude from Table 16 that when the market rates rise, due to a parallel shift, the allocation of capital is shifted from the mid-term buckets towards the 10Y-bucket, i.e. the bucket with longest maturity. The opposite can be observed when the market rates fall due to a negative parallel shift. One can also note that the allocation in the two buckets with shortest maturities are relatively unchanged after the interest rate shifts are introduced.

From Table 18 one is able to conclude that the net present value of the demand deposit for both a five and a ten year time period is seemingly small compared to the present value of the liability. However, the period for which the valuations were carried out consisted of low market interest rates and from Table 18 it is possible to observe that as a consequence, when market rates rise due to a parallel shift, the net present value of the demand deposits increases. The valuation is also dependent of the length of the time period over which the valuation is considered. This makes it difficult to properly value non-maturing products since the maturity date is unknown.

Figure 25 gives an understanding of the distribution of the valuation result for the demand deposits. It can be seen that the distribution indicates a fat left tail and this follows from the fact that $r(t)$ is allowed to assume negative values while $d(t) \geq 0 \ \forall t$. This generates the possibility for the demand deposit to have a negative value for the bank, given an unfavorable evolution of the market rates, i.e. a drop in the market rates.

All calculations of the durations for the liability cash flows produced results that were strongly oriented towards the end of the time period T. This is due to the assumption that the remaining volume is paid back at time T.

Following the calculations of the durations, the immunization portfolios of the liability were derived. Using these weights and the proposed market rate securities it is possible to calculate the present value of the immunization portfolios after shifting the yield curve. The purpose is to immunize the liabilities against parallel shifts in the zero rates. However, from Table 23 it can be seen that the present value of immunization portfolio, after introducing the parallel shifts δ_2 and δ_3, does not match the present value of the liability PV_L derived in the valuation section. The accuracy of the result declines for longer time periods as well. This might be due to the nature of this liability, where the monthly cash flows are strongly dependent on the market short rate and the volume process which both are stochastic processes. The Vasicek short rate model was not able to fully fit to the current market data which in turn decreases the accuracy of the valuation as well.

Futhurmore, following the immunization a replicating portfolio was derived. From Table 26 it is possible to conclude that the present value of the replicating portfolio does not match the present value derived in the valuation section and the accuracy declines for longer time periods, similarly to the result of the immunization portfolio. The same aspects as suggested above may improve the replicating portfolio performance. However, the replicating portfolio is not constructed to exactly match the present value of the liability but instead match the size of and the time at which the liability cash flows occur. Another fact that may contribute to the performance of both the immunization as well as the replicating portfolio is that the interest rate shifts were significantly large. Usually, this type of analysis is done for smaller shifts in the yield curve.

Deriving an immunization and/or replicating portfolio can help a risk manager to translate the stochastic cash flow produced by the liability into a portfolio of fixed income assets with known maturities. Alternatively, the immunization and/or replicating portfolio could be used in order to hedge the liability.

7 Summary and conclusions

In this report, a framework for determining a bank's liquidity risk and interest risk respectively is developed. First, a stochastic model for the market short rate is developed using a Vasicek short rate model. Thereafter, a model for the deposit volume is determined by using multiple regression analysis and the final outcome was a deposit model consisting of a constant, a dependence on the short rate and a dependence on its previous value. With these models a framework for determining the bank's liquidity risk is derived. After stressing the yield curve, different outcomes of the liquidity behaviour is presented and one can see that a market drop requires the bank to hold more short term liquidity and the other way around for a market upturn.

Furthermore, a simplified model for the bank's deposit rate is developed which consists of a linear relationship of a constant and a dependence on the short rate. This is used in order to determine a framework for the analysis of the bank's interest rate risk. By using arbitrage-free valuation of the demand deposit the net present value of the bank's liabilities was determined. This result could then be analyzed for different stress scenarios, where the yield curve was shifted. One result is that, for example, for an upturn in the market rates, the bank's liability decreases with -3.39% over a 10 year period while for a downturn in the market rates, the bank's liability increases with 1.87% over a 10 year period.

By constructing an immunization and a replicating portfolio, it is possible to translate the present value of the liability into a portfolio of market rate securities. What can be seen is that the present value of the liability increases when the market rates are shifted downwards and vice versa for an upshift. Due to the nature of non-maturing liabilities, the time horizon for which the valuation is carried out highly affects the results. However, the result of the immunization and replicating portfolio did not match the present value of the liability when the yield curve was shifted.

In order to implement the outcome of these frameworks in a bank, improvements have to be done particularly in modeling of the short rate, the deposit volume and deposit rate. The framework for determining the liquidity risk management is easier to implement as it only depends on the development of the short rate model and deposit volume model. The framework for interest rate risk however is a bit more challenging as a proper analysis of the asymmetric behaviour of the deposit rate model would have to be analyzed.

8 Future work

In order for the bank to implement this framework, there are some suggestions on improvements that should be made in order to obtain more accurate results.

First of all, all different deposit accounts have been merged into one account in this study. In the future, it would be beneficial to carry out the same procedure for the separate types of deposit accounts. In this way, one would be able to value each separate account individually, providing a better overview of the interest rate risk associated with different account types.

As the proposed deposit rate model was not fully able to describe the historical deposit rate levels, one could examine the possibility of using asymmetric models instead. There are several studies that have been done concerning asymmetric deposit rate models, such as in Maes, K. and Timmermans, T. [18], and the results have been satisfactory.

Another aspect to keep in mind for future work is that other short rate models could potentially allow for a better fit to the market data. Using a better short rate model would yield more accurate results when valuing the demand deposits. It would also allow to examine how a shift in a specific key rate affects the net present value of the demand deposit and not only a parallel shift.

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