DEGREE PROJECT IN ,
SECOND CYCLE, 30 CREDITS
*STOCKHOLM, SWEDEN 2017*

# Modeling Deposit Rates of Non-Maturity Deposits

**ANDREAS KÖRDEL**

**KTH ROYAL INSTITUTE OF TECHNOLOGY**
**SCHOOL OF ENGINEERING SCIENCES**

# Modeling Deposit Rates of Non-Maturity Deposits


## ANDREAS KÖRDEL



Degree Projects in Financial Mathematics (30 ECTS credits)
Degree Programme in Industrial Engineering and Management
KTH Royal Institute of Technology year 2017
Supervisor at Handelsbanken: Magnus Hanson
Supervisor at KTH: Camilla Landén
Examiner at KTH: Camilla Landén

*TRITA-MAT-E 2017:19*
*ISRN-KTH/MAT/E--17/19--SE*

Royal Institute of Technology
*School of Engineering Sciences*
**KTH** SCI
SE-100 44 Stockholm, Sweden
URL: www.kth.se/sci

# Abstract

The modeling of non-maturity deposits is a topic that is highly important to many banks because of the large amount of funding that comes from these products. It is also a topic that currently is in the focus of legislators. Although a non-maturity deposit may seem to be a trivial product, it has several characteristics that make it rather complex. One of the two purposes of this thesis is to compare different models for the deposit rate of non-maturity deposits and to investigate the strengths and weaknesses of the models. The other purpose is to find a new model for the deposit rate of non-maturity deposits. Several different models that are suggested in the literature are described and evaluated based on the four aspects; goodness of fit, stability, negative interest rate environment and simplicity. Three new models for the deposit rate are suggested in this thesis, one of which shows a very good performance compared to the models that can be found in the literature.

# Modellering av kundräntor för icke tidsbunden inlåning

## Sammanfattning

Modellering av icke tidsbunden inlåning är ett ämne som är mycket viktigt för många banker på grund av den stora andel finansiering som kommer från dessa produkter. Det är också ett ämne som för närvarande väcker lagstiftares intresse. Även om icke tidsbunden inlåning kan tyckas vara en trivial produkt, har den flera egenskaper som gör den komplex. Ett av de två syftena med detta arbete är att jämföra olika modeller för kundräntan i icke tidsbunden inlåning och att undersöka modellernas styrkor och svagheter. Det andra syftet är att introducera en ny modell för kundräntan i icke tidsbunden inlåning. Flera olika modeller från litteraturen beskrivs och utvärderas baserat på de fyra utgångspunkterna passform, stabilitet, negativ räntemiljö och enkelhet. Tre nya modeller av kundräntor för icke tidsbunden inlåning föreslås i detta arbete, varav en visar ett mycket bra resultat jämfört med de modeller som föreslås i litteraturen.

# Acknowledgements

I would like to give thanks to my supervisor at KTH Royal Institute of Technology, Camilla Landén, for her valuable comments and discussions throughout the study. I would also like to thank Magnus Hanson and the others at Group Risk Control at Handelsbanken for their insightful support during the process of writing this thesis. Thanks should also be given to all other people who have supported me in any way during this time.

Stockholm, May 2017
Andreas Kördel

# Contents

# List of Figures

# List of Tables

# 1. Introduction

## 1.1 Background

A non-maturity deposit (NMD) is, as the name suggests, a deposit that does not have a predetermined maturity, i.e. the deposit can be withdrawn at any time. Examples of NMDs are savings accounts, demand deposits and current accounts. While these products may seem rather simple at a first glance, they have two main features that make them difficult to model. Firstly, the customer may at any time change the balance of the deposit, by adding or withdrawing funds, often without penalty. Secondly, the bank may adjust the interest rate at any time.

Even though NMDs can be withdrawn at any time, the volumes of NMDs have historically been relatively stable. This has also been true at times when market rates change a lot. Also, deposit rates in NMDs have typically been much lower than for other types of products, although that is currently not true due to the exceptionally low interest rate environment. Therefore, NMDs have been a stable and cost-effective source of funding that is very important for many banks. This makes the modeling of NMDs a highly interesting topic for most banks.

The behavior of NMDs can be explained by the two factors, received and perceived value by the customer. For instance, Cipu and Udriste (2009) claim that an NMD with a high deposit rate relative to other NMDs and with high barriers to exit generally have longer maturities. Others claim that balances of accounts with high deposit rates are more sensitive to changes in terms of the account compared to changes in terms of other accounts. This second opinion is based on the idea that those who keep deposits for saving purposes search for the best deposit rate, while those who keep deposit for transactional purposes care more about other aspects. Therefore, deposits held for transactional purposes are usually seen as more stable even though they often have lower deposit rates.

In recent years more and more focus has been given by legislators to the handling of interest rate risks in the banking industry. The regulations focus on modeling the amount of core deposits of NMDs and on the expected maturity of this core deposit. The core deposit is the part of the NMD that would remain in the deposit even if unfavorable events would occur.

One important aspect of modeling NMDs is to understand how the net interest income of a bank will change in different scenarios. To predict the net interest income, it is important to know the dynamics of the deposit rate of NMDs. For that purpose a good model for the deposit rate is needed.

## 1.2 Problem

A difficulty that has appeared in the last few years is the extremely low interest rate environment. The short market rates are negative, while deposit rates are almost always non-negative. This means that a strategy of taking in deposits and investing them in short term market rates would be a bad business for banks, which is a new situation. It also proves difficult to model the future evolution of deposit rates when they have been floored at zero for a long time.

Models for NMDs often consist of three components; a model for market rates, a model for deposit volumes and a model for deposit rates. In some models these components are modeled separately, while in other models they are modeled in an intertwined way, e.g. with a vector autoregressive system of equations. This thesis focuses on the modelling of deposit rates.

The purpose of this thesis is twofold. The first is to make an overview of the different models for deposit rates of NMDs that can be found in the literature and to assess the eligibility of these models. The second part is to develop a new method for modelling the deposit rate of NMDs and in a similar manner assess its strengths and weaknesses.

## 1.3 Research Questions

The research question of this thesis is:

- How should the deposit rate of an NMD be modeled?

To answer this question, the following questions have to be answered first:

- What characterizes a good model?
- Which models exist in the literature today?
- Are there other possible approaches to modeling deposit rates?

## 1.4 Structure of the Thesis

In Chapter 2 the characteristics of NMDs and some of the regulations on them are described in more detail. That chapter shows some of the complexity of NMDs. In Chapter 3 existing models for deposit rates of NMDs are described and critically reviewed in. Most of the models for the deposit rates are taken from a bigger context, for instance from a valuation framework and therefore, the context where the models are used is also briefly described for most models. In Chapter 4 three new models are described and motivated. These new models are an attempt to improve the already existing models, especially when it comes to handling a negative interest rate environment. In Chapter 5 the methodology that has been used is described, including a description of the data that has been used. In Chapter 6 the results from the different models are shown and describe.  In Chapter 7 there is a discussion on the findings. The thesis ends with a conclusion and suggestions for further research in Chapter 8.

The thesis is written at Svenska Handelsbanken AB (Handelsbanken). Therefore, examples and data from Handelsbanken will be used.

# 2. Non-Maturity Deposits

*Non-maturity deposits (NMD) are usually a vital part of a bank's funding. For instance, 41.4 % of Handelsbanken's liabilities at the end of 2016 had no specified maturity. (Handelsbanken, 2017) This chapter contains a description of some of the most important characteristics of NMDs and a brief summary of the most relevant regulations on them. The described characteristics show the complexity of modeling NMDs, while the description of the regulations explains why and how banks are required to pay more attention to the modeling of NMDs.*

## 2.1 Characteristics of NMDs

At first glance an NMD seems to be a rather simple financial product, but it quickly turns out to be quite difficult to model NMDs. This is mainly due to the fact that customers may change the balance at any time and without warning, while the bank can change the deposit rate in the same manner. Blöchlinger (2015) shows that these facts cause significant option risk, where some of the options are in favor of the bank and some are in favor of the customers. Jarrow and van Deventer (1998) explain the difference between deposit rates and market rates with search/switching costs, regulatory barriers and asymmetric information. Below, the different embedded options and characteristics of NMDs are described.

### 2.1.1 Withdrawals

The most prominent characteristic of NMDs is the customers' right to withdraw their money from the deposit account at any time, often without any additional fees. When other investments yield a higher return, the value of the NMD often decrease for the customers and they can simply withdraw their money. When the difference between the deposit rate and alternative investments becomes too large, customers will move their money to other investments. Bank of Japan (2014) shows that volumes of NMDs decrease in a high interest rate environment and increase in a low interest rate environment. Bardenhewer (2007) describes the customer's possibility to withdraw money as an enormous portfolio of nested options, where the customer has the option to withdraw different amounts at different times. For instance, a customer could withdraw 10% of her deposit volume today and the other 90% next week. This portfolio would be impossible to model without a simplification.

### 2.1.2 Partial Adjustments

Blöchlinger (2015) shows that increases in interbank rates are only partly passed on to the depositors. Historically, the spread between interbank rates and deposit rates has been higher when there is a large increase in interbank rates, which implies that the value of the possibility to use partial adjustments is higher in a high interest rate environment. This possibility is in favor of the bank. However, the value of partial adjustments could be neutralized by the customers' right to withdraw money, since if they think they could get better terms somewhere else they could simply reallocate their money. Therefore, this characteristic is highly dependent on the bank's market power.

### 2.1.3 Asymmetric Adjustment Speed

It is a consensus in the literature that deposit rates are generally adjusted much faster when interest rates decrease than when they increase. This reflects customers' slow reaction to changes in conditions and the banks exploitation of customers reaction speed. Unsurprisingly, Paraschiv and Schürle (2010) find that the adjustment of the deposit rate is faster when the deviation from the equilibrium relationship between the deposit rate and market rates is larger. Neumark and Sharpe (1992) conclude that the asymmetry is bigger the bigger the banks market power is, for instance if the market concentration is high.

### 2.1.4 Discretization

Deposit rates are most often given in a discretized way, rather than on a continuous scale. For instance, interest rates are often quoted in tenths or hundreds of a percentage point and are usually not changed every day. This makes deposit rates less affected by small changes in market rates.

### 2.1.5 Caps and Floors on Deposit Rates

Some countries have regulated caps and floors on the deposit rates of NMDs. In Sweden there are no such regulations. However, there is an informal floor to the deposit rates set at zero. Although deposit rates could be negative, there seems to be a general belief that negative rates would deter customers and make them withdraw their money. The floor is in favor of depositors, while a cap would be in favor of the bank. Though it is unlikely that deposit rates fall below zero, it is more likely that fees on NMDs will increase, which is similar to negative deposit rates.

### 2.1.6 Other Aspects

Some NMDs are used by customers for purely transactional purposes and are therefore almost unaffected by the aforementioned options and characteristics. These accounts are important in businesses' or people's daily transactions and would be used almost regardless of the terms of the NMD. Some customers are poorly informed and therefore do not react to changes in conditions of their deposits. Other customers are well aware of the conditions, but get better terms for other products, for instance mortgages, when doing all their banking errands at the same bank and are thereby tied up. Some customers just want to use the same bank for all banking errands to save time compared to using several banks and therefore care relatively little about the interest rates on NMDs, as other factors dominate their choice of bank.

In many cases other factors than the terms of the NMD determines the customer's choice of bank and account type. For instance, the availability of the banking services is an important factor. While some customers still prefer to do their banking errands at a branch, the world is being more and more digitalized, making banking services available anywhere and anytime for anyone with an Internet access. The digitalization is transforming the banking industry, making customers much better informed and making it much easier for customers to transfer their money to the deposit that pays the best interest rate.

In many countries there is a deposit insurance on NMDs. In Sweden the deposit insurance was first introduced in 1996 with a coverage limit of 250 000 SEK. This limit has been increased a few times and in July 2016 the coverage limit was set to 950 000 SEK. The institutions that are included in the insurance pay a fee depending on the amount of guaranteed deposits and some risk factors. (Swedish National Debt Office, 2017) The deposit insurance should lead to lower deposit rates, as customers expect less risk premium, but for banks that have a high level of reliability the effect could be the opposite. This is since banks with lower level of reliability are also insured, which takes away the competitive advantage of being trustworthy.

There are of course other reasons for fluctuations in deposit volumes, such as people's personal circumstances and economic situation. The deposit volume of some accounts vary depending on time of the year or month. This implies that many different macroeconomic factors could affect NMDs in an intricate way. All these different characteristics make the NMD a highly complex instrument to model. One important factor in such a model is the behavior of the bank, which is the focus of this thesis.

## 2.2 Regulations

The Basel Committee on Banking Supervision (2016) divides NMDs into three different categories; retail/transactional, retail/non-transactional and wholesale. The definition of a retail deposit is that it is made by individuals at a bank. To this category it is also possible to add deposits placed by small business customers with total aggregated liabilities smaller than € 1 million, if they have similar interest rate characteristics as retail accounts and if they are managed as retail exposure. The retail deposits are categorized as transactional either if regular transactions are made in the account or if the account is non-interest bearing. In any other case the retail deposit is to be considered non-transactional. Deposits that are placed from either legal entities, sole proprietorships or partnerships are considered to be wholesale deposits.

Further, the NMDs can be divided into a stable and a non-stable part. The stable part is the portion of the NMD that is unlikely to be withdrawn. The portion of the stable part that will remain undrawn with a high likelihood even during significant changes in interest rate environment is called core deposits, while the rest of the NMD is called non-core deposit. These portions should be determined using observed volume changes over the past 10 years according to Basel Committee on Banking Supervision (2016).

NMDs are then slotted into time buckets, depending on estimated average maturity. Non-core deposits are automatically considered to be overnight deposits that should be put into the shortest time bucket. To limit overestimation of the proportion of core deposit and the time to maturity of the core deposit the Basel Committee on Banking Supervision has determined the upper limits of these metrics for the different account types. These caps are shown in Table 1.

**Table 1. Caps on core deposits and average maturity**

| | Cap on proportion of core deposits (%) | Cap on average maturity of core deposits (years) |
|---|---|---|
| **Retail/transactional** | 90 | 5 |
| **Retail/non-transactional** | 70 | 4.5 |
| **Wholesale** | 50 | 4 |

The table shows the caps on proportion of core deposits and the caps on average maturity of core deposits for different account types decided by the Basel Committee on Banking Supervision (2016).

Other interest rate-sensitive assets and liabilities in the banking book are also slotted into time buckets. The banking book consists of the banks securities that are not being actively traded and are expected to be kept until maturity. The interest rate risk in the banking book (IRRBB) can now be assessed by first netting the assets and liabilities in all time buckets and then calculating the loss in economic value of equity under certain interest rate shock scenarios. Two examples of such interest rate shocks are parallel shifts up and down.

In their model for IRRBB, the Swedish Financial Supervisory Authority (Finansinspektionen – FI) (2015) set the repricing date for NMDs to zero. The Swedish Banker's Association claims that this is a simplification that does not show the actual risk characteristics. FI motivates the assumption with the observation that there are no objective methods to determine repricing dates that could be considered to be constant over time. FI suggests that if a bank wants to reduce the capital requirement for IRRBB, they should increase the proportion of deposits that have an agreed repricing date.

The recent focus from legislators on NMDs has made banks pay more attention to the topic. For that reason this thesis is highly relevant.

# 3. Theoretical Framework

*This chapter provides an overview of different approaches to modeling NMDs and in particular to the modeling of deposit rates. The models have somewhat different purposes. Some of the articles calculate the value of NMDs, some try to find hedging strategies, some focus on the liquidity risk management and some focus on the interest rate risk management. Although they all study NMDs, they do not cover exactly the same products. Some use daily, some use monthly and some use quarterly data. Some look at specific account types at specific banks, some on aggregated data for a bank and some use aggregated data for a whole country. Despite these differences they all tackle the modelling of NMDs and are therefore of interest to this thesis. In the end of the chapter there is a section about what is thought to characterize a good model in this thesis.*

## 3.1 Models for Deposit Rates

### 3.1.1 Linear Models

Nyström (2008) models NMDs by suggesting models for the three components; market rates, deposit rates and deposit volumes. The market rates are modeled using an extended one-factor Vasicek model (see Appendix 1) and the deposit volume is modeled with behavioral models. These behavioral models are based on the behavior of individual customers, for instance by considering the customers' income. The volume is assumed to go towards a target volume, which is a fraction of the average monthly income of the customers and is supposed to cover the customers' liquidity needs. The speed of convergence to this target volume and the amount that is passed on to other investments are also modelled.

In Nyström's framework, deposit rates are allowed to be modeled in any way as long as it is a function of the market rate and/or volume. No explicit model is given, but in an example given in the article the deposit rate, $d_t$, is modeled by

$$d_t = \beta_1 r_t \tag{1}$$

where $r_t$ is a short market rate. Nyström calls the deposit rate function a policy function that is determined by the bank. In the example model, $\beta_1$ is a constant determined by the bank. The aim of the paper of Nyström is to value NMDs and to create a framework that can find the theoretically optimal policy function.

Elkenbracht and Nauta (2006) aim to stabilize the margin between the investment return and deposit rates by introducing two dynamic hedge strategies. Similarly to Nyström (2008), they do not give an explicit model for the deposit rates, but use a linear model for the deposit rate as an example,

$$d_t = \beta_0 + \beta_1 r_t \tag{2}$$

where $r_t$ is a short market rate and $\beta_0$ and $\beta_1$ are constants that should be fitted.

Bardenhewer (2007) describes an option adjusted spread (OAS) model. This approach uses the three components; term structure, deposit volume and deposit rate to model NMDs. The deposit rate is modeled by

$$\begin{gathered} d_t = \beta_0 + \beta_1 * average\ of\ 1\ M\ Libor\ over\ the\ last\ 6\ months + \\ \beta_2 * average\ of\ 5Y\ swap\ rate\ over\ the\ last\ 6\ months \end{gathered} \tag{3}$$

The volume is modeled by a withdrawal process dependent on a moving average of the short market rate, the long term market rate six months earlier and dummy variables depending on what season of the year it is. Bardenhewer stresses the importance of using an appropriate term structure, but does not restrict the model to one term structure model. With this approach Bardenhewer calculates the present value of the NMD using discounted future cash flows. The interest rate risk can then be hedged with interest rate swaps by looking at the delta profile of the NMD.

### 3.1.2 Jarrow and van Deventer

Jarrow and van Deventer (1998) use an arbitrage free approach to model NMDs. They divide the market into two parts, banks and individuals. They assume that there are significant entry barriers for banks to supply NMDs, but that both banks and individuals have unrestricted access to the treasury market. With these assumptions they derive a formula for the value of NMDs. The value at time 0, denoted $\Psi_0$, is decided by the following expression

$$\Psi_0 = E_0^Q\left[\sum_{t=0}^{\tau-1}\frac{V_t(r_t - d_t)}{B_{t+1}}\right] \tag{4}$$

where $V_t$ is the volume at time *t* and $\tau$ is the number of periods used. The value of the money market account at time *t* is denoted $B_t$ and is obtained by

$$\begin{aligned} B_t &= B_{t-1}(1 + r_{t-1}) \\ B_0 &= 1. \end{aligned} \tag{5}$$

The expectation $E_0^Q[\cdot]$ is taken under the unique risk neutral martingale measure $Q$ generated by the term structure. For a proof of equation (4) see Appendix A of Jarrow and van Deventer (1998). The valuation formula (4) has the economic interpretation of an exotic interest rate swap that receives the floating short market rate and pays the deposit rate with a principal $V_t$ in period *t*.

The present value of the NMD liability to the bank at time 0 equals the initial demand deposit less the net present value of the NMD or

$$V_0 - \Psi_0. \tag{6}$$

This value can be hedged by investing $V_0$ in the bond with shortest maturity available and by shorting the exotic interest rate swap that is represented by $\Psi_0$. Jarrow and van Deventer also present a similar model in continuous time

$$\Psi_0 = E_0^Q\left[\int_0^\tau \frac{V_t * (r_t - d_t)}{B_t}\partial t\right]. \tag{7}$$

Jarrow and van Deventer use a model for the deposit rate, $d_t$, that has a short market rate as the driving factor. They describe a model in discrete time and a similar model in continuous time. The model in discrete time is

$$\Delta d_t = d_{t-1} + \beta_0 + \beta_1 r_t + \beta_2(r_t - r_{t-1}) \tag{8}$$

or equivalently

$$d_t = d_0 + \beta_0 t + \beta_1 \sum_{j=0}^{t-1} r_{t-j} + \beta_2 (r_t - r_0)\,. \tag{9}$$

In continuous time the model is

$$\partial d_t = (\beta_0 + \beta_1 r_t)\partial t + \beta_2 \partial r_t \tag{10}$$

or

$$d_t = d_0 + \beta_0 t + \beta_1 \int_0^t r_s\, \partial s + \beta_2 (r_t - r_0) \tag{11}$$

where $\beta_0$, $\beta_1$ and $\beta_2$ are parameters that should be estimated.

Kalkbrener and Willing (2004) present a model that is based on the three components market rates, deposit rates and deposit volumes. The model is used to value NMDs and to handle interest rate and liquidity risk. The market rates are modeled with a non-parametric two-factor HJM model (see Appendix 1). Kalkbrener and Willing find that deposit volumes and market rates do not have a high correlation and therefore introduce a new stochastic factor for deposit volumes. The deposit volume is modeled with a linear trend term and an Ornstein-Uhlenbeck process, where the Ornstein-Uhlenbeck process is driven by the two Wiener processes that drives the market rate model together with a third Wiener process. Deposit rates are modeled in the same way as suggested by Jarrow and van Deventer (1998).

### 3.1.3 Asymmetric Partial Adjustment Models

O'Brien (2000) models the adjustments in deposit rates with

$$\Delta d_t = \left(\lambda^+ \mathbf{1}_{\{d_t^e > d_{t-1}\}} + \lambda^- \mathbf{1}_{\{d_t^e < d_{t-1}\}}\right)(d_t^e - d_{t-1}) + e_t \tag{12}$$

where $d_t^e = \beta_0 + \beta_1 r_t$, is an equilibrium rate that the deposit rate adjusts to, where $\beta_0$ and $\beta_1$ are parameters to estimate. The indicator function $\mathbf{1}_{\{d_t^e > d_{t-1}\}}$ is 1 if $d_t^e > d_{t-1}$ and 0 otherwise. The parameters $\lambda^+$ and $\lambda^-$ represent the adjustment speed when interest rates increase and decrease, respectively. The use of $\lambda^+$ and $\lambda^-$ makes the model take into account the asymmetric adjustment speed of the deposit rate. The equality $\lambda^+ = \lambda^-$ indicates symmetric adjustments. Finally, *e* is a zero-mean random disturbance. This model for the deposit rate is similar to the linear model suggested by Elkenbracht and Nauta (2006) with the difference being the asymmetry given by the difference between $\lambda^+$ and $\lambda^-$ and the partial adjustment that comes from $\lambda^+$ and $\lambda^-$ being in the interval [0,1].

The deposit volume, $V_t$, is modeled by

$$\log(V_t) = \alpha_0 + \alpha_1 (r_t - d_t) + \alpha_2 \log(i_t) + \alpha_3 \log(V_{t-1}) + v_t. \tag{13}$$

The volume depends on the opportunity cost, $r_t - d_t$, and a measure of income, $i_t$. The measure of income used by O'Brien (2000) is taken either from data interpolated from quarterly national income or from interpolated annual income at each bank in the study. This model also contains a zero-mean random disturbance, $v_t$. The market rate, $r_t$, is modeled by a one factor CIR model (see Appendix 1). The value of the NMD is then calculated as the discounted future cash flows.

### 3.1.4 Static Replicating Portfolio

A commonly used approach to model NMDs is to use a static replicating portfolio. The aim of this approach is to reduce the complexity of the NMD into a portfolio of simpler instruments that hopefully have similar characteristics as the NMD. The idea is to mimic the evolution of the deposit rate of the NMD using a bond portfolio. The duration of the NMD is then assumed to be the same as for the replicating portfolio.

Maes and Timmermans (2005) describe a model where the tracking error is minimized, i.e. the standard deviation of the spread between the portfolio return and the deposit rate. As an alternative approach they instead maximize the Sharpe ratio, i.e. the risk adjusted margin. Maes and Timmerman divide the deposit volume into three parts; core deposits, volatile deposits and remaining balance. The core deposit is invested in a long horizon bond (seven years) and the volatile deposit is invested in short horizon bond (one month). The remaining balance is replicated by the portfolio. However, it is unclear how they determine the ratio of the three different parts. The model specification is

$$Min\ std\left(r_t^p - d_t\right)\ or\ Max\ Sharpe\ ratio$$

$$s.t.$$

$$(i) \qquad r_t^p = \sum_{i=1}^{n} \omega_i r_{i,t} \quad for\ \ t = 1, \dots, T$$

$$(ii) \qquad \sum_{i=1}^{n} \omega_i = 1$$

$$(iii) \qquad \omega_i \geq 0 \quad for\ all\ i$$

where $r_t^p$ is the return of the replicating portfolio at time *t*, $\{\omega_i\}_{i=1}^n$ are the weights of the assets in the portfolio with return $\{r_{i,t}\}_{i=1}^n$ at time *t*, $d_t$ is the deposit rate, *n* is the number of bonds with different maturities that are chosen to be in the portfolio and $T$ is the number of periods used in the historical sample. In this model no short sales are allowed and all the weights sum up to 1. The standard deviation, $std\left(r_t^p - d_t\right)$, is estimated as the square root of the sample variance where the entire historical sample is used. The Sharpe ratio is defined as

$$\frac{\overline{r_t^p - d_t}}{std\left(r_t^p - d_t\right)} \tag{14}$$

where $\overline{r_t^p - d_t}$ is the mean of the difference between the portfolio payoff and the deposit rate.

Maes and Timmermans (2005) construct a replicating portfolio using four different assumptions and model specifications and get four quite different results. For instance, the duration varies between 1.6 and 3.7 years. This shows that the model is rather unstable with regards to the assumptions. They use portfolios consisting of assets with 3 month, 6 month, 12 month, 3 year, 5 year and 10 year maturities. Using assets with other maturities would probably give another result.

Bardenhewer (2007) also minimizes the tracking error of the replicating portfolio. However, he takes the volume, $V_t$, of the deposits into account and adds a linear, quadratic or exponential trend function of the volume, $F_t$. He also uses moving averages of the market rates.

$$ma_{i,t} = \frac{1}{N_i} \sum_{j=0}^{N_i - 1} r_{i,t-j} \tag{15}$$

where $ma_{i,t}$ is the moving average of the return of bond *i* at time *t*, *N* is the number of periods of the moving average and corresponds to the maturity time of asset *i* and $r_{i,t}$ is the return of bond *i* at time *t*. The model is specified as

$$Min\ std(r_t^p - d_t)$$

$$s.t.$$

$$(i) \qquad r_t^p = \beta_0 + \frac{F_t}{V_t} \sum_{i=1}^{n} \omega_i ma_{i,t} + \frac{A_t}{V_t} r_{1,t} \quad for\ \ t = 1, \dots, T$$

$$(ii) \qquad \sum_{i=1}^{n} \omega_i = 1$$

$$(iii) \qquad \omega_i \geq 0 \quad for\ all\ i$$

where $A_t$ is the balancing volume $V_t - F_t$ and $r_{1,t}$ is the interest rate of the shortest bond. The portfolio weights are denoted $\omega_i$ and $\beta_0$ is a parameter to be estimated.

The duration of the replicating portfolio is calculated by summing the portfolio weights multiplied by the time to maturity, $t_i$, of the respective bond

$$Duration = \sum_{i=1}^{n} \omega_i * t_i\,. \tag{16}$$

Bardenhewer (2007) also adds a liquidity constraint to the model. The maximum historical volume changes in the different estimation periods are cumulated and compared to the cumulated weights given by the model up to that period. The used weights are chosen so that the maximum cumulated weights are matched. An explanatory example of this approach is shown in Table 2.

**Table 2. Example of liquidity constraint**

| Time buckets | 1 month | 3 month | 6 month | 12 month | 2 year | 5 year | 10 year |
|---|---|---|---|---|---|---|---|
| (1) Optimal weights | 5% | 10% | 25% | 15% | 20% | 0% | 25% |
| (2) Liquidity constraint | 20% | 10% | 5% | 25% | 10% | 30% | 0% |
| (3) Row (1) cumulated | 5% | 15% | 40% | 55% | 75% | 75% | 100% |
| (4) Row (2) cumulated | 20% | 30% | 35% | 60% | 70% | 100% | 100% |
| (5) Final weights | 20% | 10% | 10% | 20% | 15% | 25% | 0% |

The table shows an example of the liquidity constraint suggested by Bardenhewer (2007). Row (1) is the optimal weights given by the model, row (2) is maximum historical volume change over a time period of the given time bucket, row (3) and (4) cumulates row (1) and (2) respectively and row (5) are the final weights given by the difference between the largest value of row (3) and (4) at the current time bucket and the largest value of row (3) and (4) at the time bucket that is one step to the left. If row (5) were to be cumulated it would be a row of the largest values from row (3) and (4).

#### 3.1.5 Ordinal Response Model

Blöchlinger (2015) takes the discretized nature of NMDs into account and suggests an ordinal response model for the deposit rate, i.e. he models deposit rates with a jump process. Firstly he suggests an ordinal decision variable

$$Y_t = X_t^T \beta + \xi_t \tag{17}$$

where $X_t$ is a vector of covariates that are known at time t, $\beta$ is a vector of parameters that are estimated using maximum likelihood and $\xi_t$ is a random effect with cumulative distribution function $F(\cdot)$. Blöchlinger suggests that $\{\xi_t\}$ should be independent and identically distributed and follow a logistic distribution. The deposit rate is then modeled by

$$d_t = d_{t-1} + \sum_{k=1}^{K} \Delta_{\mathrm{k}} \mathbf{1}_{\{\theta_{k-1} < Y_t \leq \theta_k\}} = d_{t-1} + \sum_{k=1}^{K} \Delta_{\mathrm{k}} \mathbf{1}_{\{\theta_{k-1} < X_t^T \beta + \xi_t \leq \theta_k\}} \tag{18}$$

where $\{\theta_k | k = 0, \ldots, K\}$ are cutoff values, $\theta_0 = -\infty$ and $\theta_K = +\infty$ , $\{\Delta_k | k = 1, \ldots, K\}$ are the jump sizes, which for instance could be multiples of 0.1 % and $K$ is the number of jump sizes including no jump. This gives the conditional probability function

$$\boldsymbol{P}(d_t = d_{t-1} + \Delta_k | X_t) = \boldsymbol{P}(\theta_{k-1} < Y_t \leq \theta_k | X_t) = F(\theta_k - X_t^T \beta) - F(\theta_{k-1} - X_t^T \beta) \tag{19}$$

and expected value

$$\boldsymbol{E}[d_t | d_{t-1}, X_t] = d_{t-1} + \sum_{k=1}^{K} \Delta_{\mathrm{k}} \Big( F(\theta_k - X_t^T \beta) - F(\theta_{k-1} - X_t^T \beta) \Big). \tag{20}$$

The covariates $X_t$ are chosen to capture the characteristics and optionality present in NMDs. Blöchlinger suggests that the ordinal decision variable could be written

$$Y_t = \beta_1 d_{t-1} + \beta_2 L_{3,t} + \beta_3 C_{60,t} + \beta_4 \mathbf{1}_{\{d_{t-1} < 0.005\}} + \beta_5 \mathbf{1}_{\{L_{3,t} > 0.07\}} + \beta_6 |L_{3,t} - L_{3,t-1}| + \xi_t \tag{21}$$

where $L_{3,t}$ is the three month LIBOR rate at time $t$, $C_{60,t}$ is the five year swap rate at time $t$ and $\mathbf{1}_{\{\cdot\}}$ is an indicator function.

The parameters $\theta$ and $\beta$ are estimated using Fisher's scoring algorithm (see Appendix 2). Using the log-likelihood function

$$l(\theta,\beta) = \sum_{t=1}^{T}\sum_{k=1}^{K} \mathbf{1}_{\{i_t = i_{t-1}+\Delta_k\}} \log\Big(F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)\Big) \tag{22}$$

Blöchlinger derives the score vector to be

$$\begin{aligned} \frac{\partial l(\theta,\beta)}{\partial \beta} &= \sum_{t=1}^{T}\sum_{k=1}^{K} -\mathbf{1}_{\{i_t = i_{t-1}+\Delta_k\}} \frac{f(\theta_k - X_t^T\beta) - f(\theta_{k-1} - X_t^T\beta)}{F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)} X_t \\ \frac{\partial l(\theta,\beta)}{\partial \theta_l} &= \sum_{t=1}^{T}\sum_{k=1}^{K} \mathbf{1}_{\{i_t = i_{t-1}+\Delta_k\}} \frac{\mathbf{1}_{\{k=l\}} f(\theta_k - X_t^T\beta) - \mathbf{1}_{\{k-1=l\}} f(\theta_{k-1} - X_t^T\beta)}{F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)} \end{aligned} \tag{23}$$

and Fisher information matrix to be

$$\begin{aligned} E\left[\frac{\partial l(\theta,\beta)}{\partial \beta}\frac{\partial l(\theta,\beta)}{\partial \beta^T}\right] &= \sum_{t=1}^{T}\sum_{k=1}^{K} \frac{\big(f(\theta_k - X_t^T\beta) - f(\theta_{k-1} - X_t^T\beta)\big)^2}{F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)} X_t X_t^T \\ E\left[\frac{\partial l(\theta,\beta)}{\partial \theta_n}\frac{\partial l(\theta,\beta)}{\partial \theta_l}\right] &= \mathbf{1}_{\{n=l\}} \sum_{t=1}^{T}\sum_{k=1}^{K} \frac{\Big(\mathbf{1}_{\{k=l\}} f(\theta_k - X_t^T\beta) - \mathbf{1}_{\{k-1=l\}} f(\theta_{k-1} - X_t^T\beta)\Big)^2}{F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)} X_t X_t^T \end{aligned} \tag{24}$$

with the cross-product terms

$$\begin{gathered} E\left[\frac{\partial l(\theta,\beta)}{\partial \beta}\frac{\partial l(\theta,\beta)}{\partial \theta_l}\right] = -\sum_{t=1}^{T}\sum_{k=1}^{K} \frac{(A-B) * \big(\mathbf{1}_{\{k=l\}}A - \mathbf{1}_{\{k-1=l\}}B\big)}{F(\theta_k - X_t^T\beta) - F(\theta_{k-1} - X_t^T\beta)} X_t \\ A = f(\theta_k - X_t^T\beta) \\ B = f(\theta_{k-1} - X_t^T\beta) \end{gathered} \tag{25}$$

where $f(\cdot)$ is the first derivative of $F(\cdot)$. This method assumes that the cumulative distribution function $F(\cdot)$ is differentiable and could for instance be the logistic cumulative distribution function, which is used by Blöchlinger.

Blöchlinger models logarithmic deposit volumes with an autoregressive process and the short rate as a time-heterogeneous process. He then suggests a valuation formula for NMDs and a hedging approach using deltas and vegas.

This approach to modelling deposit rates is similar to the friction models described by Paraschiv (2011). Paraschiv points out some problems that apply both to the friction models and the ordinal response model. One problem is that it requires large samples, since jumps in deposit rates do not occur very frequently and a sufficient number of jumps are required to obtain a significant result. Another drawback is that a large number of parameters have to be estimated by a complex non-linear optimization problem.

### 3.1.6 Dynamic Replicating Portfolio

Frauendorfer and Shürle (2007) formulates a dynamic replicating portfolio for NMDs through a multistage stochastic optimization. The result of this method is a dynamic replicating bond portfolio that is meant to hedge the NMD. The objective function in the optimization they use is to minimize $\int_{\Omega}\sum_{t=0}^{T} x_t^M \, dP(\omega)$, under several constraints, where $x_t^M$ is the negative surplus (or loss) at time *t*. The vector stochastic process ω contains three stochastic factors and is generated using a scenario tree. The control variables are assumed to follow a multivariate normal distribution. A scenario tree means that values are generated in time steps. From each value created in one time step, new values are created in the next time step, creating a tree of scenarios. These stochastic factors drive the processes for the market rates, deposit rates and deposit volumes.

The method models the evolution of the market rates using a two factor extended Vasicek model (see Appendix 1). Frauendorfer and Shürle models the deposit rate similarly to Blöchlinger's (2015) ordinal response model described above. They introduce a control variable that is driven by the stochastic factors that were generated by the scenario tree. Depending on the control variable and some threshold values the deposit rate makes a jump from one time period to the other. The allowed deposit rate increments are chosen and includes 0 for no jump. The deposit volume, $V_t$, is modeled with a positive linear trend and the stochastic variables, $\eta_1$, $\eta_2$ and $e_t$, that are generated by the scenario tree. The evolution of the deposit volume is modeled by

$$\ln V_t = \ln V_{t-1} + \beta_0 + \beta_1 t + \beta_2 \eta_{1,t} + \beta_3 \eta_{2,t} + e_t. \tag{26}$$

The model becomes very complex due to the fact that many parameters have to be estimated and due to the generation of a scenario tree.

### 3.1.7 Other Models

This section briefly mention some of the other approaches to modeling NMDs.

Sheehan (2013) firstly separates retained deposit volumes from total deposit volumes. Retained volumes are defined as the total volume in the accounts that are open at the beginning of the sample period, while the total volume is the total volume in all accounts. Retained volumes, total volumes and market rates are forecasted using a vector autoregressive system of equations. The system contains $3i$ equations, where $i$ is the number of different account types in the bank. Each equation contains at least 7 parameters that should be estimated. This leads to that a large number of parameters are supposed to be estimated.

Hutchison and Pennacchi (1996) values the rents earned by banks in NMDs. They assume that a rational bank would at all times set deposit rates so that the expression

$$(r_t - d_t - c_t)V_t \tag{27}$$

is maximized, where $r_t$ is the market rate, $c_t$ is the cost of issuing the deposit for the bank and $V_t$ is the volume of the deposit. Function (27) would be maximized by

$$d_t = r_t - c_t - \frac{V_t}{\partial V / \partial d}. \tag{28}$$

In perfect competition the elasticity $\partial V / \partial d$ would go towards infinity and make the deposit rate equal to $r_t - c_t$.

Paraschiv and Schürle (2010) formulates a model for deposit rates and for the deposit volumes. The purpose of their study is to investigate the dynamics of deposit rates and for the deposit volumes. The specification for the movement in the deposit rates are

$$\Delta d_t = \beta_0 + \beta_1 \Delta d_{t-1} + \beta_2 \Delta r_{t-1} + \beta_3 \Delta r_{t-1}^{long} + \beta_4 EC_{t-1} + e_t \tag{29}$$

where $\Delta r_t^{long}$ is the change in a long market rate at time $t$, $EC_t$ is the error correction term at time $t$ and $e_t$ is a residual term. The purpose of the error correction term is to make the process go towards a long-run equilibrium. They also experiment with adding a threshold value. For instance, if $\Delta r_{t-1}$ is above a certain threshold they might add $\beta_5 \Delta r_{t-1}$ to (29).

De Jong and Wielhouwer (2003) also formulate a model with error correction for the deposit rate. Their model is

$$\partial d_t = \beta_1 (r_t - \beta_0 - d_t) \partial t + \sigma \partial W_t \tag{30}$$

where $r_t - \beta_0$ is the long-run value and $W_t$ is a Wiener process. The purpose of this study is to measure interest rate risk with duration.

Dewachter et al. (2006) suggests a multi-factor model for valuation and risk management of NMDs. The value is the expectation of discounted future cash flows. Deposit rates are modeled as depending on the term structure and on a spread factor. The short rate is modeled as the sum of N latent term structure factors $\{f_{i,t}\}_{i=1}^{N}$

$$r_t = \sum_{i=1}^{N} f_{i,t}. \tag{31}$$

The deposit rates are modeled with the same latent factors and a spread factor. The dynamics of the deposit volume is modeled with the difference between the deposit rate at time *t* and a constant withdrawal rate, *w*. The model is

$$dV_t = (r_t - w) V_t dt. \tag{32}$$

## 3.2 What Characterizes a Good Model?

To be able to evaluate the models it is necessary to determine what it is that characterizes a good model. This can be done in several ways and it is up to the reader to decide what is most important. In this thesis the following criteria are assumed to represent a good model:

- **Goodness of fit**
  The model should have a good fit with the historical sample.
- **Stability**
  The model should be stable with regards to assumptions and depending on what historical sample is being used to calibrate the model.
- **Negative interest rate environment**
  The model should be able to handle a negative interest rate environment.
- **Simplicity**
  The model should be possible to be implement in a realistic way. If it is too simple it might not be usable because of bad performance, but if it is too complex it will not be used either.

# 4. New Models

*As a contribution to the research on NMDs this section presents three new models for the deposit rates of NMDs. The purpose is to find a model that fulfills the criterions stated in Section 3.2. In Section 4.1, a motivation for the models and some mathematical preliminaries are given. Thereafter, the three models are described in detail.*

## 4.1 Motivation and Mathematical Preliminaries

In recent years, market rates have been at a historically low level. Although market rates have been negative, the extremely low rates have not been passed on to the customers. Instead the deposit rates have been floored at zero. Therefore, the new models will assume that deposit rates will not be lower than zero. This will simply be done using a maximum function

$$d_t = \max(predicted\ d_t\ , 0). \tag{33}$$

Since deposit rates have been floored at zero despite the fluctuations of the negative market rates, it is not possible to draw any conclusions regarding the behavior of the deposit rates in a positive interest environment from just looking at them during a negative interest environment. Therefore, the periods where the deposit rates are zero are excluded from the historical sample.

In the model the deposit rate is driven by a short market rate. This is due to the observation that deposit rates follow the movement of short-rates rather well. However, since deposit rates are usually much less volatile than market rates, the model will use moving averages, $ma_t$, of the short market rate,

$$ma_t = \frac{1}{N}\sum_{j=0}^{N-1} r_{t-j} \tag{34}$$

where $N$ is the number of days for the moving average. The length of the moving average will be approximately one month, or more precisely 21 trading days. This will reduce some of the spikes that occur in the short-rates that are not passed on to the deposit rates.

## 4.2 First Model

In the first and simplest new model the deposit rate is assumed to follow a linear function of the moving average of the short market rate. The deposit rate is not allowed by the model to be negative. This gives the model for the deposit rate

$$d_t = \max(\beta_0 + ma_t, 0) \tag{35}$$

where $\beta_0$ is a parameter that should be estimated. The parameter calibration is made during the part of the historical sample where the deposit rate is larger than zero. The model implies that the bank wants to have a constant margin $\beta_0$ compared to the reference rate, which is the moving average of the short market rate, but the bank refuses to impose negative deposit rates.

## 4.3 Second Model

The second model takes into consideration the discretized nature of deposit rates and the asymmetric adjustment speed. To discretize the deposit rate process, the model makes the estimated deposit rate change in steps instead of in a continuous manner. The asymmetric adjustment speed is taken into account by using different jump sizes for when interest rates increase and when they decrease. Small jump sizes implies a faster adjustment than large jump sizes. When the deposit rate does jump it is set to the same value as in the first model. The model is specified by

$$d_t = \begin{cases} \max(\beta_0 + ma_t, 0) & if\ ma_t - d_{t-1} > \beta_0 + \theta_{up} \quad or \quad ma_t - d_{t-1} < \beta_0 - \theta_{down} \\ d_{t-1} & otherwise \end{cases} \tag{36}$$

where $\theta_{up}$ is the median jump size when interest rates increase and $\theta_{down}$ is the median jump size when interest rates decrease. Both are taken from a historical sample. The median is used instead of the mean to stop extreme events from having a too large impact on the model.

This is not a jump process, but rather a discretization of a continuous process. This means that the model is not expected to have a better fit than the first model. However, this model is supposed to move in a similar manner to the deposit rate

## 4.3 Third Model

The third model is similar to the first model, but the moving average term is now multiplied by a factor, $\beta_1$. The motivation for this is that the spread between the deposit rate and the reference rate is assumed to be bigger when the rates are higher, which would be true if $\beta_1$ is between zero and one. The model becomes

$$d_t = \max(\beta_0 + \beta_1 ma_t, 0) \tag{37}$$

where $\beta_0$ and $\beta_1$ are parameters that should be estimated.

# 5. Methodology

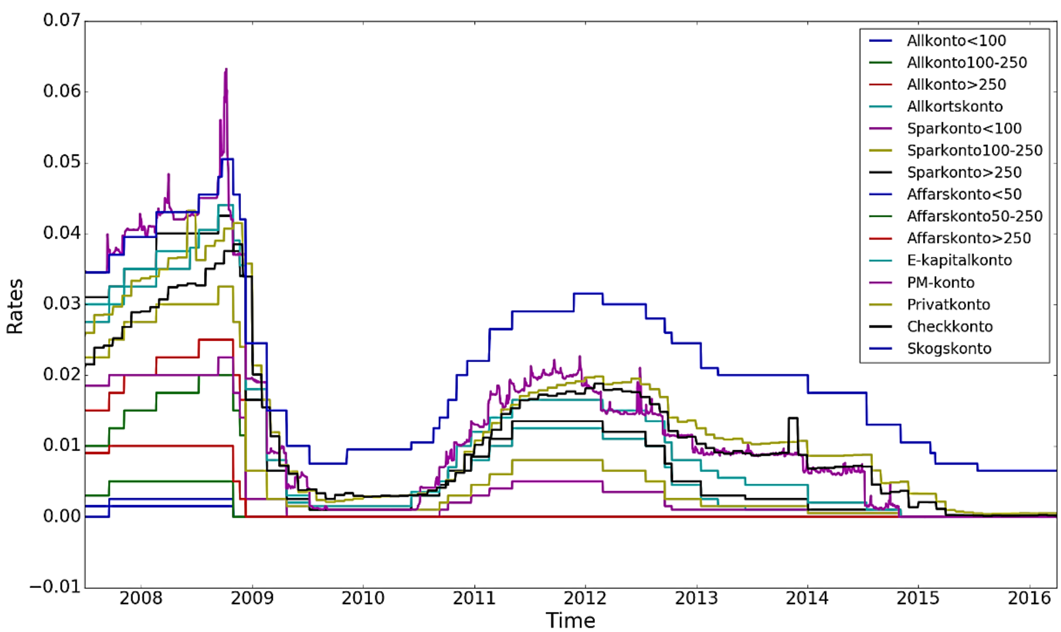
*In this chapter the methodology used in the study is presented. Firstly in Section 5.1, a description of the data is given. Secondly in Section 5.2, a description of the method to compare different models is presented. Finally in Section 5.3, there is a summary of the different models that will be compared. The full model specifications are given in Chapter 3 and Chapter 4.*

## 5.1 Description of the Data

Most of the data that is being used has been provided by Handelsbanken. The data consists of 1 week, 1 month, 3 month, 6 month, 1 year, 2 year, 3 year, 4 year and 5 year swap rates and deposit rates from 10 different NMD types. Three of the accounts are divided into three categories depending on the amount of money the customer has in the deposit. The data consists of daily rates from 06-29-2007 to 03-31-2016. The historical deposit rates for the different account types are shown in Figure 1. The account types are described below.

Daily interest rates of the Stockholm Interbank Offer Rate (STIBOR) and the Swedish Repo rate is retrieved from Sweden's central bank (Riksbanken). STIBOR rates are overnight, 1 week, 1 month, 2 month, 3 month and 6 month rates.

**Figure 1. Historical deposit rates**




The figure shows the historical deposit rates of the different account types at Handelsbanken.

**Allkonto** – This is an account with the purpose of being a transactional account with a low deposit rate for retail customers. This account type is divided into three different categories depending on the volume (less than 100 kSEK, between 100 kSEK and 250 kSEK and more than 250 kSEK). Historically, the more that is in the deposit, the higher the deposit rate is. However, since the financial crisis in the year 2008 the deposit rates have been floored at zero for all the categories. The three categories have recently been merged into one.

**Allkortskonto** – This account is connected to the Allkort, which is a credit card that allows for 45 days free credits and bonuses on purchases. For volumes below 100 000 SEK the account has a deposit rate that usually is higher than for the Allkonto, although it currently is zero. For the part of the volume that is above 100 000 SEK the deposit rate is zero.

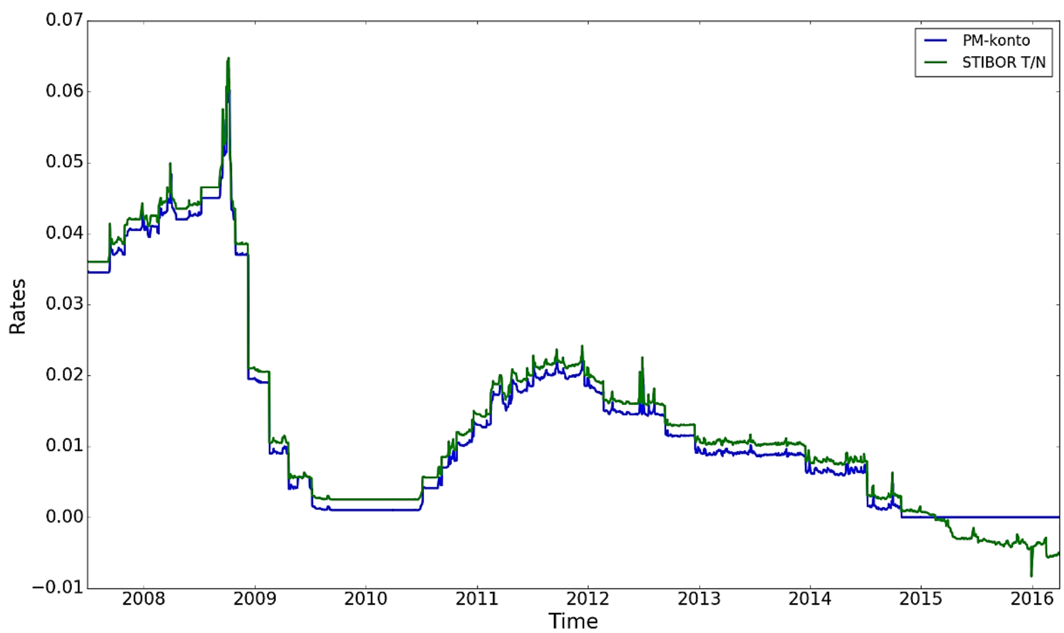
**Sparkonto** – This is a savings account for retail customers with a deposit rate that is usually higher than for the Allkonto. This account type also has the three categories, with the same limits, as the Allkonto.

**Affärskonto** – This is a transactional account for businesses. It has historically provided the lowest deposit rate of the ones presented here, however, since the financial crisis in the year 2008 the deposit rates have been floored at zero. This account type also has three categories just as the two previous account types, but with other limits (less than 50 kSEK, between 50 kSEK and 250 kSEK and more than 250 kSEK). Just as for the Allkonto, the three categories have recently been merged into one.

**E-kapitalkonto** – This account type is similar to the Sparkonto. Historically the E-kapitalkonto has had a somewhat higher deposit rate than the Sparkonto, though they currently both have a deposit rate at zero. The deposit rate is only paid if the E-kapitalkonto has a volume of at least 100 000 SEK.

**PM-konto** – This is a money market account that is used by businesses, especially large businesses, for transactional or investment purposes. The deposit rate is set in relation to the STIBOR T/N (overnight) rate. However, the deposit rate has remained at zero even though STIBOR T/N has been negative, as seen in Figure 2. This is evidence that Handelsbanken is reluctant to use negative deposit rates.

**Figure 2. Deposit rate of the PM-konto and STIBOR T/N**




The figure shows the historical deposit rate of the PM-konto together with STIBOR T/N, which is the reference rate of this account. Especially interesting is the right part of the graph, where the reference rate is negative, but the deposit rate is floored at zero. Source: Sweden's Central Bank (2017).

**Privatkonto** – This account is used by retail customers both for savings and transactions. The deposit rates are set individually for each customer. In this thesis, aggregated data is used for these accounts.

**Checkkonto** – This account is similar to the Privatkonto, but for businesses. The deposit rates are set individually. In this thesis aggregated data is used.

**Skogskonto** – This account type has the purpose to smooth out incomes from foresting. The reason to have this type of account is to avoid large taxes the year that the timber is sold. The account has a rather high deposit rate compared to the other account types.

## 5.2 Evaluation Methods

To evaluate the different models, the four different aspects described in Section 3.2 are considered. These are goodness of fit, stability, negative interest rate environment and simplicity.

### 5.2.1 Goodness of Fit

The goodness of fit will be measured with the coefficient of determination, $R^2$. The definition of $R^2$ is

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \tag{38}$$

where the total sum of squares, $SS_{tot}$, is defined as

$$SS_{tot} = \sum_{t=0}^{T} \left(d_t - \bar{d}\right)^2 \tag{39}$$

$$\bar{d} = \frac{1}{T} \sum_{t=0}^{T} d_t \tag{40}$$

and the residual sum of squares, $SS_{res}$, is defined as

$$SS_{res} = \sum_{t=0}^{T} (actual\ d_t - estimated\ d_t)^2 \tag{41}$$

A high $R^2$ means a good fit. For a perfect fit $R^2$ is equal to 1.

### 5.2.2 Stability

The model should be stable with regards to assumptions and which historical sample is being used. To test the stability of the models, an out-of-sample analysis will be conducted. That means that parameters will be estimated in one time period and evaluated in another. Also, different input assumptions will be tested for the models where that is applicable.

### 5.2.3 Negative Interest Rate Environment

Most models do not pay any attention to negative interest rates. This is probably a consequence of the fact that most literature on the subject is at least a few years old, while negative interest rates are a rather new phenomena. The handling of a negative interest rate environment will be evaluated by looking at the errors when the short market rate is negative in the historical sample.

### 5.2.4 Simplicity

To evaluate the simplicity of a model is a somewhat subjective task. Therefore, it is inevitable that the opinion of the author will be reflected in this evaluation. The simplicity will be evaluated by looking at how easy the model is to implement, how intuitive the outlines of the model are, how it converges (if applicable) and the time it takes to calibrate the parameters.

## 5.3 Choice of Models

The following models will be evaluated:

A. Linear models
   1. $d_t = \beta_1 r_t$ , as suggested by Nyström (2008)
   2. $d_t = \beta_0 + \beta_1 r_t$, as suggested by Elkenbracht and Nauta (2006)
   3. $d_t = \beta_0 + \beta_1 * average\ of\ 1\ M\ swap\ rate\ over\ the\ last\ 6\ months + \beta_2 * average\ of\ 5Y\ swap\ rate\ over\ the\ last\ 6\ months$ , as suggested by Bardenhewer (2007)
   4. $d_t = \beta_0 + \beta_1 * average\ of\ 1\ M\ swap\ rate\ over\ the\ last\ 1\ month + \beta_2 * average\ of\ 5Y\ swap\ rate\ over\ the\ last\ 1\ month$, (almost) as suggested by Bardenhewer (2007)

B. Jarrow and van Deventer (1998):
   1. $d_t = d_0 + \beta_0 t + \beta_1 \sum_{j=0}^{t-1} r_{t-j} + \beta_2(r_t - r_0)$

C. Asymmetric partial adjustment models as suggested by O'Brien (2000):
   1. $\Delta d_t = \left(\lambda^+ \mathbf{1}_{\{d_t^e > d_{t-1}\}} + \lambda^- \mathbf{1}_{\{d_t^e < d_{t-1}\}}\right)(d_t^e - d_{t-1}) + e_t$

D. Static replicating portfolio
   1. Model suggested by Maes and Timmermans (2005):

$$Min\ std\left(r_t^p - d_t\right)$$
$$s.t.$$
$$(i) \qquad r_t^p = \sum_{i=1}^{n} \omega_i r_{i,t} \quad , for\ \ t = 1, \ldots, T$$
$$(ii) \qquad \sum_{i=1}^{n} \omega_i = 1$$
$$(iii) \qquad \omega_i \geq 0 \quad for\ all\ i$$

   2. Model suggested by Maes and Timmermans (2005), but with moving averages of the market rates:

$$Min\ std\left(r_t^p - d_t\right)$$

$$s.t.$$

$$(i) \qquad \sum_{i=1}^{n} \omega_i ma_{i,t} = r_t^p, for\ \ t = 1, \dots, T$$

$$(ii) \qquad \sum_{i=1}^{n} \omega_i = 1$$

$$(iii) \qquad \omega_i \geq 0\ for\ all\ i$$

$$(iv) \qquad ma_{i,t} = \frac{1}{N_i} \sum_{j=0}^{N_i - 1} r_{i,t-j}$$

E. Ordinal response model as suggested by Blöchlinger (2015):

$$d_t = d_{t-1} + \sum_{k=1}^{K} \Delta_\mathrm{k} \mathbf{1}_{\{\theta_{k-1} < X_t^T \beta + \xi_t \leq \theta_k\}}$$

1. $X_t = \left[d_{t-1}, L_{3,t}, C_{60,t}, \mathbf{1}_{\{d_{t-1} < 0.005\}}, \left|L_{3,t} - L_{3,t-1}\right|\right]^T$
2. $X_t = \left[d_{t-1}, r_t, \mathbf{1}_{\{d_{t-1} < 0.005\}}\right]^T$

F. New models:

1. $d_t = \max(\beta_0 + ma_t, 0)$
2. $d_t = \begin{cases} \max(\beta_0 + ma_t, 0) & if\ ma_t - d_{t-1} > \beta_0 + \theta_{up}\ \ or\ \ ma_t - d_{t-1} < \beta_0 - \theta_{down} \\ d_{t-1} & otherwise \end{cases}$
3. $d_t = \max(\beta_0 + \beta_1 ma_t, 0)$

See Chapter 3 and Chapter 4 for detailed model specifications. From here on the models will be denoted with the letter and number they are given above, for instance B1 for the Jarrow and van Deventer model.

## 5.4 Valuation

To go one step further in the study of NMDs, a simple valuation of the NMDs will be conducted. For this a simplified version of the framework suggested by Jarrow and van Deventer (1998) will be used (see Section 3.1.2). The simplification is to assume that the volumes of the NMDs are constant. The valuation then simply becomes

$$\Psi_0 = E_0^Q \left[\sum_{t=0}^{\tau-1} \frac{V_t * (r_t - d_t)}{B_{t+1}}\right] = E_0^Q \left[\sum_{t=0}^{\tau-1} \frac{(r_t - d_t)}{B_{t+1}}\right] * V \qquad (42)$$

where $V$ is the constant volume. In this valuation, the end date, $\tau$, will be in 10 years and 50 years, respectively. After the end date the volumes of the NMDs is assumed to be zero. The term structure will be modeled by the Vasicek model (see Appendix 1).

One reason to value NMDs could be to determine whether or not a certain deposit rate strategy is good or bad for the bank. If the valuation is low, the bank may want to consider another strategy for the deposit rate.

# 6. Results

*In this chapter the findings from the study is presented. First of all the goodness of fit of the different models and for the different accounts is shown. Then results from each model is presented. For convenience, plots will only display the results when using the Sparkonto >250. Lastly, the result from a simple valuation of the NMDs is presented for some of the models.*

## 6.1 Goodness of Fit

In this section two tables of the goodness of fit measured in $R^2$ of the different models and for the different account types are presented.

Table 3 shows the goodness of fit when the whole historical sample is used. Table 4 shows an out-of-sample analysis, where the parameters are estimated using the historical sample from 07-01-2007 to 06-30-2011, while $R^2$ is measured for the historical sample from 07-01-2011 to 06-30-2015. These results for each model are described further in the respective sections below.

**Table 3. Goodness of fit**

| R-squared | A1 | A2 | A3 | A4 | B1 | C1 | D1 | D2 | E1 | E2 | F1 | F2 | F3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Allkonto <100** | 0.633 | 0.701 | 0.593 | 0.691 | 0.860 | 0.702 | -580 | -290 | - | - | -4.15 | -0.182 | 0.874 |
| **Allkonto 100-250** | 0.677 | 0.754 | 0.655 | 0.748 | 0.765 | 0.755 | -1.38 | -0.357 | - | 0.665 | 0.987 | 0.974 | 0.987 |
| **Allkonto >250** | 0.685 | 0.760 | 0.665 | 0.755 | 0.789 | 0.761 | -0.122 | 0.299 | - | 0.560 | 0.986 | 0.976 | 0.987 |
| **Allkortskonto** | 0.920 | 0.932 | 0.864 | 0.933 | 0.923 | 0.939 | 0.868 | 0.911 | 0.824 | 0.772 | 0.973 | 0.969 | 0.982 |
| **Sparkonto <100** | 0.860 | 0.893 | 0.780 | 0.876 | 0.956 | 0.894 | -0.269 | 0.296 | - | - | 0.602 | 0.592 | 0.962 |
| **Sparkonto 100-250** | 0.889 | 0.913 | 0.812 | 0.903 | 0.835 | 0.914 | 0.633 | 0.775 | - | 0.448 | 0.893 | 0.902 | 0.980 |
| **Sparkonto >250** | 0.860 | 0.893 | 0.780 | 0.876 | 0.958 | 0.948 | 0.907 | 0.931 | 0.592 | 0.661 | 0.982 | 0.976 | 0.986 |
| **Affärskonto <50** | 0.551 | 0.618 | 0.503 | 0.591 | 0.318 | 0.621 | -237 | -118 | - | - | -0.496 | -0.550 | -0.060 |
| **Affärskonto 50-250** | 0.631 | 0.703 | 0.585 | 0.673 | 0.749 | 0.703 | -49.9 | -24.4 | - | - | 0.320 | 0.261 | 0.893 |
| **Affärskonto >250** | 0.670 | 0.742 | 0.638 | 0.719 | 0.860 | 0.742 | -7.98 | -3.71 | - | - | 0.782 | 0.750 | 0.935 |
| **E-kapitalkonto** | 0.965 | 0.966 | 0.884 | 0.968 | 0.964 | 0.970 | 0.919 | 0.949 | 0.929 | 0.910 | 0.977 | 0.974 | 0.985 |
| **PM-konto** | 0.983 | 0.983 | 0.878 | 0.971 | 0.986 | 0.985 | 0.973 | 0.974 | - | - | 0.988 | 0.988 | 0.988 |
| **Privatkonto** | 0.910 | 0.940 | 0.922 | 0.980 | 0.953 | 0.962 | 0.824 | 0.886 | 0.950 | 0.925 | 0.889 | 0.893 | 0.963 |
| **Checkkonto** | 0.901 | 0.932 | 0.921 | 0.972 | 0.949 | 0.959 | 0.699 | 0.829 | 0.919 | 0.855 | 0.782 | 0.788 | 0.957 |
| **Skogskonto** | 0.505 | 0.923 | 0.823 | 0.955 | 0.965 | 0.927 | 0.824 | 0.847 | 0.580 | 0.873 | 0.834 | 0.859 | 0.925 |

The table shows the goodness of fit measured in $R^2$ for the different models when applied to different NMDs. The historical sample used is from 06-29-2007 to 03-31-2016. Values are missing where the model did not converge.

**Table 4. Goodness of fit, out-of-sample analysis**

| R-squared | A1 | A2 | A3 | A4 | B1 | C1 | D1 | D2 | E1 | E2 | F1 | F2 | F3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Allkonto <100** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Allkonto 100-250** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Allkonto >250** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Allkortskonto** | 0.739 | 0.781 | 0.857 | 0.895 | 0.065 | 0.520 | 0.405 | 0.652 | 0.616 | 0.280 | 0.817 | 0.591 | 0.751 |
| **Sparkonto <100** | 0.792 | 0.817 | 0.821 | 0.871 | -1.00 | -0.618 | -7.71 | -1.69 | - | - | -0.007 | 0.065 | 0.755 |
| **Sparkonto 100-250** | 0.792 | 0.824 | 0.855 | 0.892 | -1.38 | 0.031 | -1.34 | -0.116 | - | 0.091 | 0.668 | 0.572 | 0.893 |
| **Sparkonto >250** | 0.807 | 0.832 | 0.886 | 0.915 | 0.080 | 0.624 | 0.551 | 0.696 | 0.375 | -1.11 | 0.824 | 0.522 | 0.829 |
| **Affärskonto <50** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Affärskonto 50-250** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **Affärskonto >250** | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **E-kapitalkonto** | 0.877 | 0.888 | 0.939 | 0.949 | 0.851 | 0.869 | 0.799 | 0.891 | -2.52 | 0.197 | 0.958 | 0.884 | 0.898 |
| **PM-konto** | 0.980 | 0.981 | 0.949 | 0.972 | 0.843 | 0.960 | 0.957 | 0.966 | - | 0.056 | 0.986 | 0.986 | 0.989 |
| **Privatkonto** | 0.813 | 0.937 | 0.969 | 0.966 | 0.368 | 0.791 | 0.847 | 0.925 | 0.861 | 0.561 | 0.385 | 0.658 | 0.716 |
| **Checkkonto** | 0.880 | 0.929 | 0.951 | 0.956 | 0.820 | 0.820 | 0.809 | 0.935 | 0.725 | 0.726 | 0.391 | 0.487 | 0.822 |
| **Skogskonto** | 0.268 | 0.945 | 0.969 | 0.969 | 0.725 | 0.499 | 0.938 | 0.931 | 0.674 | 0.770 | 0.033 | 0.423 | 0.485 |

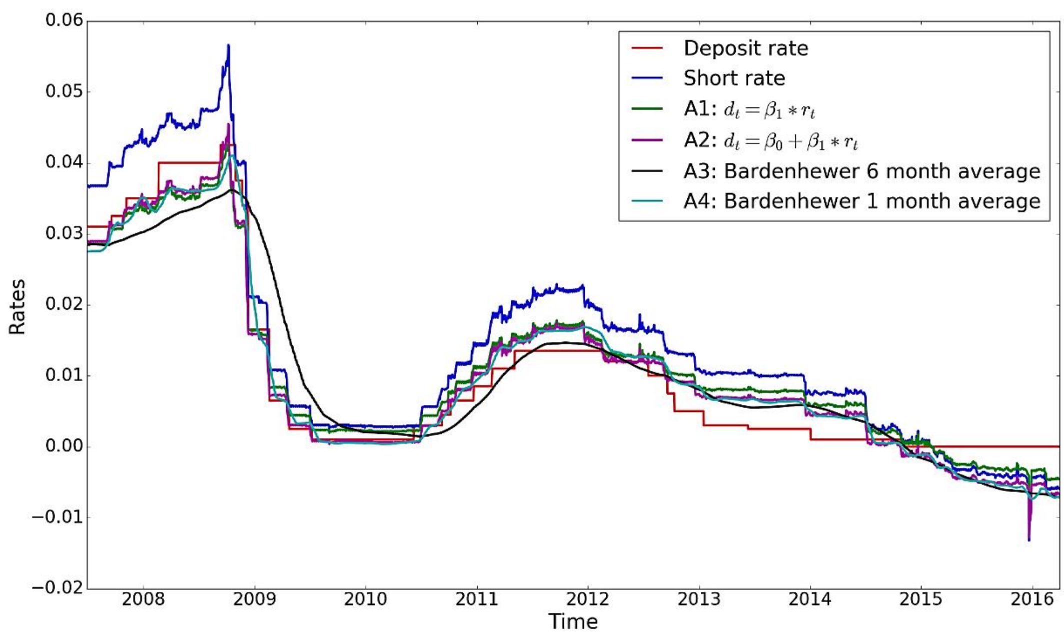
The table shows the goodness of fit measured in $R^2$ in an out-of-sample analysis for the different models when applied to different NMDs. The parameters are estimated using the historical sample from 07-01-2007 to 06-30-2011, while $R^2$ is measured for the historical sample from 07-01-2011 to 06-30-2015. Noteworthy is that the later historical samples used here are more flat than the earlier ones. This lowers the values of $R^2$. When switching the time periods, the value of $R^2$ generally becomes larger. Some of the NMDs are not possible to do this analysis on, since the deposit rate has been zero during the whole period from 07-01-2011 to 06-30-2015. For those NMDs, values are missing. Values are also missing when the model did not converge.

## 6.2 Linear Models

The linear models are very simple to implement and they give a rather good fit. Figure 3 shows the curve fitted with the historical sample of the Sparkonto >250. It shows that the linear model with two parameters (A2) somewhat better matches the deposit rate curve than the one parameter model (A1). This is, of course, no surprise. For some account types the A2 model has a much better fit than the A1 model. The model that was suggested by Bardenhewer (A3) has a rather poor fit. It seems that when using a six month moving average the model reacts too slowly to new events. The observation that the six month moving average is too slow raises the question of how a shorter moving average would work. It is shown that the one month moving average model (A4) has a better fit than the six month moving average model (A3) for almost every account type. For some of the account types the model A4 has a better fit than the model A2, but overall they perform similarly.

Although the models work well in a non-negative interest rate environment, they have a poor fit in a negative interest rate environment. The models blindly follow the reference rate, although the deposit rate is floored at zero. The out-of-sample analysis shows that the linear models are quite stable for different historical samples. The improved Bardenhewer model (A4) is the best model of them all in the out-of-sample analysis.

**Figure 3. Linear models**



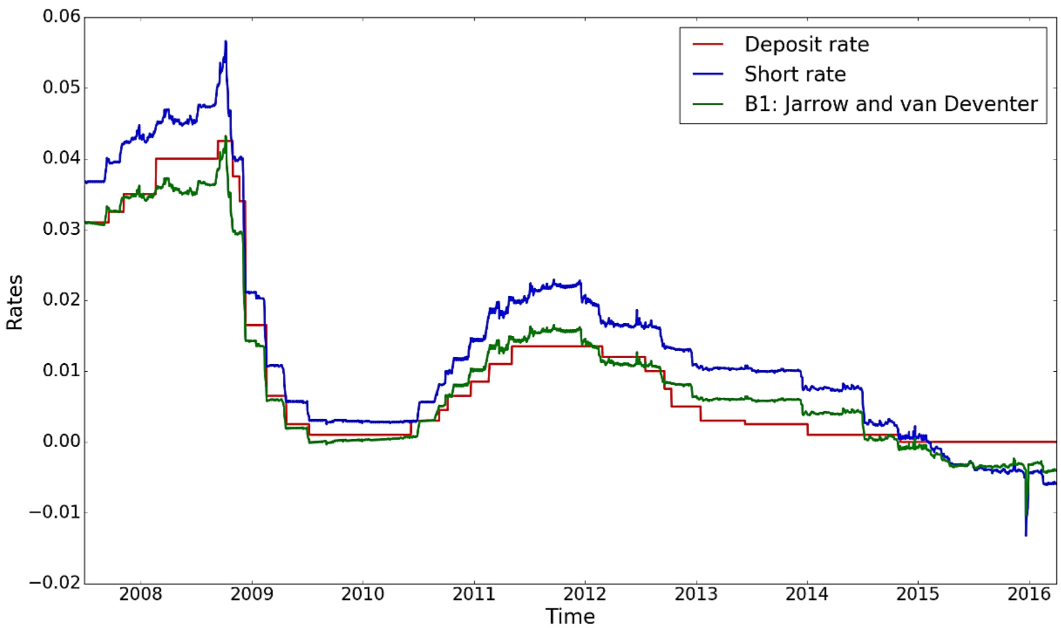

The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the fitted linear models.

## 6.3 Jarrow and van Deventer

The deposit rate model suggested by Jarrow and van Deventer (1998) is rather simple to implement and judging from the performance in the test of goodness of fit it is among the best models in this study. However, the model has the same problem as the linear models when it comes to handling negative interest rate environments. When the deposit rate is floored the model may still predict a negative deposit rate.

The out-of-sample analysis shows the weakness of the model. The model performs quite well a few times. Sometimes, however, it falls far from the real deposit rate. This is something that especially happens when interest rates are low and the deposit rate is zero over a long time period. This issue occurs because an error in the beginning of the prediction period has consequences for the whole prediction. This sensitivity makes the model quite unstable.

**Figure 4. The Jarrow and van Deventer model**

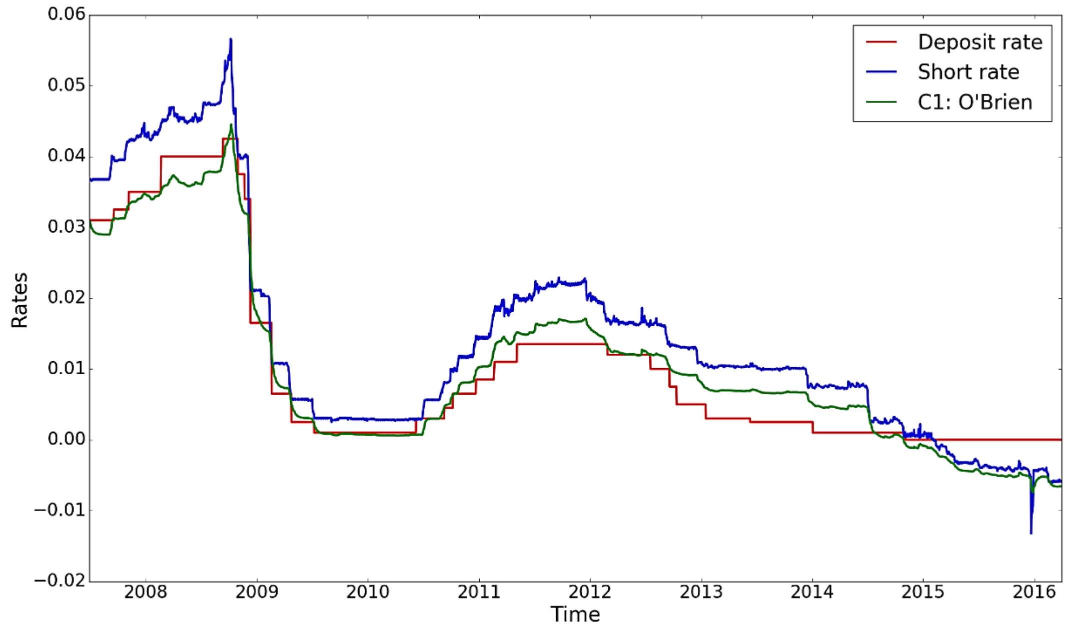



The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the fitted Jarrow and van Deventer model.

## 6.4 Asymmetric Partial Adjustment Model

In Figure 5 the fitted curve of the asymmetric partial adjustment model is shown. The curve moves similarly to the A2 model. The difference is that the asymmetric partial adjustment model moves more smoothly. Although this model is a bit more complex than the linear models, it is still quite simple to implement. The model has the same issues with a negative interest rate environment as the linear models and the Jarrow and van Deventer model. The asymmetric partial adjustment model has a mediocre performance in the test of goodness of fit and in the out-of-sample analysis.

**Figure 5. The asymmetric partial adjustment model**




The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the fitted asymmetric partial adjustment model.

## 6.5 Static Replicating Portfolio

The static replicating portfolio has a good fit for a few of the account types. These account types are the ones with a deposit rate that moves quite a lot. For the account types that have a more flat deposit rate the model works very bad. The model has even bigger problems with a negative interest rate environment than the other models. When interest rates are floored at zero the model fails to replicate the deposit rate.

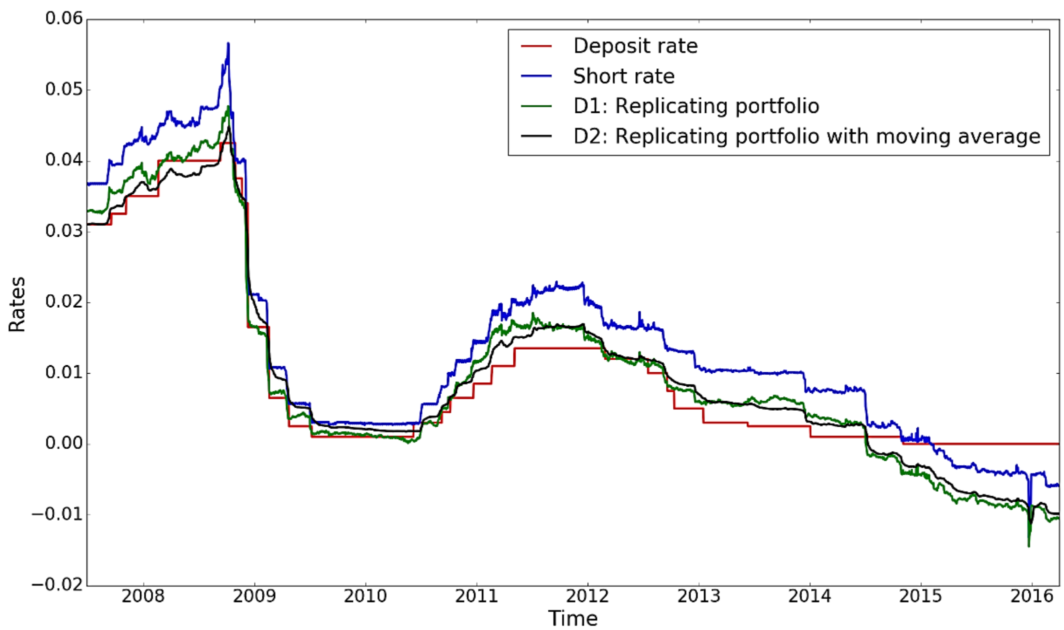
The model is sensitive to the choice of historical sample. This becomes apparent when looking at the duration for the replicating portfolio when calibrating the portfolio weights for different time periods. For instance, the duration of the D1 model for the Skogskonto calibrated over a five year period varies between 0.70 and 12.66 months depending on which time period is chosen. The D2 model is somewhat more stable in regards to which historical sample is used. The duration of the two models and the different account types are shown in Table 5.

**Table 5. Durations according to the static replicating portfolio**

| Duration (months) | **Duration D1** | **Duration D2** |
|---|---|---|
| **Allkonto <100** | 46.99 | 60.00 |
| **Allkonto 100-250** | 33.66 | 44.54 |
| **Allkonto >250** | 28.50 | 36.90 |
| **Allkortskonto** | 11.86 | 16.38 |
| **Sparkonto <100** | 28.27 | 38.08 |
| **Sparkonto 100-250** | 21.96 | 27.48 |
| **Sparkonto >250** | 10.60 | 12.71 |
| **Affärskonto <50** | 46.62 | 60.00 |
| **Affärskonto 50-250** | 45.84 | 60.00 |
| **Affärskonto >250** | 42.40 | 59.66 |
| **E-kapitalkonto** | 9.80 | 13.36 |
| **PM-konto** | 1.09 | 3.62 |
| **Privatkonto** | 6.78 | 16.44 |
| **Checkkonto** | 14.48 | 22.81 |
| **Skogskonto** | 3.32 | 13.10 |

The table shows the calculated durations of the NMDs using the two different replicating portfolios, D1 and D2.

**Figure 6. Static replicating portfolio**




The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the return of the static replicating portfolio with and without moving averages.

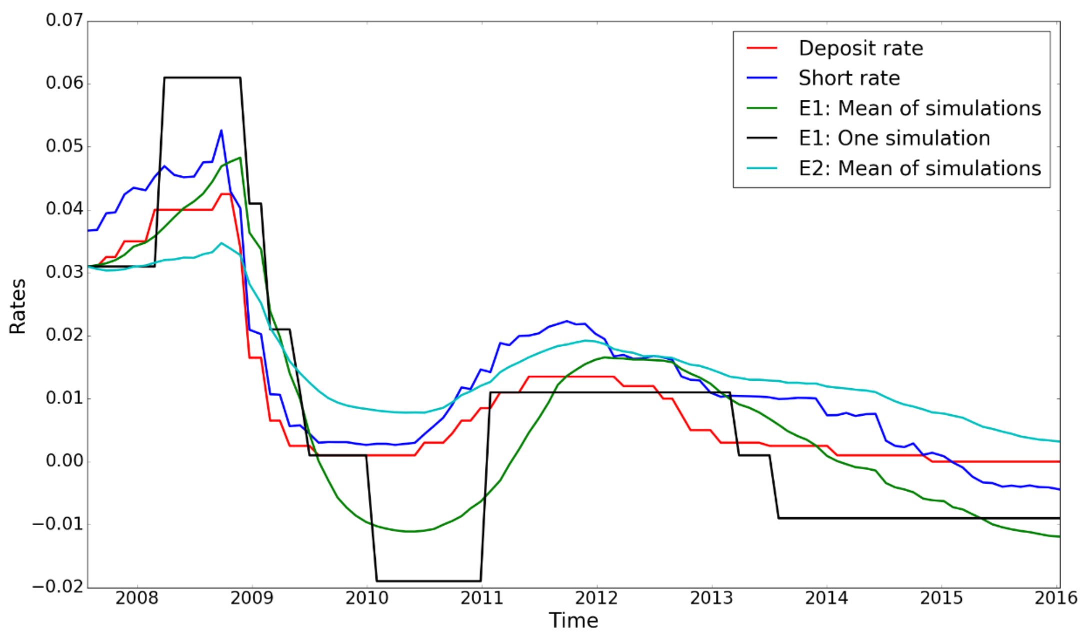
## 6.6 Ordinal Response Model

The ordinal response model suggested by Blöchlinger (2015) almost never converges when using daily data, therefore monthly data is used for this model. The reason that the model does not converge for daily data is probably because there are too many days without jumps in the deposit rate. When using monthly data there is a higher proportion of jumps. For the same reason, the model does not converge for the account types that have been floored at zero for a long time.

To be able to compare the results from other models, deposit rates are simulated over the historical sample period. The mean of the simulations at each point in time are then used to calculate $R^2$. The results from one simulation and the mean of 10 000 simulations are shown in Figure 7.

The model is difficult to implement. Fisher's scoring algorithm, that is explained in Appendix 2 and Section 3.1.5 is a complex method and it is very sensitive to the start values that are used and the model does not converge for every account type. In most cases this is due to the fact that there are too few jumps in the historical sample. To get the model to converge, a large sample is needed and when there are few jumps an even larger sample is needed. The instability of the model is also seen in the bad performance in the out-of-sample analysis.

**Figure 7. The ordinal response model**



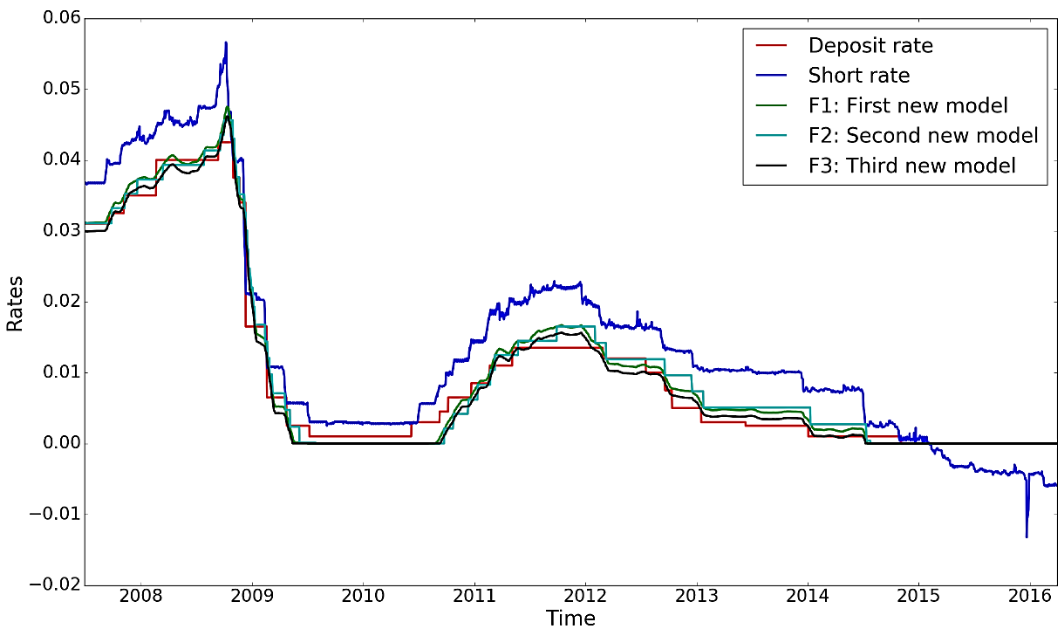

The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the mean of 10 000 simulations from two versions of the ordinal response model (E1 and E2) and one simulation from the model E1.

## 6.7 New Models

The new suggested models for the deposit rate have good performances for most account types. The test for goodness-of-fit shows that the first and the second new models work very well for some of the account types, but work quite poorly for a few of the account types. The third new model, on the other hand, performs among the best for almost every account type. The reason for this is the fact that spreads are different when interest rates are higher.

The out-of-sample analysis shows similar results. It shows that the third new model is quite stable with regards to which historical sample is used for estimating the parameters. The models are rather simple to implement and have an intuitive interpretation. The strength of the models, at least in the used historical sample, is the handling of a negative interest rate environment.

**Figure 8. The new models**




The figure shows the short market rate and the deposit rate of the Sparkonto >250 together with the fitted new models.

## 6.8 Valuation

The three models A2, B1 and F3 are chosen to test the valuation described in Section 5.4. The models A2 and F3 are chosen because of their good performance and B1 is chosen because the valuation formula, just as the model, is taken from Jarrow and van Deventer (1998). The valuation assumes that volumes will be constant over time and that all the money will be withdrawn from the NMDs after 10 and 50 years respectively. The value is taken as the mean of 10 000 simulations of the dynamics of the short rate. The short rate is simulated with a one factor Vasicek model, where the parameters have been estimated using the historical sample and the method described in Appendix A. However, the long term short rate is set to 2 % instead of the estimated long term short rate, since this particular historical sample would give an unrealistically low estimation of the long term short rate. The results from the valuation is presented in Table 6.

**Table 6. Valuation**

| Value (as % of volume) | **A2 (10 years)** | **A2 (50 years)** | **B1 (10 years)** | **B1 (50 years)** | **F3 (10 years)** | **F3 (50 years)** |
|---|---|---|---|---|---|---|
| **Allkonto <100** | 0.866 | 38.848 | -0.637 | 26.569 | 0.761 | 39.752 |
| **Allkonto 100-250** | 2.735 | 35.036 | -2.276 | -16.371 | 0.645 | 38.837 |
| **Allkonto >250** | 3.382 | 33.986 | -10.606 | -178.401 | 0.732 | 39.199 |
| **Allkortskonto** | 1.995 | 16.173 | -3.375 | -40.598 | -1.629 | 7.452 |
| **Sparkonto <100** | 2.133 | 29.432 | -16.182 | -220.372 | 0.760 | 39.366 |
| **Sparkonto 100-250** | 2.357 | 23.330 | -9.637 | -133.047 | -0.809 | 16.477 |
| **Sparkonto >250** | 1.842 | 13.864 | -7.784 | -71.169 | -2.02 | 4.624 |
| **Affärskonto <50** | 0.825 | 38.570 | 2.381 | 48.474 | 0.602 | 39.553 |
| **Affärskonto 50-250** | 1.264 | 38.422 | -0.811 | 17.925 | 0.747 | 39.573 |
| **Affärskonto >250** | 1.923 | 36.337 | -13.811 | -241.140 | 0.739 | 38.839 |
| **E-kapitalkonto** | 0.568 | 8.636 | -3.357 | -26.092 | -2.840 | -0.427 |
| **PM-konto** | -0.109 | 2.202 | -6.563 | -36.042 | -3.692 | -7.784 |
| **Privatkonto** | -2.599 | -1.823 | 2.527 | 22.882 | -4.439 | -7.004 |
| **Checkkonto** | -2.284 | 2.767 | 7.341 | 54.475 | -3.819 | -2.420 |
| **Skogskonto** | -10.470 | -35.290 | -9.936 | -79.887 | -10.744 | -36.522 |

The table shows the valuation of the different account types and three different models for the deposit rate. In the valuation the volume is assumed to be constant over time and the result presented in the table is the simulated value as percentage of this volume. The valuation is the mean from 10 000 simulations and all the money is assumed to be withdrawn after 10 and 50 years respectively.

# 7. Discussion

*In this chapter the results presented in the previous chapter is discussed. The discussion is based on the four aspects described in Section 3.2; goodness of fit, stability, negative interest rate environment and simplicity. Further, the valuation technique suggested by Jarrow and van Deventer (1998) and the duration calculation of the static replicating portfolio is discussed. The discussion is ended with a link to Chapter 2 and some comments on the characteristics of NMDs that are described there.*

## 7.1 Goodness of Fit

The goodness of fit varies a lot over the different models and the different account types. The account types with the most flat deposit rates have the lowest $R^2$. This is since $R^2$ measures how good a model fits compared to a horizontal line and a horizontal line has a good fit if the deposit rate is very flat.

The linear models have a rather good and stable goodness of fit for the different account types both for the whole historical sample and in the out-of-sample-analysis. The Jarrow and van Deventer model and the asymmetric partial adjustment model perform similarly to the linear models when the whole sample is used, but show a bad performance for some of the account types in the out-of-sample analysis. The static replicating portfolio has a very bad performance for the account types with a flat deposit rate. This is due to the fact that the portfolio weights have to sum up to one and there are no market rates that have been that flat in the historical sample, even when using moving averages. The static replicating portfolio where moving averages is used is generally slightly better than the one without. The ordinal response model does not converge for every account type, but when it does it performs similarly or worse than the other models.

The new models generally have a good performance in the test of goodness of fit. The third new model performs a lot better than the first and the second new model. This shows that it takes more than just the short market rate and a constant margin to explain the dynamics of the deposit rate. The first new model performs better than the second new model. This is not surprising, since the second model does not add anything that would improve the goodness of fit compared to the first new model. The third new model has the best performance of all models when testing for the goodness of fit over the whole historical sample.

## 7.2 Stability

The out-of-sample analysis shows how stable the different models are with regards to which historical sample is used to calibrate the parameters. In the test it can be seen that the linear models, the static replicating portfolio and the third new model perform similarly in the out-of-sample analysis as in the test for the whole historical sample. Therefore, these models are considered to be stable with regards to which historical sample is used to calibrate the parameters. The other models perform significantly worse in the out-of-sample analysis than when the whole sample is used and are therefore considered to be more unstable.

The static replicating portfolio is quite unstable with regards to different assumptions and inputs. This is described further in Section 7.6. When the parameters in the ordinal response model are calibrated, start values are required as an input. The output of the model depends on which start values are used and the convergence of the model is very unstable for different start values. The model is also highly dependent on what jump sizes are used and which covariates are chosen, which can be seen in the difference between the outputs of the two model specifications presented in this thesis.

## 7.3 Negative Interest Rate Environment

It is clear that the models for the deposit rate suggested in the literature are constructed in a non-negative interest rate environment. They work well when market rates are positive and the deposit rates actually move, but when market rates are negative and deposit rates are floored at zero, the models fail. The last few years, interest rates have been extraordinarily low. This has led to a need for new models for deposit rates, which can handle this type of interest rate environment. The new models suggested in this thesis tackle this issue and they have proven to do it quite well. However, they are based on the rather crude assumption that banks will not pass on the negative interest rates to their customers. Only time will tell if this assumption will hold true over a prolonged time period. Figure 1 and Figure 2 show that at least Handelsbanken is very reluctant to have negative deposit rates.

## 7.4 Simplicity

Most of the models are quite simple both when it comes to interpretation and implementation. The linear models are the most simple as they just follow the short market rate or a combination of the short and long market rate. A moving average of the market rates can be used to smooth out the daily fluctuations that are not passed on to customers. The moving average is shown to be a good tool for the purpose of smoothing out the fluctuations, but when using too long moving averages the models react too slowly to changes in the reference rates.

The new models suggested in this thesis are just variations of the linear models. They also follow a moving average of the short market rate, but are modified in such a way that they do not allow negative deposit rates. This is an intuitive improvement of the linear models. The second new model is a discretization of the first new model. It does not add anything to improve the goodness of fit, stability or the handling of a negative interest rate environment, it is even a bit more complex. The only advantage of the second new model over the first new model is that the trajectories created by the second new model look more like the trajectory of a real deposit rate. The third new model adds a factor in front of the reference rate compared to the first new model. This can be motivated with an assumption that the spread between the reference rate and the deposit rate is larger when interest rates are higher. The good performance of the third new model shows that this is a reasonable assumption.

The static replicating portfolio is also similar to the linear models. The difference is that the static replicating portfolio comes from an investment strategy that is supposed to mimic the NMD. The fact that it comes from an investment strategy restricts the model from having negative portfolio weights (no short sales are allowed) and that the portfolio weights have to sum to one. These restrictions are intuitive for the investment strategy, but reduce the models ability to explain the deposit rate dynamic.

The asymmetric partial adjustment model suggested by O'Brien (2000) takes a linear model and makes the deposit rate move slower. The model also has different adjustment speeds for when interest rates rise and when interest rates fall. This makes the modeling a bit more complex, but not enough to become a big issue. The model suggested by Jarrow and van Deventer (1998) is somewhat different from the other models, since it has a linear trend term depending on time and it takes into account the evolution of the reference rate. There is no clear intuition in modeling the deposit rate like that and it also turns out that the model performs quite poorly.

The most complex model studied in this thesis is the ordinal response model suggested by Blöchlinger (2015). It takes into consideration the discretized nature of deposit rates and it is flexible enough to involve any parameters that might be relevant for a certain NMD. The difficulty of the model is in the implementation of it. The calibration of the parameters takes a rather long time and requires good start values.

## 7.5 Valuation

The results from the valuation of the NMDs differ a lot for the different account types and the different deposit rate models. This fact clearly shows that the value produced by the formula is unreliable. However, the valuation shows an interesting comparison between the account types. For instance, when comparing the different categories of the different categories of the Sparkonto using the linear model and the new model it can be seen that the Sparkonto >250 has the smallest value, which is what is expected, since this category pays the highest deposit rate. The Jarrow and van Deventer model shows a result that is inconsistent with the other models for many of the account types. This may be explained by the bad performance of the model in the out-of-sample-analysis, which makes the model unreliable in the valuation process.

The Skogskonto has the lowest valuation, which is because the valuation compares the deposit rate with a short market rate and since the Skogskonto pays the highest deposit rate it gets the lowest valuation. The problem with this valuation is that the Skogskonto is used as a more long term investment and should probably be compared to a longer market rate. The same problem as for the Skogskonto could to some extent apply to all accounts, since the valuation does not consider the stability of the deposits, but instead compares them to the shortest possible market rate regardless of their expected maturity. Also, the simplification in this thesis does not consider the dynamics of the deposit volume, which is an important aspect in valuing NMDs.

There are two reasons why the valuations over ten years are so low. Firstly, there are a lot fewer years to add to the value when using ten years compared to fifty years. Secondly, the valuation begins in an extremely low interest rate environment, which means that the simulated short rate process needs some time before the short rate is positive. This leads to a negative cash flow in the first years of the valuation.

By looking at the valuations it may be possible to determine which accounts are better or worse for the bank, a high valuation implying that the account is good for the bank. However, such an assessment could only be done for the NMDs standalone. For instance, the value could be completely different if considering how the NMDs affect the customer´s behavior when it comes to the use of other banking services. Such an analysis would be very complex.

## 7.6 Duration of the Static Replicating Portfolio

To calculate the duration of NMDs, many banks use a static replicating portfolio. (Maes and Timmermans (2005)) It is a simple way to get an estimate of the duration and that is probably why it is so popular. The question is how reliable this estimation is. It seems that the method ranks the duration of the different NMDs in a good way. The NMDs that are held for transactional purposes are assumed to have longer durations, which is something that the model shows. However, there are some aspects that would suggest that this method of calculating the duration is not reliable.

One of these aspects has been discussed earlier and is the bad fit of the model for the account types with an almost flat deposit rate. This shows that the duration that the model produces for these account types is very unlikely to be the actual duration. Another aspect associated with the same account types is that some of them get the maximum duration allowed by the model. That means that if bonds with longer time to maturity where used by the model the duration would probably be even longer.

The strongest argument against the use of this approach is the effect different assumptions have on the result. One assumption is the use of moving averages, which improves the goodness of fit. The use of moving averages also makes the estimated duration longer. Even Maes and Timmermans (2005) show that different assumptions give quite different durations.

Although the method of using a static replicating portfolio for estimating the duration of NMDs seems to rank the duration of the NMDs in a good way, it is unlikely that the method estimates the true durations.

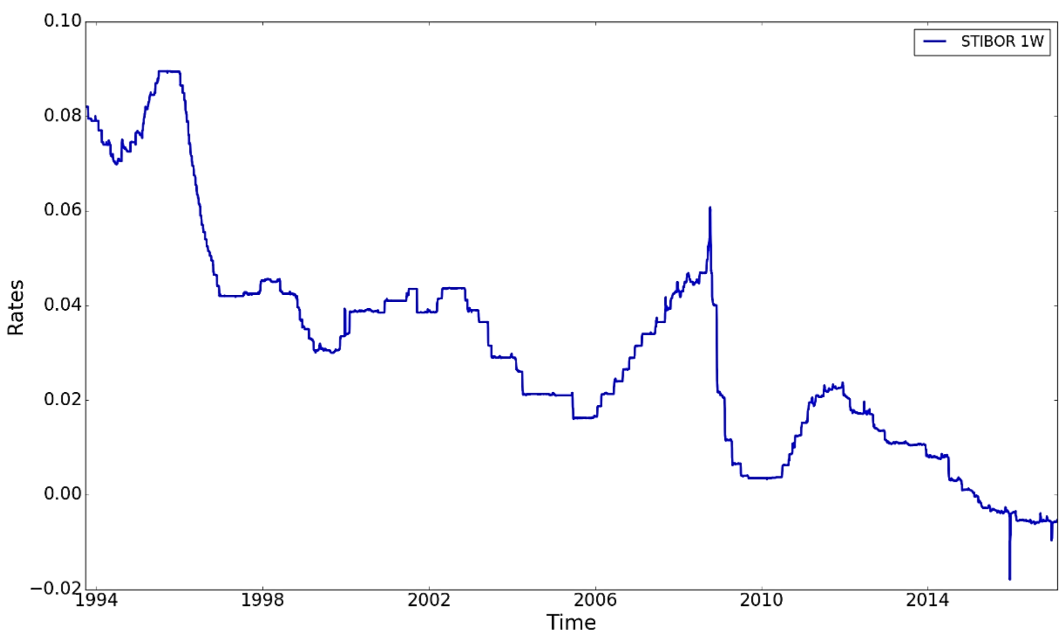
## 7.7 Comments on the Characteristics of NMDs

The discussion is ended by reconnecting to Chapter 2 and the characteristics of NMDs that are described there. There is seemingly no reason not to agree with the common opinion that the most prominent characteristics that define NMDs are the customers' right to withdraw money and the banks' right to change the deposit rate.

A common conclusion is that there is an asymmetric adjustment speed of deposit rates when interest rates go up and down, respectively. With the limited historical sample that is used in this study, this conclusion cannot be verified. Although deposit rates fall faster on average than they rise on average, the market rates also often falls faster than they rise. This can be seen in Figure 9. This means that just looking at deposit rates, there may be an asymmetric adjustment speed, but in a context where interest rates drive deposit rates there may not be such an asymmetry. With this said, this thesis provides no proof of the hypothesis.

Most of the NMDs observed in this study have a deposit rate which moves in a discretized way. Most models do not take this discretized nature into consideration and there is seemingly no relevant reason to do that, at least if the purpose of the model is to value the NMD. One characteristic that is highly relevant for the modeling is the informal floor on deposit rates set at zero. However, no cap on deposit rates have been observed in this study.

**Figure 9. STIBOR 1 week**




The figure shows the dynamics of the STIBOR 1 week rate since 1994. Source: Sweden's Central Bank (2017).

# 8. Conclusion and Further Research

## 8.1 Conclusion

The modeling of NMDs is a highly important topic for most banks, since they are to a large extent funded by NMDs. An NMD may seem to be a trivial financial product, but certain characteristics make them rather complex to model. An integral part of modeling NMDs is to model the deposit rate. This thesis has contributed to the research on modeling deposit rates of NMDs by comparing different models and evaluating their performances when applied to data from Handelsbanken. This thesis also suggests three new models for the deposit rate of NMDs, one of which shows a very good performance.

The evaluation of the models has been done on the basis of four aspects; goodness of fit, stability, negative interest rate environment and simplicity. What may be surprising, is that simplicity does not automatically mean a compromise of the performance of the model. On the contrary, the simplest models often have the best performances.

The model that has the best overall performance is the third new model that has been suggested in this thesis. One reason for this is that the new suggested models have a way of handling a negative interest rate environment while the other models, which are suggested in the literature, were created at least a few years ago when negative market rates were not an issue. The older models, therefore, do not handle this problem. The good handling of the negative interest rate environment gives the third new model a good fit to the historical sample. The model is also intuitive and simple. One drawback of the new models is that they are all based on the assumption that banks will not make the deposit rates negative.

## 8.2 Further Research

The modeling of NMDs is a comprehensive subject and there is much to be done with it. For instance, this thesis has briefly touched upon the valuation of NMDs. Regulations focus mainly on modeling the proportion of core deposits in NMDs and the average maturity of the core deposits. Therefore, it is reasonable that researchers should focus on the modeling of core deposits.

This thesis has focused on modeling deposit rates of NMDs. The models for the deposit rates could be a part of a bigger framework for modeling NMDs or they could be used as standalone models for predicting the future deposit rates or to understand the behavior of banks. Future research should put the models into a bigger context. One of the contributions of this thesis is the new models that are suggested. These need to be further evaluated and only time will tell if the assumption of non-negative deposit rates will hold.

# References

Basel Committee on Banking Supervision. 2016. *Standards, Interest rate risk in the banking book*. Bank for international settlements.

Bank of Japan. 2014. *Survey on Core Deposit Modeling in Japan: Toward Enhancing Asset Liability Management*. Financial System and Bank Examination Department.

Bardenhewer, M. 2007. *Chapter 10, Modeling Non-maturing Products*, In Matz, L. and Neu, P. (eds.), Liquidity Risk Measurement and Management: A Practitioner's Guide to Global Best Practices. John Wiley & Sons, pp. 220-256.

Björk, T. 2009. *Arbitrage Theory in Continuous Time*. Oxford University Press.

Blöchlinger, A. 2015. *Identifying, valuing and hedging of embedded options in non-maturity deposits*. Journal of Banking & Finance 50, pp. 34–51.

Cipu , E. and Udriste, S. 2009. *Estimating Non-Maturity Deposits*. Proceedings of the 9th WSEAS International Conference on Simulation, Modelling and Optimization, pp. 369-374.

De Jong, F. and Wielhouwer, J. 2003. *The Valuation and Hedging of Variable Rate Savings Accounts*. Astin Bulletin, 33 (2), pp. 383-397.

Dewachter, H., Lyrio, M. and Maes, K. 2006. *A Multi-Factor Model for the Valuation and Risk Management of Demand Deposits*. National Bank of Belgium.

Elkenbracht, M. and Nauta, B-J. (2006). *Managing interest rate risk for non-maturity deposits* [Online]. risk.net. Available at: [http://www.risk.net/risk-management/1500326/managing-interest-rate-risk-non-maturity-deposits](http://www.risk.net/risk-management/1500326/managing-interest-rate-risk-non-maturity-deposits)

Frauendorfer, K. and Shürle, M. 2007. *Dynamic Modeling and Optimization of Non-maturing Accounts*. In: Matz, L. and Neu, P. (eds.), Liquidity Risk Measurement and Management. John Wiley & Sons, pp. 327-359

Givens, G. and Hoeting, J. 2013. *Optimization and solving nonlinear equations*. In: Computational Statistics. 2$^{nd}$ ed. John Wiley & Sons. pp. 21-57.

Handelsbanken. 2017. *Risk och kapitalhantering 2016 – information enligt Pelare 3*.

Hutchison, D. and Pennacchi, G. 1996. "Measuring Rents and Interest Rate Risk in Imperfect Financial Markets: The Case of Retail Bank Deposits". Journal of Financial and Quantitative Analysis 31, pp. 399-417.

Jarrow, R. and van Deventer, D. 1998. *The arbitrage-free valuation and hedging of demand deposits and credit card loans*. Journal of Banking and Finance 22, pp. 249–272.

Kalkbrener, M. and Willing, J. 2004. *Risk management of non-maturing liabilities*. Journal of Banking and Finance 28, pp. 1547–1568.

Maes, K. and Timmermans, T. 2005. "Measuring the interest rate risk of Belgian regulated savings deposits". National Bank of Belgium, Financial Stability Review, pp. 137-152.

Neumark, D. and Sharpe, S. 1992. *Market Structure and the Nature of Price Rigidity: Evidence from the Market for Consumer Deposits*. The Quarterly Journal of Economics 107 (2), pp. 657-680.

Nyström, K. 2008. *On deposit volumes and the valuation of non-maturing liabilities*. Journal of Economic Dynamics and Control 32, pp. 709-756.

O'Brien, J. 2000. *Estimating the value and interest rate risk of interest-bearing transactions deposits*. Working Paper. Division of Research and Statistics, Board of Governors of the Federal Reserve System.

Paraschiv, F. 2011. *Modeling Client Rate and Volumes of Non-maturing Savings Accounts*. Dissertation no. 3924. University of St. Gallen, School of Management, Economics, Law, Social Sciences and International Affairs.

Paraschiv, F. and Schürle, M. 2010. *Modeling client rate and volumes of non-maturing accounts*. Institute for Operational Research and Computational Finance, University of St.Gallen.

Sheehan, R. 2013. *Valuing Core Deposits*. Journal of Financial Services Research 43, pp. 197–220.

Sweden's Central Bank (Sveriges Riksbank). 2017. *Sök räntor & valutakurser* [Online]. Available at: http://www.riksbank.se/en/Rantor-och-valutakurser/Sok-rantor-och-valutakurser/ [Accessed: April 11, 2017].

Swedish Financial Supervisory Authority (Finansinspektionen). 2015. *FI's methods for assessing individual risk types within Pillar 2*. Memorandum. FI Ref. 14-14414.

Swedish National Debt Office (Riksgälden). 2017. *Insättningsgarantin* [Online]. Available at: https://www.riksgalden.se/Dokument_sve/insattningsgarantin/insattningsgarantin_infoblad.pdf [Accessed: March 29, 2017].

# Appendix 1 – Interest Rate Models

This appendix is mainly based on Björk (2009).

## A1.1 Forward Rates

An interest rate model is a model that describes the future evolution of interest rates. This can be done by describing the dynamics of the (continuously compounded and annualized) short rate, $r_t$. Under the risk-neutral measure $Q$ and with $\{F_t\}_{t\geq 0}$ being the natural filtration for the process, the price of a zero-coupon bond, $P(t,T)$, at time *t* maturing at time $T$ with payoff 1 is given by

$$P(t,T) = E^Q\left[e^{-\int_t^T r_s ds}\middle|F_t\right]. \quad (43)$$

The instantaneous forward rates, $f(t,T)$, are given by

$$f(t,T) = -\frac{\partial}{\partial T}\ln\big(P(t,T)\big). \quad (44)$$

Thus with a no-arbitrage argument and some technical conditions, a model for the short rate is also a model for forward rates.

## A1.2 Interest Rate Models

Below, some models for the short interest rate is presented. They are all driven by Wiener processes, denoted $W_t$.

Some of the most used one-factor models are:

- The Ho-Lee model: $\partial r_t = \theta_t \partial t + \sigma \partial W_t$
- The Vasicek model: $\partial r_t = k(\theta - r_t)\partial t + \sigma \partial W_t$
- The extended Vasicek model: $\partial r_t = k_t(\theta_t - r_t)\partial t + \sigma_t \partial W_t$
- The CIR model: $\partial r_t = a(b - r_t)\partial t + \sqrt{r_t}\sigma \partial W_t$
- The extended CIR model: $\partial r_t = (\theta_t - \alpha_t r_t)\partial t + \sigma_t\sqrt{r_t}\partial W_t$
- The Dothan model: $\partial r_t = a r_t \partial t + \sigma r_t \partial W_t$
- Black-Derman-Toy: $\partial r_t = \theta_t r_t \partial t + \sigma_t r_t \partial W_t$

The one factor models can be extended to two factor models. For instance, a two-factor Vasicek model is defined as

$$r_t = \beta_0 + \beta_1\eta_{1,t} + \beta_2\eta_{2,t} \quad (45)$$

where

$$\partial\eta_{i,t} = k(\theta - r_t)\partial t + \sigma\partial W_t\,, \quad for \quad i = 1,2. \quad (46)$$

An alternative approach is to directly model the dynamics of instantaneous forward-rates. This can be done with the Heath-Jarrow-Morton (HJM) framework, which assumes that the forward rate can be found by solving the stochastic differential equation

$$\begin{aligned} \partial f(t,T) &= \alpha(t,T)dt + \sigma(t,T)\partial W_t \\ f(0,T) &= f^*(0,T) \end{aligned} \tag{47}$$

where $f^*(0,T)$ is the observed forward rate curve at time $0$.

The short rate is then given by

$$r_t = f(t,t). \tag{48}$$

Kalkbrener and Willing (2004) use a non-parametric version of the HJM-framework where the drift, $\alpha(t,T)$, is defined as

$$\alpha(t,T) = \sum_{i=1}^{2} \sigma(t,T)\left(\int_t^T \sigma_i(t,y)\partial y - \lambda_i\right) \tag{49}$$

where $\lambda_i$ are constants. Now it is enough to specify the volatility functions, $\sigma_i(t,y)$, which Kalkbrener and Willing define with piecewise constant functions.

## A1.3 Calibration of the One-Factor Vasicek Model

The parameters in the one-factor Vasicek model can be estimated using the maximum likelihood method. The log-likelihood of the one-factor Vasicek model is

$$l(k,\beta,\theta) = -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(r_i - r_{i-1} * e^{-k} - \theta\left(1 - e^{-k}\right)\right)^2 \tag{50}$$

where n is the number of observation days. The parameters is estimated as

$$k,\beta,\theta = \underset{k,\beta,\theta}{\operatorname{argmax}}\, l(k,\beta,\theta). \tag{51}$$

In the model, $\theta$ can be interpreted as the long-term mean short-rate that the model will evolve around, $k$ is the speed of reversion towards $\theta$ and $\sigma$ is the instantaneous volatility.

# Appendix 2 – Fisher's Scoring Algorithm

This appendix is based on Givens and Hoeting (2013).

## A2.1 Maximum Likelihood Estimation

The idea behind maximum likelihood estimation is to find the parameters that maximize the probability of a particular outcome. Given the outcome $x_1, \dots, x_p$ of the independent and identically distributed stochastic variables $X_1, \dots, X_p$ with density function $f(x|\boldsymbol{\theta})$ the parameters $\boldsymbol{\theta}$ can be estimated with the maximum likelihood estimator (MLE) of $\boldsymbol{\theta}$. The MLE is the parameters that maximizes the likelihood function

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{p} f(x_i|\boldsymbol{\theta}) \tag{52}$$

It is often more convenient to use the log-likelihood function

$$l(\boldsymbol{\theta}) = \ln L(\boldsymbol{\theta}) \tag{53}$$

Finding the MLE of $\theta$ is equivalent to solving

$$l'(\boldsymbol{\theta}) = \mathbf{0} \tag{54}$$

where

$$l'(\boldsymbol{\theta}) = \left(\frac{dl(\boldsymbol{\theta})}{d\theta_1}, \dots, \frac{dl(\boldsymbol{\theta})}{d\theta_k}\right) \tag{55}$$

is called the score function and has the property

$$E[l'(\boldsymbol{\theta})] = \mathbf{0} \tag{56}$$

## A2.2 Newton-Raphson Iterations

A common numerical approach to finding roots is through Newton-Raphson iterations. The method is based on linear Taylor expansion:

$$f(x) \approx f'(x_0)(x - x_0) + f(x_0). \tag{57}$$

This is a tangent to the function *f(x)* that has a root at

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{58}$$

This leads to the iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{59}$$

The multivariate equivalence is

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - J^{-1}(\boldsymbol{x}_n) f(\boldsymbol{x}_n) \tag{60}$$

where $J(\boldsymbol{x}_n)$ is the Jacobian of $f(\boldsymbol{x}_n)$

$$J(\boldsymbol{x}_n) = \begin{bmatrix} \frac{\partial f^1}{\partial x_n^1} & \cdots & \frac{\partial f^1}{\partial x_n^p} \\ \vdots & \ddots & \vdots \\ \frac{\partial f^p}{\partial x_n^1} & \cdots & \frac{\partial f^p}{\partial x_n^p} \end{bmatrix}. \tag{61}$$

## A2.3 Fisher's Scoring Algorithm

Fisher's scoring algorithm can be used to estimate MLE parameters. Fisher's scoring algorithm is given by replacing the Jacobian in Newton-Raphson iterations by the Fisher information matrix

$$I(\boldsymbol{\theta}) = E[l'(\boldsymbol{\theta}) l'(\boldsymbol{\theta})^T] = -E[l''(\boldsymbol{\theta})] \tag{62}$$

where

$$l''(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial^2 l(\boldsymbol{\theta})}{\partial \theta_1^2} & \cdots & \frac{\partial^2 l(\boldsymbol{\theta})}{\partial \theta_p \partial \theta_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 l(\boldsymbol{\theta})}{\partial \theta_1 \partial \theta_p} & \cdots & \frac{\partial^2 l(\boldsymbol{\theta})}{\partial \theta_p^2} \end{bmatrix}. \tag{63}$$

The iteration is then

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + I^{-1}(\boldsymbol{\theta}_n) l'(\boldsymbol{\theta}_n). \tag{64}$$

The standard errors of the estimates is the square root of the diagonal elements of the inverted Fisher information matrix at the maximum likelihood estimates.

TRITA -MAT-E 2017:19
ISRN -KTH/MAT/E--17/19--SE