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Micro-Level Loss Reserving in Economic Disability Insurance

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Abstract

In this thesis we provide a construction of a micro-level reserving model for an economic disability insurance portfolio. The model is based on the mathematical framework developed by Norberg (1993). The data considered is provided by Trygg-Hansa. The micro model tracks the development of each individual claim throughout its lifetime. The model setup is straightforward and in line with the insurance contract for economic disability, with levels of disability categorized by 50%, 75% and 100%. Model parameters are estimated with the reported claim development data, up to the valuation time τ. Using the estimated model parameters the development of RBNS and IBNR claims are simulated. The results of the simulations are presented on several levels and compared with Mack Chain-Ladder estimates. The distributions of end states and times to settlement from the simulations follow patterns that are representative of the reported data. The estimated ultimate of the micro model is considerably lower than the Mack Chain-ladder estimate. The difference can partly be explained by lower claim occurrence intensity for recent accident years, which is a consequence of the decreasing number of reported claims in data. Furthermore, the standard error of the micro model is lower than the standard error produced by Mack Chain-Ladder. However, no conclusion regarding accuracy of the two reserving models can be drawn. Finally, it is concluded that the opportunities of micro modelling are promising however complemented by some concerns regarding data and parameter estimations.

Keywords: Micro Model, IBNR, RBNS, Loss Reserving

Sammanfattning

I detta examensarbete ges ett förslag på uppbyggnaden av en mikro-modell för reservsättning. Modellen är baserad på det matematiska ramverket utvecklat av Norberg (1993). Data som används är tillhandahållen av Trygg-Hansa och berör försäkringar kopplade till ekonomisk invaliditet. Mikro-modellen följer utvecklingen av varje enskild skada, från skadetillfälle till stängning. Modellen har en enkel struktur som följer försäkringsvillkoren för den aktuella portföljen, med tillstånd för invaliditetsgrader om 50%, 75% respektive 100%. Modellparametrarna är estimerade utifrån den historiska utvecklingen på skador, fram till och med utvärderingstillfället τ. Med hjälp av de estimerade parametrarna simuleras den framtida utvecklingen av RBNS- och IBNR-skador. Resultat av simuleringarna presenteras på flera nivåer och jämförs med Mack Chain-Ladder estimatet. Den simulerade fördelningen av sluttillstånd och tid mellan rapportering och stängning, följer mönster som stöds av rapporterad data. Den estimerade slutkostnaden från mikro-modellen är betydlig lägre än motsvarande från Mack Chain-Ladder. Skillnaden kan delvis förklaras av en låg skadeintensitet för de senaste skadeåren, vilket är en konsekvens av färre rapporterade skador i data. Vidare så är standardfelet lägre för simuleringarna från mikro-modellen jämfört med standardfelet för Mack Chain-Ladder. Däremot kan inga slutsatser angående reservsättningsmetodernas precision dras. Slutligen, framförs möjligheterna för mikro-modellering som intressanta, kompletterat med några svårigheter gällande datautbud och parameterestimering.

Svensk Titel: Reservsättning för Ekonomisk Invaliditet på Mikronivå

Nyckelord: Micro modell, IBNR, RBNS, Reservsättning

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1 Introduction

1.1 Background

The insurance business is built upon the idea that a collective of individuals together share the risk of unfortunate events. Thus, if one individual gets exposed to such an event, where the consequences impairs his or her economical situation, the collective can compensate that loss. The role of the collective has been taken by insurance companies. These institutions gather premiums from large groups of individuals who in return get insured to be compensated if they would face different unlikely and unfortunate events.

The revenue of insurance companies are based on the premiums collected, while the expenses arise from having to compensate the covered customers. Thus, companies at least need to gather premiums that can cover the losses of compensation for future accidents. At the time of gathering premiums, the losses arising from the collective of individuals are unknown. Therefore the sizes of individual premiums must reflect the future distribution of losses, derived from separate unfortunate events or accidents. The expected size of future losses is affected by individual risk characteristics as well as the number of individuals who are covered.

An individual who has signed an insurance contract can file for compensation, in the event of facing accidents. Such a request of compensation arriving at an insurance company is referred to as a claim.

Reserving in the insurance business is the process of setting aside capital to cover the losses for claims that have occurred in the historical accident periods. At a certain stopping time τ, the premiums collected must cover the liabilities (both paid and outstanding) originated from before that point in time (Norberg, 1993). Some parts of the liability at τ might include payments that are made in the future, however, the insurance companies are not allowed to forecast future premiums to cover those outstanding liabilities. Thus, reserving in insurance comes down to making estimations and predictions of the unknown future development of claims that have occurred during the current or previous accident periods. This involves predicting development of reported but not settled claims as well as unreported claims.

A common method for reserving in the insurance business is the Chain-Ladder model, specially for claims of non-life insurance. Advantages of this method are that it is easily utilized and suited for observing trends over aggregated claims on a portfolio level. However, Chain-Ladder lacks in its ability to account for individual claim characteristics. Furthermore the Chain-Ladder model requires that historical trends are representative of the future development of reserves and the method is not suitable for claim portfolios that are volatile. In recent years other reserving methods have been explored. Methods with focus on the individual claim characteristics such as reporting delay, payment delays and payment sizes etc. These types of methods are referred to as micro models. The purpose of designing micro models is to be able to utilize information that aggregated methods such as Chain-Ladder can not. However, aggregated methods such as Chain-Ladder are the most common both in practice and in literature. In recent years studies on micro modelling approaches in reserving have increased in volume. With possibilities of producing methods with enhanced estimations of reserves, micro modelling is in an ongoing evaluation of performance. The demand of designing and evaluating such micro models might be particularly high for insurance portfolios with characteristics unsuitable for aggregated models. Characteristics such as slow developing or volatile.

1.2 Problematization

The problem we aim to investigate is that of evaluating the performance of micro modelling in reserving. By using findings and frameworks produced within the field of micro modelling, we aim to perform a case study of such a method. The purpose is to evaluate the performance and convenience of implementing such procedures in the process of reserving. Thus, we mean to apply concepts of micro modelling on a set of insurance data particularly characterized by a slow development, which is often inappropriate for traditional methods such as Chain-Ladder.

The disadvantages of traditional reserving methods are known. Furthermore the possibilities of managing those disadvantages by implementing alternative reserving methods have been discussed and evaluated by studies on micro modelling. However, the conventional approach in practice is still constituted by aggregated models. Therefore the area of micro modelling might not yet be fully explored. By adapting the concepts in the field, we hope that our model design, implementation and evaluation will contribute to the knowledge of micro model usability in reserving.

1.3 Reserving Techniques

Consider standing at the end time of an accident period, from here on referred to as the evaluation date τ. At that time we have data describing the claims that have occurred within historical periods, given that they have been reported. In the data of those reported claims there might be some that have been settled, which implies that the ultimate cost of those claims is known . Other reported claims might still be open at the evaluation date. Therefore the development of those claims have an unobserved part, which is the development beyond τ. Finally, there might also have occurred claims during the accident period which not yet have been reported at the evaluation date. Due to reporting delays of these claims the entire development is unobserved. Since insurance companies must set a reserve for the ultimate cost of all the claims originated from a certain accident period, the unobserved claim developments must be estimated. Thus, reserving becomes a prediction problem, where estimation of future development is necessary to set an accurate reserve.

As presented the traditional approach for reserve estimations are aggregated portfolio methods. In such methods the historical loss developments are used to calculate development factors. These factors are then used to estimate the loss development over the expected lifetime of the claim portfolio. The losses and factors are based on accumulated claims and therefore trends and developments illustrates the behavior of all accidents lumped together, i.e on portfolio level.

Micro modelling instead focus on the individual claim level. This involves modelling individual claim traits such as occurrence of claims, reporting delay, payment delays, payment sizes and settlement delay among others. Thus, the approach builds upon estimating parameters and distributions of the various characteristics of claims, based on historical data. By using the estimations future development of claims are simulated separately. From individual simulations the developed portfolio of claims can be aggregated to an estimated ultimate cost for the entire portfolio. Thus, the micro estimate could also be compared to any alternative estimate, for example an estimate produced by some Chain-Ladder technique.

The components of micro modelling and Chain-Ladder methods will be presented in detail in later chapters.

1.4 Outline of Thesis

The structure of this thesis is as follows. The Introduction in Section 1 is followed by Section 2 where a literature review is presented, describing earlier work published in the area of reserving, relevant for this study. In Section 3 a description of the portfolio and model layout is given. In Section 4 the theoretical framework which this thesis is based on is presented. This includes both theory regarding insurance as well as mathematics. In Section 5 we present the data used, followed by Section 6 where explanations of how the estimations of necessary parameters are given. The first results are those of the estimation of parameters and distributions which are presented in Section 7. In Section 8 the simulation procedures are described before the simulation results on claim- and portfolio-level are presented in Section 9. Finally Section 10 presents conclusions and discussions on areas of future research.

2 Literature Review

The literature within the field of stochastic loss reserving, focus mainly on macro models such as Chain-Ladder. However, in recent years a number of articles and studies focusing on the micro modelling approach to reserving have been published. In this section we present some of the literature that constitutes the framework of micro modelling as well as other relevant theories and models in the area of reserving and insurance.

Norberg (1986) published a paper tackling the issue of predicting IBNR-claims (Incurred but not reported). In the study he used a wide framework and various specifications of model assumptions. As data was grouped annually, basic model assumptions included yearly risk measures of exposure as a known quantity. Furthermore, each year was paired with quantities representing the latent general risk conditions which were assumed to be unobservable random elements. The total amount of claims occurring during an accident year was assumed to be Poisson distributed.

In 1993 Norberg published a follow up paper, where assuming a continuous time line, claim generating was modelled by a non-homogeneous Marked Poisson Process. This setup again implied the total amount of claims to follow a generalized Poisson distribution. By categorizing claims into four classes: Settled, Reported but not settled (RBNS), Incurred but not reported (IBNR) and Covered but not incurred, Norberg proved that the four classes follow independent Marked Poisson Processes. Therefore, total outstanding liability could be estimated by the sum of the predictors for each category. In another follow up study Norberg (1999) revisited the modelling of a position dependent Marked Poisson Process and added some theoretical results. In particular the decomposition of categories was further generalized.

The reserve-modelling of Marked Point Processes is adapted in several studies. Arjas (1989) presented structural ideas on how Point Process- and martingale theory could by applied to the modelling and estimation of claim reserves. Reserving as a prediction problem based on assumptions and available information was discussed and investigated.

Arjas & Haastrup (1996)studied claims reserving from the Marked Point Process perspective. The insurance data considered was a dental claim portfolio. By implementing a non-parametric Bayesian approach estimates of posterior distribution and distribution of outstanding liabilities were simultaneously estimated through Markov Chain Monte Carlo Integration. Individual claim components included occurrence time, reporting delay and a process describing payments and settlement.

With a similar framework to that of Arjas & Haastrup (1996), a case study on data from a European insurance company was conducted in Antonio & Plat (2014). Data from two separate insurance portfolios were applied. In contrast to the study conducted by Arjas & Haastrup (1996), Antonio & Plat (2014) used a semi parametric approach. Parameters describing individual claim components such as intensities, distributions and hazard rates were estimated from maximizing the likelihood of observed data. Using the decomposition from Norberg (1993) IBNR- and RBNS-claims were simulated separately and summed together for estimations of outstanding liabilities.

In Jin (2013) the model specifications from Antonio & Plat (2014) were extended, to handle changing development patterns. The case study was conducted on data from a workers compensation insurance portfolio. The performance of the micro model approach was evaluated and compared to the performance of an Over-Dispersed-Poisson Chain-ladder method as well as the observed real life development. Furthermore, the author presented discussions on how to incorporate elements to consider the phenomena of inflation in micro and macro reserving models.

England & Verall (2002) published a report presenting various stochastic techniques for loss reserving that had been developed at that time. The authors presented a number of aggregated models such as extensions of Chain-Ladder or Bornhutter-Ferguson, where cumulative or incremental payments for portfolio accident periods were considered. Furthermore, some micro-focused approaches were discussed where number of claims for a period was modelled by a Poisson distribution, similar to the approach presented in Norberg (1993).

In Andersen (2010) the approach of micro modelling loss reserving was investigated on insurance data from a danish portfolio for workers compensation (loss of earning capacity). The author constructed a model of states, representing certain events occurring during evaluation and lifetime of such claims. The modelling and parameter estimations were focused on the state transitions, in combination with distributions of loss of earning capacity. The aspect of reporting delay was disregarded by assuming no lag between accident and reporting for all claims.

Pigeon et al. (2014) developed a stochastic model based on individual claim data of payments and incurred losses. From the model expressions for expected ultimate loss were derived. For validation the authors performed a case study and compared reserve estimates from different distributional assumptions as well as from other reserving techniques such as Chain-Ladder.

3 Portfolio and Model Layout

In this study we have chosen to construct a micro model for an insurance portfolio of economic disability. This choice of portfolio is motivated by the few states of disability categorizations, which makes it suitable for modelling. Furthermore the lifetime development of such a portfolio is quite slow, which also makes it an interesting target for examination of micro modelling performance, as an alternative to traditional aggregated methods.

Economic disability arises from unfortunate events where the consequences directly impact individuals ability to work and provide for themselves. The payments for this type of insurance contract is constructed so that fixed levels of compensation are predetermined to some fixed states described by different levels of severity of disability. Thus, such a portfolio enables the modelling to focus on the drivers of the claim payments i.e the states which are reached and at what times. The times and sizes of payments for a claim are determined by the events of reaching states of disability. Therefore, modelling the state change development of claims enables estimations of the portfolio reserve.

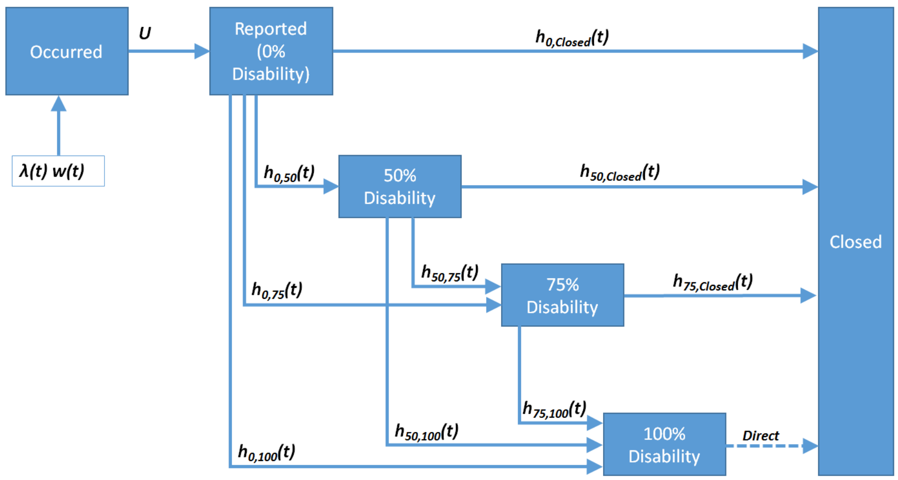


Figure 1: Model of states describing claim development and disability levels.

Figure 1 illustrates the state model, constructed for the portfolio under consideration in this thesis. The arrows represent the flow of a claim, i.e which jumps between states that are considered. The variables presented in the figure will be introduced in Sections 4-6.

Table 1 presents the model variables of individual claims development. Our

Variable	Description
T_i	Occurrence time of claims i assuming no claims occur at the exact same time
U_i	Reporting delay of claim i, i.e time from occurrence until reporting
V_{ij}	Time until next state change, the jth, state change of claim i
S_{ij}	Associated state of state change j of claim i, i.e *Reported*, 50% , 75%, 100% or *Settled*

Table 1: Variables.

model design is illustrated in Figure 1 and can be described as follows. Claim

i, occurs at time T_i. Trivially, at time T_i claim i immediately reaches the state *Occurred*. Each claim has a reporting delay U_i before it reaches the state *Reported*, at time T_i+U_i. As the state *Reported* is reached, the claim is available for investigation and determination of disability level. The construction of the insurance contract is such that there are three different levels/states of disability that generates payments for a claimant. These levels are 50%, 75% and 100% disability. The total liability of claim i is determined by which states it reaches and at what times it does so.

The first state change of claim i (after reporting) is described by the pair (V_{i1}, S_{i1}). V_{i1} represents the time since reporting of the first state change. S_{i1} represents the associated state of the change. Hence, at time $T_i + U_i + V_{i1}$ claim i reaches state S_{i1}.

During the lifetime of a claim different states can be reached at separate times. However, the model is constructed so that a claimant can only change state to retain a higher disability level. Furthermore if a claimant reaches the state/level of 100% disability the claim is assumed to close immediately. Thereby reaching state 100% could be interpreted as the event of settling together with a payment. The state *Closed* could however be reached from the states *Reported*, 50% and 75% as well. Events of settlement directly from the states *Reported*, 50% or 75% could be interpreted as settlement without additional payment. Events of reaching states 50% or 75% could accordingly be interpreted as intermediate payments.

With a portfolio design as presented above the parameter estimations for the purpose of micro modelling are mainly focused on occurrence intensity, reporting delay and state change intensities. As in Norberg (1986), the occurrence referrers to the event which gave rise to the claim, namely the time of the accident. In the following section we present the theoretical framework used for the estimations in this thesis.

4 Theoretical Framework

4.1 Types of Claims

In previous research as well as in practice claims are divided into categories depending on their status at the reserving evaluation date τ. In a micro model approach this is specially relevant since the different categories of claims and their lifetime development are handled separately. This categorization of claims is presented in Table 2.

Settled claims are claims that have been closed before τ. Thereby the data describing these claims is complete in the sense that the entire development including occurrence time, reporting delay and state changes to settlement is observed. The fully observable data implies no predictions are required.

Claim type	Description
Settled	Claim is closed and the ultimate liability is determined
Reported but not settled	Claim is open and reported but no ultimate cost is determined
Incurred but not reported	Claim has occurred but is not yet reported to the insurance company

Table 2: Types of claims.

Reported but not settled (RBNS) claims have been reported before τ, however the full development to settlement is not yet determined. For these claims the observable data includes occurrence time, reporting delay and possibly information about intermediate state changes. In the reserving scenario the unknown future development of these claims needs to be estimated.

Incurred but not reported (IBNR) claims originate from accident periods previous to τ. However due to extensive reporting delays these claims have not yet been reported. Thereby the data available at τ shows no record of these claims and they are completely unobservable. Since the reserving should account for all claims that have occurred, IBNR claims must be included in the prediction. For this sub-class the entire development must be predicted. This involves predicting number of IBNR-claims, their respective occurrence times, reporting delays and development from reporting to settlement.

Figure 2 illustrates the lifetime of events for some claim i. Depending on where the evaluation date τ is placed, claim i would belong to 1 of the 3 categories:

1. if $\tau = \tau_1$, claim i is an IBNR-claim.
2. if $\tau = \tau_2$, claim i is a RBNS-claim.
3. if $\tau = \tau_3$, claim i is a Settled claim.

categorizations are subject to that claim i settles at the third state change.

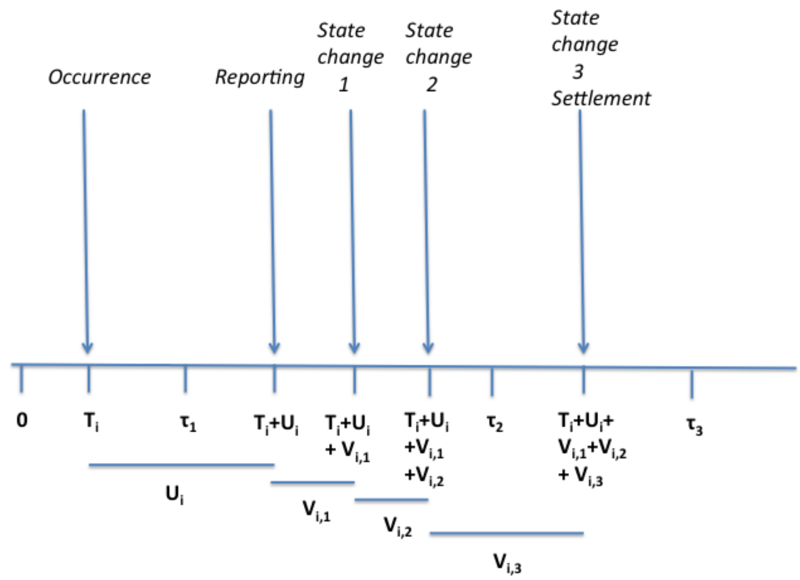


Figure 2: Illustration of lifetime for claim i.

4.2 Poisson Marked Point Process

The definitions and notation presented in this section are taken from the framework presented by Norberg (1993). The claim- occurrence and -development process can be modelled by a Marked Poisson Point Process. From this it follows that the occurrence of claims comes from a Poisson process. Furthermore each occurrence of a claim, at time T_i, is coupled with a mark $Z_i(t)$, which in itself is a stochastic process describing the development of claim events and event times. Therefore a specific claim i is of the form $C_i = (T_i, Z_i)$. The mark Z_i, is considered to be of the form $Z_i = (U_i, V_i, Y_i(t))$, where U_i describes the reporting delay, V_i is the time delay from reporting to settlement and $Y_i(t)$ is the accumulated payments of claim i up to time t after reporting. Hence, $Y_i(V_i)$ is equal to the total payment from a claim. With these defined variables the entire lifetime development of claim i is modelled.

The total process of a claim portfolio is said to be a random collection of pairs $(T_i, Z_i)_{i=1,..,N<\infty}$ where the T_i's are assumed to come from a in-homogeneous Poisson distribution with intensity $w(t)$, $t > 0$. The marks $\{Z_i\}_{i>0}$ are assumed to come from a family of mutually independent elements that are also independent of the Poisson process, where $Z_t \sim P_{Z|t}$. Thus the insurance portfolio is a Marked Poisson Process with position depending marking and can be written as

$$\{(T_i, Z_i)\}_{i=1,.....,N} \sim Po(w(t), P_{Z|t}; t > 0),$$

where $w(t)$ represents the risk exposure, and can be seen as a simple measure of volume or size of business. However $w(t)$ can also be modelled to include additional information reflecting the composition of the portfolio. Since the exposure relates to the risk arising from insurance contracts set up before the break up point τ, in practise $w(t) = 0$ for t large enough. In this scenario it is sufficient to assume that the total exposure is finite

$$W = \int_0^\infty w(t)dt < \infty. \tag{1}$$

Antonio & Plat (2014) extended the modelling of the exposure to include two parameters, $w(t)$ & $\lambda(t)$. In their study the exposure $w(t)$ was chosen as premiums collected, as was also suggested by Norberg (1993). However, the premium measure was complemented by an additional risk measure $\lambda(t)$, which was estimated using maximum likelihood (MLE) of occurrence data. By incorporating $\lambda(t)$ to the exposure of premiums, additional information such as seasonal trends could be captured in the claim occurrence intensity. The Poisson intensity of claim occurrence thus became $w(t)\lambda(t)$.

4.3 Intensity of Claim Process

By following the framework presented and used by Norberg (1993), Arjas & Haastrup (1996) and Antonio & Plat (2014) among a few, the claim process is a Position Dependent Poisson Marked Point Process. From this it follows that the occurrence times of claims T_i follows a Poisson process with non-homogeneous intensity measure $\lambda(t)w(t)$. The function $\lambda(t)$ should capture trends of claim occurrence that the exposure measure can not.

For the purpose of modelling the development of the categorized claims Norberg (1993) proved that the different categories of claims can be assumed to come from independent Marked Poisson Processes.

From earlier we have defined U_i to describe the reporting delay of claim i. Further we let X_i describe the development of claim i after reporting. Hence, X_i describes the times and types of state changes, which occurs for claim i. In our model those states are $\{50\%, 75\%, 100\%, Close\}$.

If we let $P_{U|t}$ and $P_{X|t,u}$ denote the distributions of U and X respectively we can relate back to the concept of a Marked Poisson Process. Each occurrence of a claim T_i is coupled with a mark Z_i that should describe the development pattern of the claim occurrence. The distribution $P_{Z|t}$, of the mark variable Z_i could be described using the distributions of the reporting delay $P_{U|t}$ and the state change development $P_{X|t,u}$. Note that the reporting delay distribution

$P_{U|t}$ is conditional on the occurrence time t. The state change distribution $P_{X|t,u}$ is conditional on the occurrence time as well as the reporting delay.

Using these defined distributions to describe the mark Z the Poisson process of reported claims have measure (Antonio & Plat, 2014)

$$w(dt)\lambda(dt)P_{U|t}(\tau - t)\mathbb{1}_{(t\in[0,\tau])}\frac{P_{U|t}(du)\mathbb{1}_{(u\leq\tau-t)}}{P_{U|t}(\tau - t)}P_{X|t,u}(dx).$$

Reported claims are on the set defined by $C^r = \{(t, u, x)|t \leq \tau, t + u \leq \tau\}$. I.e. random combinations of occurrence time, reporting delay and claim development such that

1. The occurrence dates of the claims happened before the evaluation date τ.
2. The reporting dates of the claims happened before or at the evaluation date τ.

The Poisson process of IBNR on the other hand, have measure (Antonio & Plat, 2014)

$$w(dt)\lambda(dt)(1 - P_{U|t}(\tau - t))\mathbb{1}_{(t\in[0,\tau])}\frac{P_{U|t}(du)\mathbb{1}_{(u>\tau-t)}}{(1 - P_{U|t}(\tau - t))}P_{X|t,u}(dx).$$

Thus IBNR claims are on the set defined by $C^r = \{(t, u, x)|t \leq \tau, t + u > \tau\}$. I.e. random combinations of occurrence time, reporting delay and claim development such that

1. The occurence dates of the claims happened before the evaluation date τ.
2. The reporting dates of the claims happened after the evaluation date τ.

4.4 Likelihood Function of Claim Process

In reserving the ultimate costs of claims that have occurred up until the current time τ should be estimated. Those claims that have been reported up to τ are observable data. Denote the observable part of the process

$$(T_i^O, U_i^O, X_i^O)_{i\geq1}.$$

The likelihood of the observed part of the claim process is given by (Antonio & Plat, 2014)

$$L \propto \{\prod_{i\geq 1} w(T_i^O)\lambda(T_i^O)P_{U|t}(\tau - T_i^O)\} \times exp(-\int_0^\tau w(t)\lambda(t)P_{U|t}(\tau - t)dt)\times$$
$$\{\prod_{i\geq 1} \frac{P_{U|t}(dU_i^O)}{P_{U|t}(\tau - T_i^O)}\} \times \prod_{i\geq 1} P_{X|t,u}^{\tau - T_i^O - U_i^O}(dX_i^O). \quad (2)$$

The observed part is the data of settled and RBNS claims from the portfolio. The evaluation date τ is 2018-01-01, all data of occurrence, reporting delay and state changes must be dated before or at that point in time for it to be observable. The likelihood presented in (2) has three parts, each connected to different elements in the claim development process.

1. The first product of (2) describes the likelihood of the observed claim occurrences. The reporting of the occurred claims are of course dependent of the reporting delay being smaller than the time remaining to τ.
2. The second product of (2) describes the likelihood of the observed reporting delays.
3. The third product of (2) describes the likelihood of observed state changes. This part of the likelihood will be further extended in Section 6 when the concepts of survival analysis and hazard rates have been introduced.

4.5 Survival Analysis

In our model insured individuals can move between different states. Therefore it is necessary to determine the intensities of such transitions, in order to simulate future development. In the scenario where the different transitions are assumed to be independent of each other and only dependent on time, the intensity estimations becomes that of a standard survival analysis estimation. The transition between two states can be seen as a model of lifetime, where the event of transitioning from the first state to second is the event of dying. The theory of survival analysis and hazard rates presented in this section is taken from Norberg (2002).

In this section we denote the survival time by T. This notation is only used in the presentation of the theory and is not to be mistaken for the claim occurrence time variable.

For a population of individuals being born into state 1 the lifetime before dying to state 2 varies between the individuals. The cumulative distribution for the survival time variable T is given by

$$F(t) = P(T \leq t).$$

In survival analysis it is often appropriate to refer to the survival function

$$\tilde{F}(t) = P(T > t) = 1 - F(t).$$

If we assume that F(t) is absolutely continuous then the density of T is given by

$$f(t) = \frac{d}{dt}F(t) = -\frac{d}{dt}\tilde{F}(t).$$

The mortality intensity, or hazard rate, for the survival of an individual is given by the derivative of $-ln\tilde{F}(t)$

$$\mu(t) = -\frac{d}{dt}ln(\tilde{F}(t)) = \frac{f(t)}{\tilde{F}(t)}.$$

By integrating from 0 to t and using $\tilde{F}(0) = 1$ we get

$$\tilde{F}(t) = e^{-\int_0^t \mu(s)ds}.$$

Further we can express the density of the lifetime T as

$$f(t) = e^{-\int_0^t \mu(s)ds}\mu(t).$$

Estimating $\mu(t)$ could be done by trying to fit CDF and PDF to the data. However this could be problematic in our case, specially since we have a censored survival data, due to the unobserved development of RBNS claims and the multiple states in our model. Another approach of estimating $\mu(t)$ could instead be to find the maximum-likelihood estimator $\hat{\mu}(t)_{ML}$.

4.5.1 MLE of Transition Hazard Rate

If we consider a constant μ and $T_1, ..., T_n$ as n observed survival times. Then the likelihood function of μ, assuming independence among observations, is given by

$$L(\mu) = \prod_{i=1}^{n} f(T_i) = \prod_{i=1}^{n} e^{-\int_0^{T_i} \mu ds}\mu = \prod_{i=1}^{n} e^{-T_i\mu}\mu = \mu^n e^{-\sum_{i=1}^{n} T_i\mu}. \tag{3}$$

Our objective is to estimate μ by maximizing the likelihood of our observations $T_1, ..., T_n$, i.e maximize the likelihood function with respect to μ. Maximizing the likelihood function is equivalent to maximizing the logarithm of the likelihood function. Taking the logarithm gives us

$$lnL(\mu) = nln(\mu) - \sum_{i=1}^{n} T_i\mu. \tag{4}$$

To analytically solve for the maximum likelihood estimator we take the derivative of (4) with respect to μ

$$\frac{d}{d\mu} lnL(\mu) = \frac{n}{\mu} - \sum_{i=1}^{n} T_i. \tag{5}$$

Since the second derivative $\frac{d^2}{d\mu^2} lnL(\mu) = -\frac{n}{\mu^2} < 0$ we have a maximum. Setting (5) equal to zero and solving for μ we get the maximum likelihood estimator as

$$\hat{\mu_{ML}} = \frac{n}{\sum_{i=1}^{n} T_i}. \tag{6}$$

Hence the MLE intensity is given by the total number of transitions divided by the total time of exposure before transition. Note that (6) is the estimator for a constant μ without considering censored waiting times. However, this constant estimation of μ can be translated in to a piecewise constant estimation $\hat{\mu}(t) = (\hat{\mu}^1, \ldots, \hat{\mu}^K)$ on the partition $0 = t_0, t_1, \ldots, t_{K-1}, t_K = \tau$ of the observed time interval $[0, \tau]$ as

$$\hat{\mu}_{ML}^k = \frac{\sum_{i=1}^{n} \mathbb{1}_{\{T_i \in (t_{k-1}, t_k]\}}}{\sum_{i=1}^{n} min(t_k - t_{k-1}, T_i - t_{k-1}) \mathbb{1}_{\{T_i \in (t_{k-1}, \tau]\}}}, \tag{7}$$

$k = 1, \ldots, K.$

The estimate of the piecewise constant hazard rate is given by the number of transitions in the intervals $(t_{k-1} - t_k)$ divided by the total time of exposure to transition in the same interval. Thus we from (7) have an estimate of the piecewise constant hazard rate $(\hat{\mu}_{ML}^1, ..., \hat{\mu}_{ML}^K)$.

However the estimates given by (7) are still not adjusted for the presence of censored observations of the lifetimes in a state. Due to the cutoff time at which the reserves of claims should be calculated, we do not have observable data of the entire lifetime of each claim. Therefore, not taking censoring into account and using (7) as the estimates for our state transition intensities, would lead to an overestimation of the hazard rates.

To incorporate the censoring in the estimations we instead consider censored survival times $T_i^{cen} = min(T_i, c)$, where c is the censoring time and $i = 1, ..., n$ runs over all observed survival times in a state. Further consider $\Delta_i = \mathbb{1}_{\{T_i^{cen} = T_i\}}$, as an indicator of whether T_i^{cen} is a real survival time or a censored time. For the censored survival times we get the following likelihood function for the constant hazard rate μ

$$L^c(\mu) = \prod_{i=1}^{n} e^{-\int_0^{T_i^{cen}} \mu ds} \mu^{\Delta_i} = \prod_{i=1}^{n} e^{-T_i^{cen} \mu} \mu^{\Delta_i} = \mu^{\sum_{i=1}^{n} \Delta_i} e^{-\sum_{i=1}^{n} T_i^{cen} \mu}. \tag{8}$$

Using the same procedure as before, taking the logarithm and maximizing over μ we get

$$\hat{\mu}_{ML} = \frac{\sum_{i=1}^{n} \Delta_i}{\sum_{i=1}^{n} T_i^{cen}}. \tag{9}$$

Thus, similar to before the hazard rates are estimated by the number of real transition (not censored times) divided by the total time exposed to transition. Using (9) together with (7) we arrive at a piecewise constant estimate as

$$\hat{\mu}_{ML}^{k} = \frac{\sum_{i=1}^{n} \Delta_i \mathbb{1}_{\{T_i^{cen} \in (t_{k-1}, t_k]\}}}{\sum_{i=1}^{n} min(t_k - t_{k-1}, T_i^{cen} - t_{k-1}) \mathbb{1}_{\{T_i^{cen} \in (t_{k-1}, \tau]\}}}, \tag{10}$$

$k = 1, \dots, K$.

The maximum likelihood estimator given by (10), of the censored survival times, on piecewise constant form is what we use for our hazard estimations in the thesis.

4.6 Distribution Fitting to Data

For the objective of implementing the micro model framework we are faced with the task of distribution fitting. In particular, we aim to find parameters of a specific distribution to describe the model element of reporting delay U. In this section we present concepts which will be applied in the process of finding the appropriate distribution.

The parametric modelling approach is as presented by Hult et al. (2012) based upon three steps:

1. Select parametric family of distribution.
2. Estimate parameters.
3. Validate the resulting distribution.

In the process of finding candidate parametric families it is often appropriate to inspect graphical illustrations of data, i.e raw plots or histograms. The graphical investigation often generates knowledge on which distributional characteristics to look for in the candidate parametric families.

4.6.1 QQ-Plots

In the procedure of finding a distribution to observed data, quantile-quantile-plots are a useful graphic tests. Consider we have data $x_1, \dots, x_n$ as observations of the random variables $X_1, \dots, X_n$, which are assumed to be independent identically distributed (IID). The distribution F of X_i is unknown and what we wish to find. A common approach for finding F is to suggest a reference distribution and test if the observations $x_1, \dots, x_n$ could constitute a sample of that reference distribution. One such test is the QQ-plot where the quantiles of the reference distribution are plotted against the sample (empirical quantiles).

Let $x_{1,n} \geq \dots \geq x_{n,n}$ denote the sample ordered by value. Then the QQ-plot are the points

$$\Big\{\Big(F^{-1}(\frac{n-k+1}{n+1}), x_{k,n}\Big) : k = 1, \dots, n\Big\}. \tag{11}$$

Let F_n denote the empirical distribution function of the sample. Then (11) could be rewritten as

$$\Big\{\Big(F^{-1}(\frac{n-k+1}{n+1}), F_n^{-1}(\frac{n-k+1}{n+1})\Big) : k = 1, \dots, n\Big\}. \tag{12}$$

The QQ-plot should be approximately linear if the data are generated by a distribution similar to the reference distribution. Furthermore the QQ-plot should also be linear if the data are transformed by an affine transformation, which would imply the data is in the same scale-location family as the reference distribution. If the data are a sample from the reference distribution then the intercept and slope of the line should be 0 & 1 respectively. With an affine transformation of data we would have $F_n^{-1}(p) = \mu + \sigma F^{-1}$ and the location- and scale-parameters can be estimated from the qq-plot (Hult et al. 2012).

4.6.2 Maximum Likelihood Estimation of Parameters

In the procedure of estimating the parameters of the candidate distributions, maximum likelihood is a viable approach. If we again consider $x_1, ..., x_n$ to be observations of the IID random variables $X_1, ..., X_n$, for which we wish to find the parametric distribution. Having identified a parametric family, the X_k's have the density function f_θ where θ (parameter(s)) is unknown. Finding the MLE of θ is done by finding the θ which maximizes the likelihood of the data

$$\hat{\theta} = argmax_\theta(\prod_{k=1}^{n} f_\theta(X_k)).$$

Using the strictly increasing property of the logarithm, it is often appropriate to find the equivalent estimator which maximizes the log-likelihood

$$\hat{\theta} = argmax_\theta(\sum_{k=1}^{n} ln(f_\theta(X_k))).$$

It is worth nothing that the maximum likelihood estimator of θ is not equivalent to making the QQ-plot as linear as possible (Hult et al., 2012).

4.7 Chain-Ladder

In this section we present the theoretical framework of the traditional Chain-Ladder reserving method. This method is later implemented for the purpose of comparison with our micro model.

The Chain-ladder model uses aggregated claim data over different development periods. By constructing run-off triangles and calculating development factors future development of the aggregated data is estimated. The triangle construction simplifies notation and allows for both cumulative and incremental data (England & Verall, 2002).

Consider we have incremental claim data of a portfolio

$$\{C_{ij}; i = 1, \dots, n; j = 1, \dots, n - i + 1\}. \tag{13}$$

In the triangle index i corresponds to the row and describes the accident period (year, quarter, month etc). Index j corresponds to the column and describes the development periods.

For an accident period i the cumulative claim loss is therefore defined by

$$D_{ij} = \sum_{k=1}^{j} C_{ik}. \tag{14}$$

Table 3 illustrates the run-off triangle with observed cumulative claim data.

Acc per / Dev per	1	2	3	4
1	$D_{1,1}$	$D_{1,2}$	$D_{1,3}$	$D_{1,4}$
2	$D_{2,1}$	$D_{2,2}$	$D_{2,3}$	
3	$D_{3,1}$	$D_{3,2}$		
4	$D_{4,1}$			

Table 3: Run-off triangle of data.

Let $\{\lambda_j : j = 2, \dots, n\}$ denote the development factors between development periods $j - 1$ and j. The estimates of the volume-weighted Chain-Ladder development factors are then given by

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} D_{ij}}{\sum_{i=1}^{n-j+1} D_{i,j-1}}. \tag{15}$$

Applying the development factors to the latest cumulative claim amounts we get the forecasted ultimate claim amounts $D_{i,n}$ as:

$$\hat{D}_{i,n-i+2} = D_{i,n-i+1}\hat{\lambda}_{n-i+2},$$

$$\hat{D}_{i,k} = \hat{D}_{i,k-1}\hat{\lambda}_k,$$

$k = n - i + 3, n - i + 4, + \ldots, n.$

and the reserve $\hat{R}_i$ is given by:

$$\hat{R}_i = D_{i,n-i+1}(\hat{\lambda}_{n-i+2} \times \hat{\lambda}_{n-i+3} \cdots \times \hat{\lambda}_n - 1). \quad (16)$$

Table 4 illustrates the development of the run-off triangle using (15) as the development factor estimates.

Acc per / Dev per	1	2	3	4
1	D_{11}	D_{12}	D_{13}	D_{14}
2	D_{21}	D_{22}	D_{23}	$D_{23}\hat{\lambda}_4$
3	D_{31}	D_{32}	$D_{32}\hat{\lambda}_3$	$D_{32}\hat{\lambda}_3\hat{\lambda}_4$
4	D_{41}	$D_{41}\hat{\lambda}_2$	$D_{41}\hat{\lambda}_2\hat{\lambda}_3$	$D_{41}\hat{\lambda}_2\hat{\lambda}_3\hat{\lambda}_4$

Table 4: Development of run-off triangle.

The Chain-Ladder model is used to predict ultimate losses for specific accident periods of aggregated insurance claim portfolios. Since the standard Chain-Ladder model described above produces point estimates of the ultimate losses it might be relevant to examine how the variability of the estimates can be incorporated. To analyze variability in the sense of variance or standard errors distributional characteristics of the claim development must be determined. England & Verall (2002) presented some of the common distributions used with Chain-Ladder in loss reserving.

4.7.1 The Mack Chain-Ladder Model

The Mack Chain-Ladder model is a method for distribution-free estimations of the standard errors (SE), of the Chain-Ladder forecast, under three conditions. The model was published by Thomas Mack in 1993 (Mack, 1993).

To forecast the amounts $\hat{D}_{i,k}$ for $k > n - i + 1$ the Mack model assumes:

$$E[D_{i,k}|D_{i,1}, D_{i,2}, \ldots, D_{i,k-1}] = D_{i,k-1} * \lambda_k, 1 \leq i \leq n, n - i + 1 < k \leq n, \quad (17)$$

$$Var(D_{i,k}|D_{i,1}, D_{i,2}, \ldots, D_{i,k-1}) = D_{i,k-1}\sigma_k^2, \quad (18)$$

$$\{D_{i,1}, \ldots, D_{i,n}\}, \{D_{j,1}, \ldots, D_{j,n}\}, \text{ are independent for } i \neq j. \quad (19)$$

Under these three assumptions the mean squared error $\hat{mse}(\hat{R}_i)$ can be estimated by:

$$\hat{mse}(\hat{R}_i) = \hat{D}_{i,n}^2 \sum_{k=n-i+2}^{n} \frac{\hat{\sigma}_k^2}{\hat{\lambda}_k^2} \Big(\frac{1}{\hat{D}_{i,k-1}} + \frac{1}{\sum_{j=1}^{n-k-1} D_{j,k-1}} \Big), \tag{20}$$

where

$$\hat{\sigma}_k^2 = \frac{1}{n-k} \sum_{i=1}^{n-k+1} D_{i,k-1} \Big(\frac{D_{i,k}}{D_{i,k-1}} - \hat{\lambda}_k \Big)^2, 2 \leq k \leq n-1, \tag{21}$$

$$\hat{\sigma}_n^2 = min(\hat{\sigma}_{n-1}^4 / \hat{\sigma}_{n-2}^2, min(\hat{\sigma}_{n-2}^2, \hat{\sigma}_{n-1}^2)). \tag{22}$$

From definition, the square root of an estimator of the mean squared error is the standard error of $\hat{R}_i$.

$$s.e.(\hat{R}_i))^2 = \hat{mse}(\hat{R}_i).$$

The standard error of the total reserve $\hat{R}$ is often of interest. Due to correlation in the estimators of $\hat{\lambda_k}$ and $\hat{\sigma}_k$, can not simply add $(s.e.(\hat{R}_i))^2$. Instead the mean squared error of the total reserve can be estimated by:

$$\hat{\text{mse}}(\hat{R}) = \sum_{i=2}^{n} \Big\{ (s.e.(\hat{R}_i))^2 + \hat{D}_{i,n} \Big(\sum_{j=i+1}^{n} \hat{D}_{j,n} \Big) \sum_{k=n+2-i}^{n-1} \frac{2\hat{\sigma}_k^2 / \hat{\lambda}_k^2}{\sum_{j=1}^{n-k} D_{j,k-1}} \Big\}. \tag{23}$$

5 Data

Our data consists of claim data and exposure data.

5.1 Claim Data

The claim data consists of 6328 economic disability claims, dating from 2000-01-01 to 2017-12-15. For each claim the date of the accident and reporting are available. For all the closed claims we have the settlement date and the full development with maximum 3 decisions and decision dates. For the open claims the development up to 2017-12-31 is available with maximum 2 decisions. 5468 of the 6328 claims are closed, 4005 are closed with 0% economic disability and 1463 of the claims are closed with a disability degree of either 50%, 75% or 100%. The remaining 860 claims are open, of them 682 are open without any decision made and 178 are open with a disability degree of either 50% or 75%. In the table below you can see examples of claims with different types of development.

ID	Acc date	Reg. date	S. date	D1	Date D1	D2	Date D2	D3	Date D3
21	04-12-09	17-10-23	17-11-06	-	-	-	-	-	-
22	08-08-18	15-12-01	-	50%	17-05-31	-	-	-	-
23	03-08-01	10-11-12	11-12-15	100%	11-12-07	-	-	-	-
23	03-01-01	07-10-29	14-06-13	50%	12-05-09	50%	14-06-11	-	-
43	03-06-04	07-06-04	15-08-31	50%	08-01-07	25%	11-06-24	25%	15-08-26

Table 5: Example of claim developments from data.

5.1.1 Settled Claims

As earlier mentioned, for settled claims the full development is known. In this section we look closer on the settled claim data. Firstly by examining the amount distribution of settled claims over the accident years 2000-2017, and secondly by examining the end state distribution for the settled claims. End state is defined as the last state a claim was stationed in before entering the state *Closed*.

5.1.1.1 Distribution Over Accident Years

In Table 6 the occurrence amounts of settled claims over accident years 2000-2017 are displayed. The number of settled claims is around 500 for the accident years 2000-2005, with a peak in 2004. From 2004 to 2017 we have a decreasing trend, from 667 settled claims for accident year 2004 to 5 settled claims for accident year 2017. The big gap between early and recent accident years is partly an effect of the reporting delay and the slow development of the portfolio. However, the volatility in the amount of claims could also be due to changes in claim occurrence intensity.

Accident year	# of Settled claims
2000	567
2001	547
2002	489
2003	506
2004	667
2005	509
2006	436
2007	360
2008	288
2009	262
2010	247
2011	188
2012	108
2013	112
2014	85
2015	60
2016	32
2017	5
Total	5468

Table 6: The distribution of settled claims over accident years.

5.1.1.2 End State Distribution & Time to Settlement

In Table 7 the end state distribution for settled claims is displayed. The pattern suggests that the intensity of jumping to state *Closed* from *Reported* is dominant relative other destinations.

End state	# of claims	% of total claim
0%	4005	73.2%
50%	526	9.6%
75%	107	2.0%
100%	830	15.2%
Total	5468	100%

Table 7: The end state distribution of the settled claims (at τ = 2018-01-01).

In Table 8 the average times to settlement are displayed, together with the values for the quantiles 2.5% & 97.5%. Time to settlement is defined as the time difference in days between registration date and settlement date. There is a clear pattern where claims having the end state 50%, 75% or 100% on average are open longer than claims with the end state 0%. The large difference in the quantiles implies a wide distribution of the time to settlement. Considering the extensive delays displayed it is evident that the portfolio is slow developed, even more so since the reporting delay is not included in this measure.

End state	Time to settlement	Quantile 2,5%	Quantile 97,5%
0%	881.4	0	3965
50%	1995.9	5	5417
75%	1738.4	5	4810
100%	1791.3	5	5223

Table 8: Average times to settlement for settled claims (days).

5.1.2 RBNS Claims

For RBNS claims the injury date and registration date are known, as well as the possible development up to time τ= 2018-01-01. This subclass consists of all claims in the data which have not yet been settled. In this section we display the occurrence distribution of RBNS claims over the accident years 2000-2017, together with distribution of current state at time τ, i.e. which state each claim belongs to at time τ.

5.1.2.1 Distribution Over Accident Years

In Table 9 the distribution of the number of RBNS claims per accident year is displayed. In a perfect scenario the number of RBNS-claims should increase as time approaches τ, given that the intensity of total claim occurrence is constant over the entire period. However, if the intensity of how claims occur varies over the accident periods then at least the proportion of RBNS claims relative to total amount of reported claims (settled + RBNS) should increase as time approaches τ. As the third column of Table 9 illustrates, this is not the case, except of the substantial increase in the two most recent accident years. The proportion of RBNS claims is quite low for all accident years before 2016. These RBNS claims originated from earlier accident years might arise from the most extreme accidents, with respect to the time requirement for the process of reporting and decisions on disability. Therefore, those extreme claims who constitute the proportion of RBNS-claims might not follow the logical increasing trend over accident years. Other reasons for the irregular behavior of the proportion of RBNS claims could be related to factors such as changes in reporting delay or capacity of the claim-handler.

Accident year	# of RBNS claims	$\frac{\#RBNS}{\#RBNS+\#settled}$
2000	35	5.8%
2001	58	9.6%
2002	51	9.4%
2003	108	17.6%
2004	121	15.3%
2005	114	18.3%
2006	141	24.4%
2007	57	13.7%
2008	25	8.0%
2009	30	10.3%
2010	38	13.3%
2011	21	10.1%
2012	4	3.6%
2013	7	5.9%
2014	6	6.6%
2015	5	7.7 %
2016	20	38.5 %
2017	19	79.1 %
Total	860	

Table 9: The distribution of RBNS claims over accident years.

5.1.2.2 Current States of RBNS Claims

Table 10 displays the current state distribution for the RBNS claims at the evaluation date of 2018-01-01. By model construction obviously no RBNS claims can be stationed in state 100%. Approximately 20% of the RBNS claims have already been given a disability level > 0%, and can in the future development either settle at the current level or at a higher level. The proportion (79.3%) of RBNS claims which are stationed in state *Reported* at τ have the possibility of settling in each of the model states.

Current state	# of claims	% of total RBNS claims
0%	682	79.3%
50%	170	19.8%
75%	8	0.9%
Total	860	100%

Table 10: The current state distribution of the RBNS claims (at τ = 2018-01-01).

5.2 Exposure Data

As a measure of exposure we have chosen the yearly earned premium, ranging over accident years 2000-2017. Earned premium is normally viewed as a good proxy for exposure, as size of total premiums is correlated with number of insured individuals and thereby the exposure to number of accidents. However, collected premiums is not a perfect measure of exposure as pricing is often based on packages of various moments of insurances. Therefore changes in yearly premiums could be derived from changes of other insurance moments in the package, rather than changes in exposure to economic disability claims in particular.

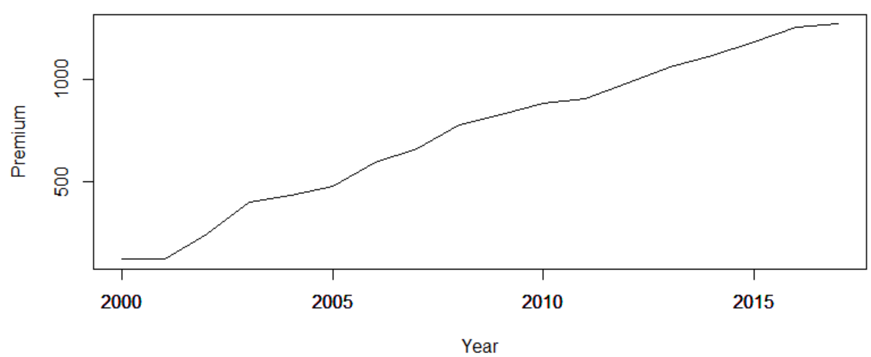


Figure 3: Yearly premiums expressed in millions.

Figure 3 displays the development of the yearly premiums in the period 2000-2017. The premiums seem to follow a somewhat linear increasing trend over the years. Thus, the exposure of the portfolio is larger for more recent accident years than earlier accident years. By observing the exposure isolated the intuition is that claim occurrence intensity should be higher for later accident years. However, this intuition is not taking the effect of the intensity parameter $\lambda(t)$ into account.

6 Estimation of Parameters

From the likelihood-function of the observed claim development there are some parameter's in need of estimation. With the purpose of being able to simulate future development of past accident years the following estimations are necessary.

6.1 Reporting Delay

One important part of the claim process is the distribution of the reporting delay. The reporting delay distribution $P_{U|t}$ is necessary both in the aspect of being able to simulate reporting delays for IBNR claims but also for the task of estimating $\lambda(t)$, in the part of the likelihood corresponding to the claim occurrence T_i.

In the process of estimating and fitting a distribution to the reporting delay data we start by studying visualizations of the empirical distribution.

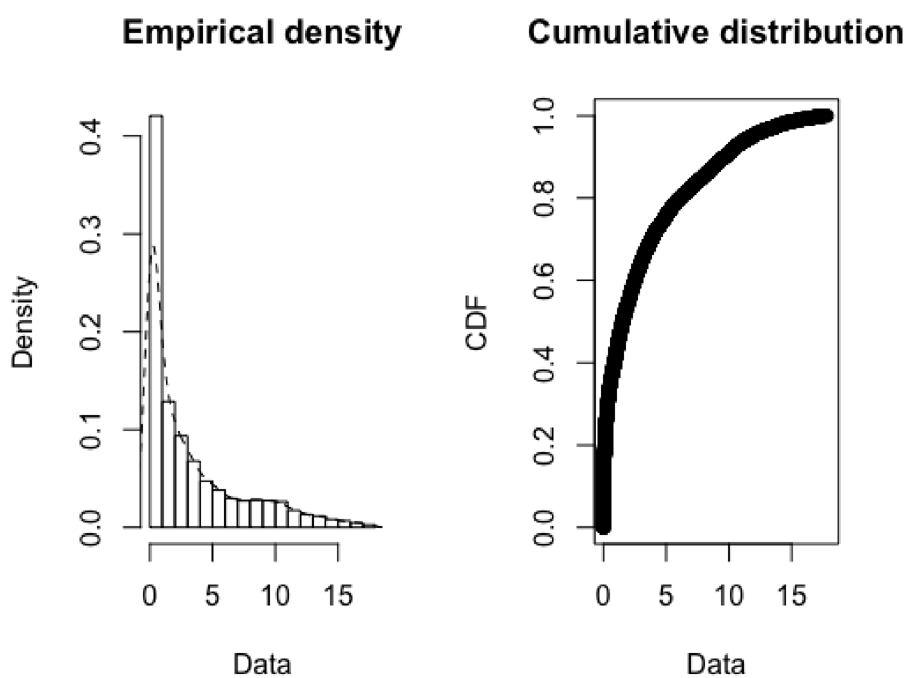


Figure 4: Empirical distribution and density of the reporting delay expressed in years.

From studying the graphic representation of the reporting delay data in Figure 4 it is evident that the development of claims in the particular portfolio of economic disability is very long lived. The recorded reporting delays often exceed a year and although many delays are limited to a few years the distribution is very heavy tailed with a considerable amount of delays exceeding as much as ten years.

Statistic	Value
Max	17.80274
Min	0
Mean	3.223463
Median	1.520548
Estimated sd	3.949637
Estimated skewness	1.444318
Estimated kurtosis	4.299173

Table 11: Summary statistics of reporting delay empirical distribution.

From Table 11 the heavy tailed feature of the reporting delay distribution is illustrated, both by the large kurtosis and also by the fact that the mean is considerably larger than the median. Furthermore the distribution of reporting delays is obviously non-negative. Therefore we are in our distribution fitting considering non-negative parametric families which are also characterized by heavy tails.

With the distributional features described we can use MLE to estimate parameters of candidate distributions such as Weibull, Pareto & Gamma. By comparison of the fitted distributions with their respective parameter estimates we choose the candidate who best represents the recorded reporting delays.

As a remark we fit the distribution of reporting delay to the observed data. The right censoring, arising from the time of observation τ, obviously bounds the reporting delay data to a maximum of 18 years. Therefore the sample of reporting delays is not a perfect random sample, as too extreme values are unobservable. This could generate an underestimation of the tail relative to the true reporting delay distribution. However, due to a long time window of observation where most observed reporting delays arise from claims originated from early accident years, we have a fairly good chance of capturing even extreme reporting delays. With this in mind we deem the approach of fitting on observed reporting delay data as a good approximation.

6.2 Claim Occurrence Intensity

From the likelihood function of the observed development, the part corresponding to the occurrence of claims is given by

$$\prod_{i\geq 1} w(T_i^O)\lambda(T_i^O)P_{U|t}(\tau - T_i^O) \times exp(-\int_0^\tau w(t)\lambda(t)P_{U|t}(\tau - t)dt).$$

Using the fitted distribution of the reporting delays U we can estimate the occurrence intensity $\lambda(t)$ by maximizing the likelihood of reported claims. As the measure of exposure $w(t)$ we use total yearly premiums. Therefore we follow the approach of Antonio & Plat (2014) and specify a piecewise constant estimation of $\lambda(t)$. Hence, $\lambda(t) = \lambda_y$ for $t \in [d_y, d_{y+1})$, where $y = 1, \ldots, n$ and d_y is the first day of year y. The time of evaluation is therefore $\tau \in [d_n, d_{n+1})$. The exposure $w(t) = w_y$ is obviously constant on yearly intervals as well. The approach of modelling occurrences of claims on annual basis is deemed appropriate due to the slow development of our particular portfolio. As discussed by Norberg (1986) grouping data on annual basis might be better suited for long-tailed businesses in contrast to short-tailed businesses where a narrower time intervals might be adequate.

If we let $NC(y)$ denote the number of claims that have occurred in year y, the part of the likelihood (2) related to occurrences of claims becomes

$$\begin{aligned}&\Big\{\prod_{i\geq 1} P_{U|t}(\tau - T_i^O)\Big\} \times \Big\{(\lambda_1 w_1)^{NC(1)} \times \ldots \times (\lambda_n w_n)^{NC(n)}\Big\}\times \\ &exp(-w_1\lambda_1 \int_{d_1}^{d_2} P_{U|t}(\tau - t)dt) \times \ldots \times exp(-w_n\lambda_n \int_{d_n}^{\tau} P_{U|t}(\tau - t)dt). \quad (24)\end{aligned}$$

When maximizing (24) over $\lambda(t)$ we can separate and maximize over the λ_y's individually. The likelihoods to maximize becomes

$$L(\lambda_y) = (\lambda_y w_y)^{NC(y)} \times exp(-w_y\lambda_y \int_{d_y}^{d_{y+1}} P_{U|t}(\tau - t)dt). \quad (25)$$

Taking logarithm and the derivative with respect to λ_y of (25) yields

$$\frac{\delta}{\delta\lambda_y} lnL(\lambda_y w_y) = \frac{NC(y)}{\lambda_y} - w_y \int_{d_y}^{d_{y+1}} P_{U|t}(\tau - t)dt. \quad (26)$$

By setting (26) equal to zero and solving for λ_y we get the MLE of the piecewise constant estimation of $\lambda(t)$

$$\lambda_y = \frac{NC(y)}{w_y \int_{d_y}^{d_{y+1}} P_{U|t}(\tau - t)dt}, \tag{27}$$

for years $y = 1, \ldots, n$.

6.3 State Development & Hazard Rates

As presented in Section 3 we consider a model with a number of states that describes the level of disability determined for a claimant. Once a claim has been reported it reaches the initial state *Reported* and is from that point eligible for a disability evaluation. For the modelling of this process of claim development, including transfers between states, survival analysis with hazard rates for the lifetime within a state is appropriate.

The lifetime of a claim in a state (before it transfers to a another state) is described by a hazard rate $h_{a,b}(t)$, going from state a to state b. Since the model of states is constructed in such a way that given a certain disability level a claim has more than one possible outcome in reevaluation, each state requires more than one hazard rate. For example in the state *Reported*(0%) a claim has four possible outcomes in evaluation. It could go to 50%, 75%, 100% or get closed at 0%. Therefore the modelling of the lifetime in state *Reported* requires four hazard rates: $h_{0,50}(t)$, $h_{0,75}(t)$, $h_{0,100}(t)$ and $h_{0,Close}(t)$.

In total we need to estimate 9 different hazard rates

$$h_{a,b}(t) : \begin{cases} b = \{50, 75, 100, Close\}, & \text{if } a = 0, \\ b = \{75, 100, Close\}, & \text{if } a = 50, \\ b = \{100, Close\}, & \text{if } a = 75. \end{cases}$$

We follow the approach of Antonio & Plat (2014) and use a piecewise constant estimation of the hazard rates

$$\hat{h}_{a,b}(t) = (\hat{h}^1_{a,b}, ..., \hat{h}^n_{a,b}),$$

where $\hat{h}^k_{a,b}$ is constant on the interval $[t_{k-1}, t_k)$ and $0 = t_0 < t_1 < \ldots < t_{n-1} < t_n = \tau$.

By constructing the piecewise estimations such that the intervals, on which the hazard rates are constant, are equal in length for each state we get the total hazard rate of leaving state a as

$$\hat{h}_{a,Tot}(t) = \sum_b \hat{h}_{a,b}(t).$$

For the estimations of the piecewise constant hazard rates we need the waiting times $W_{a,b}$ as the time spent in state a before leaving for state b. Note that we have to take censored waiting times into account, both since the observational period is limited to the development of claims up to 2018-01-01, but also because in each state a, claims have several possible jumps they could make.

Consider the waiting times $W_{0,50,i}$ corresponding to $h_{0,50}(t)$, namely the time spent in state *Reported* before jumping to state 50%. Those claims who have done this particular jump constitute the observational part of the actual waiting times. Further, we have to include censored waiting times for all claims that have spent time in *Reported* but then jumped to any of the other possible destinations $\{75\%, 100\%, Close\}$. This is necessary since before these claims jumped from *Reported*, they were at risk of jumping to 50%. Lastly, we also need to include censored waiting times for all claims that during the observational period have not made it further than the state *Reported*. As before these claims were all at risk of jumping to 50% until the time of censoring.

The similar censored and observational waiting time scheme apply to all combinations of possible jumps between states a and b. If we denote the waiting time, either observed or censored, for claim i by $W^{Cen}_{a,b,i}$ and let $\delta(W^{Cen}_{a,b,i})$ be an indicator equal to 1 if $W^{Cen}_{a,b,i}$ is observed and equal to 0 otherwise. Then we have

$$(W^{Cen}_{a,b,i}, \delta(W^{Cen}_{a,b,i})) = \begin{cases} (W_{a,k,i}, 1), & \text{if } k = b, \\ (W_{a,k,i}, 0), & \text{if } k \neq b, \\ (\tau - T^O_i - U^O_i, 0), & \text{if } W_{a,k,i} > \tau - T^o_i - U_i. \end{cases}$$

6.3.1 Expansion of Likelihood

In Section 4.4 the likelihood function of the observed claim development was presented. Now that the approach of modelling claim development by hazard rates have been adopted, the part of the likelihood corresponding to state changes can be extended. From earlier this part of the likelihood was written as

$$\prod_{i \geq 1} P^{\tau - T^O_i - U^O_i}_{X|t,u}(dX^O_i). \tag{28}$$

In previous studies, such as those conducted by Antonio & Plat (2014), Jin (2013) or Arjas & Haastrup (1996), the claim development has included some sort of distribution for payment sizes. Since our modelling focuses on the drivers of payments, i.e. the states of disability that are determined through out the lifetime of a claim, the entire claim process after reporting is described by the hazard rates. Using the hazard rates $h_{a,b}(t)$ and the observed times within states $V^O_{i,j}$, (28) can be extended.

For simplicity of notation let NE_i denote the number of evaluations observed for claim i, where an evaluation is defined to start as soon as a claim reaches a new state. Trivially the first evaluation for each claim starts when the state *Reported* is reached. Furthermore, model setup implies $max(NE_i) = 3$. Now 28 becomes

$$\prod_{i:T_i^O+U_i^O\leq\tau}\prod_{j=1}^{NE_i}\prod_a\prod_b h_{a,b}(V_{i,j})^{I_{\{a,b,i,j\}}}\times exp(-\int_0^{\tau_i}\sum_a h_{a,Tot}(u)I_{\{a,i,j\}}du). \tag{29}$$

Here $I_{\{a,b,i,j\}}$ is an indicator being equal to 1 if $V_{i,j}$ is a waiting time for the jump from state a to state b. The upper limit of the integral is given by $\tau_i = min(\tau - T_i^O - U_i^O, V_{i,j})$. $I_{\{a,i,j\}}$ is an indicator being equal to 1 if $V_{i,j}$ is a waiting time spent in state a.

To summarize, the likelihood of the claim development runs over all observed claims i. j runs over all state transitions observed for claim i. $h_{a,b}(V_{i,j})^{I_{\{a,b,i,j\}}}$ evaluates the likelihood of jumping from a to b after a waiting time of $V_{i,j}$ after arriving to a. $exp(-\int_0^{\tau_i}\sum_a h_{a,Tot}(u)I_{\{a,i,j\}}du)$ evaluates the likelihood of staying in state a for the time $V_{i,j}$.

Note that if a claim stays in state a until the time of censoring, $\tau - T_i^O - U_i^O$, then the likelihood of staying in a for that amount of time would be evaluated.

7 Estimation Results

7.1 Reporting Delay

In the distribution fitting of the reporting delay we note that some claims are reported immediately at the same day as the accident. Therefore, our first approach is to use a degenerate probability for those claims with a reporting delay equal to 0 days. For those claims with reporting delays that exceed 0 days we use MLE to fit positive-valued and heavy tailed distributions to data. The distributions considered were Gamma-, Weibull- and Pareto-distributions. The results from the fitting are illustrated in Figure 5 & 6.

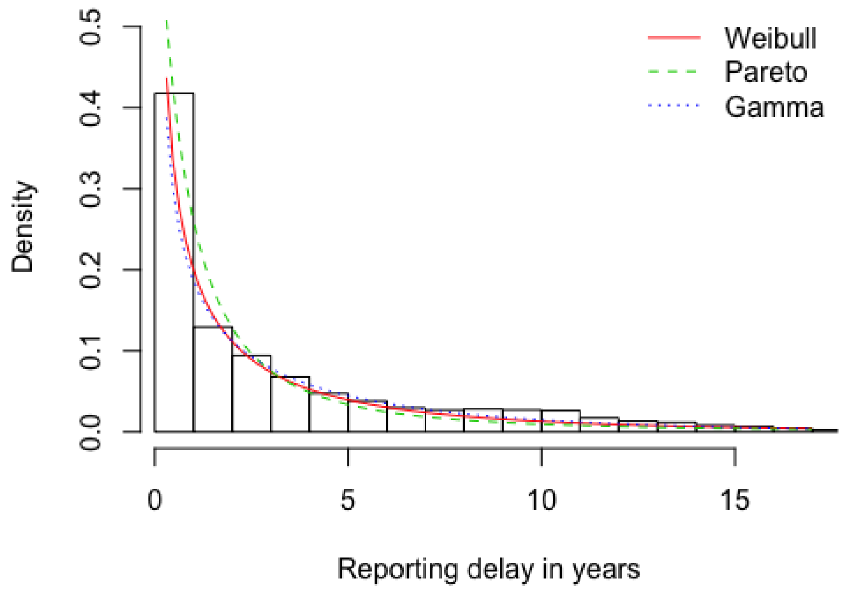
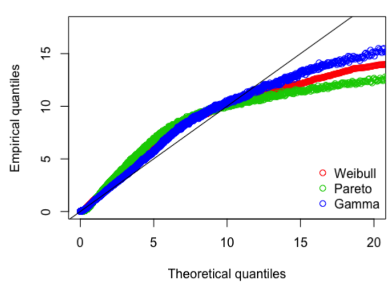
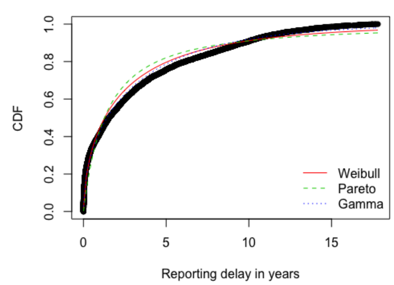


Figure 5: Histogram & theoretical densities for reporting delay distribution.



(a) QQ-plots of fitted distributions.



(b) Empirical vs theoretical CDFs.

Figure 6: QQ-plots and CDF.

As can be seen in the QQ-plots all fitted distributions are very heavy tailed compared to the empirical quantiles of reporting delays. Using any of these fitted distributions would overestimate the probability of extreme reporting delays.

However, the fitted Gamma-distribution seem to deviate the least in the tail, compared to the data.

For all three distributions there are two parameters in need of estimation. Each of Gamma, Weibull and Pareto have one scale- and one shape-parameter. In Table 12 parameter MLE's are illustrated.

	Gamma	Pareto	Weibull
Shape est	0.479417	1.266857	0.6085554
Shape st.err	0.007024706	0.05791646	0.006314671
Scale est	6.757480	1.738531	2.3324251
Scale st.err	0.157839138	0.13670154	0.050713371

Table 12: Parameter estimates from MLE of reporting delay.

Criterion	Gamma	Pareto	Weibull
AIC	**24273.82**	26527.54	24564.24
BIC	**24287.32**	26541.04	24577.73

Table 13: Akaike information criterion & Bayesian information criterion.

Table 13 shows a comparison of AIC & BIC for the MLE's of the parameter fittings for the three distributions. Based upon these criterion's the fitted Gamma-distribution seems to be the best fit for the reporting delay data.

Since the QQ-plots and both information criterion's indicate that the fitted Gamma-distribution are the best choice to represent the distribution of the reporting delay, we choose that alternative. The Gamma-distribution is fitted to the reporting delays > 0 days. For the reporting delays $= 0$ days we use a degenerate probability. Thus, the CDF of the reporting delay U becomes

$$F_U(u) = P(U \leq u) = p_0 + (1 - p_0)F_{gamma}(u : \alpha^g, \theta^g), \tag{30}$$

where, p_0 is the degenerate probability of a reporting delay being equal to 0 and is given by

$$p_0 = \frac{\#\{delays = 0\}}{\#\{delays\}} = 0.005056091.$$

$F_{gamma}(u : \alpha^g, \theta^g)$ is the CDF of a Gamma-distribution with α^g (Shape) and θ^g (Scale) as specified by the MLE above

$$(\alpha^g, \theta^g) = (0.479417, 6.757480).$$

It seems quite problematic that the estimated reporting delay deviates extensively in the right tail. Complications from using the gamma fit, with overestimated tail, would be that predicted reporting delays for IBNR claims from early accident years become unrealistically long. With this in mind we consider an alternative approach of modelling the reporting delay distribution.

From observing Figure 5 the empirical reporting delay behaves quite smooth. Therefore we consider using the empirical distribution. Specially for smaller reporting delays, data is compact and could be modelled using the empirical distribution. However in the tail, data is more sparse and thus we want to infer some fitted distribution that could smooth the tail behaviour.

In the alternative distribution fitting we use the empirical distribution for delays up to 10 years. This part of the empirical distribution corresponds to quantiles up to 0.908818. Thus if we let $F_U^{Emp}(u)$ denote the empirical CDF of the reporting delay, we have

$$F_U^{Emp}(10) = 0.908818.$$

In the tail fitting we thus have to model the delays exceeding 10 years. To get a smooth fit we estimate distribution parameters on reporting delay data which exceeds 8 years. We use MLE to estimate parameters for a Weibull distribution.

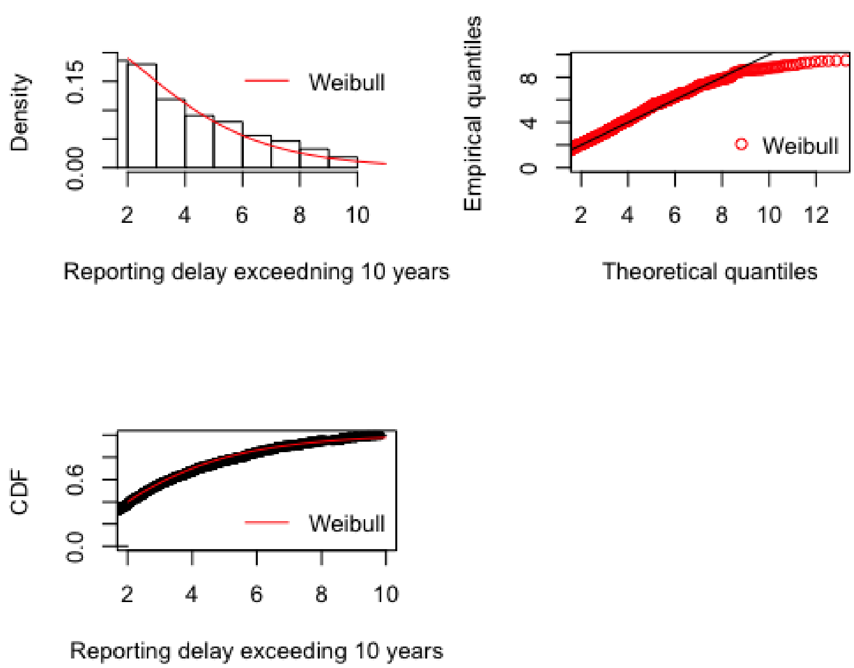


Figure 7: Weibull fit vs tail data. X-axis are the number of years exceeding 8 years of reporting delay.

In Figure 7 the results from the MLE fitting are illustrated. The Weibull fit, representing the right tail of the reporting delay, seem to be a good approximation of the most extreme delays. When comparing the QQ-plots this estimation with empirical probabilities for U up to 10 years and a Weibull distribution for $U > 10$ years seems to overestimate the extreme delays much less then the first approach with empirical probability for $U = 0$ years and a Gamma fit for the rest of the delays. Therefore using the second approach should affect the predictions of the IBNR- claims in such a way that the number of predicted IBNR claims for earlier years decreases. Further the simulated reporting delays for IBNR claims should decrease, specially for claims from earlier accident years.

	Weibull (for $U > 8$)
Shape est	1.239373
Shape st.err	0.03306081
Scale est	3.438841
Scale st.err	0.09536521

Table 14: Parameter estimates from MLE for tail of reporting delay.

Table 14 shows the parameter estimates for the Weibull distribution, representing the reporting delays that exceeds 8 years.

The Weibull distribution is for smoothness fitted to data of reporting delays that exceeds 8 years. Since we by setup only are interested in the distribution of delays that exceeds 10 years, we need to cut the Weibull distribution at 10 years. Let $P^*_{U^{Tail}}(U^{Tail} \leq u)$ denote the Weibull distribution for the reporting delays exceeding 8 years, where $U^{Tail} = U - 8$. Cutting the tail distribution at 10 years we get

$$\begin{aligned} P^*_{U^{Tail}}(U^{Tail} \leq u | U^{Tail} > 10 - 8) = \\ \frac{P^*_{U^{Tail}}(U^{Tail} \leq u) - P^*_{U^{Tail}}(U^{Tail} \leq 2)}{1 - P^*_{U^{Tail}}(U^{Tail} \leq 2)} \\ = \frac{F^{Wei}_{U^{Tail}}(u) - F^{Wei}_{U^{Tail}}(2)}{1 - F^{Wei}_{U^{Tail}}(2)}, \end{aligned} \tag{31}$$

where $F^{Wei}_{U^{Tail}}(u)$ is the CDF of the Weibull distribution with parameters as presented in Table 14.

Using (31) we get the combined distribution, of the empirical probabilities and the Weibull tail, as

$$F_U(u) = P(U \leq u) = \begin{cases} F_U^{Emp}(u), & \text{if } u \leq 10, \\ F_U^{Emp}(10) + (1 - F_U^{Emp}(10)) \frac{F_{UTail}^{Wei}(u-8) - F_{UTail}^{Wei}(2)}{1 - F_{UTail}^{Wei}(2)}, & \text{if } u > 10. \end{cases} \tag{32}$$

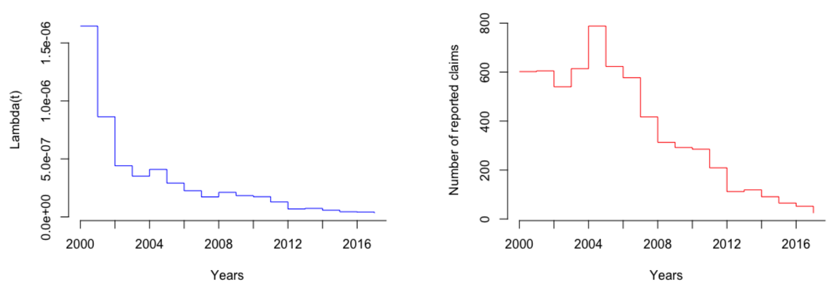
Due to the more realistic behavior in the tail we from this point and on use the estimated distribution presented in equation (32). Note that the chosen parametric distribution still overestimates the tail relative to the data. However, not as distinct as the first approach. The overestimation of the tail will partially have a countering effect on the underestimation arising from the censoring of data, presented in Section 6. On the other hand the actual underestimation effects from the censoring is unknown, therefore fitting a distribution with overestimated tail cannot be interpreted as a perfect adjustment.

7.2 Occurrence Intensity

The MLE of the piecewise constant $\lambda(t)$ is given by:

$$\lambda_y = \frac{NC(y)}{w_y \int_{d_{y-1}}^{d_y} P_{U|t}(\tau - t)dt}.$$

The MLE of λ_y depends on the observed occurrences of claims, the estimated reporting delay distribution and the exposure.



(a) Lambda(t).

(b) Number of reported claims.

Figure 8: Lambda and reported claims.

From studying Figure 8a, it can be concluded that $\lambda(t)$ is decreasing in years. With a decreasing $\lambda(t)$, the increase in claim occurrence intensity, suggested by the increasing exposure (Figure 3), is neutralized. Despite that the portfolio has a larger exposure towards economic disability claims for later years the trend of $\lambda(t)$ indicates that claim occurrence would not be expected to follow the same increasing pattern. The risk variable $\lambda(t)$ suggests that there exists other factors affecting claim intensity rather than just the exposure of collected premiums. One possible explanation could be changes in policy, regarding which damages/diagnostics that are classified to be eligible for economic disability compensation. Another possible explanation could be policy changes in pricing.

Figure 8b displays the number of reported claims per accident year. The number of reported claims are around 600 claims for the accident years 2000 to 2007 before starting to decrease for later years. This is partially a consequence of the long reporting the delay.

From using the framework of Marked Poisson Processes with piecewise constant estimations of $\lambda(t)$ and $w(t)$, the total number of claims occurring in year y follows a Poisson distribution with intensity $w_y \lambda_y \int_{d_y}^{d_{y+1}} (1 - P_{U|t}(\tau - t))dt$. These intensities, based upon previous estimations of $P_{U|t}(t)$ and $\lambda(t)$ are presented in Figure 9. The plotted intensities suggest that the expected number of claims occurring per accident year is on a steep decreasing trend from 2006 and forward.

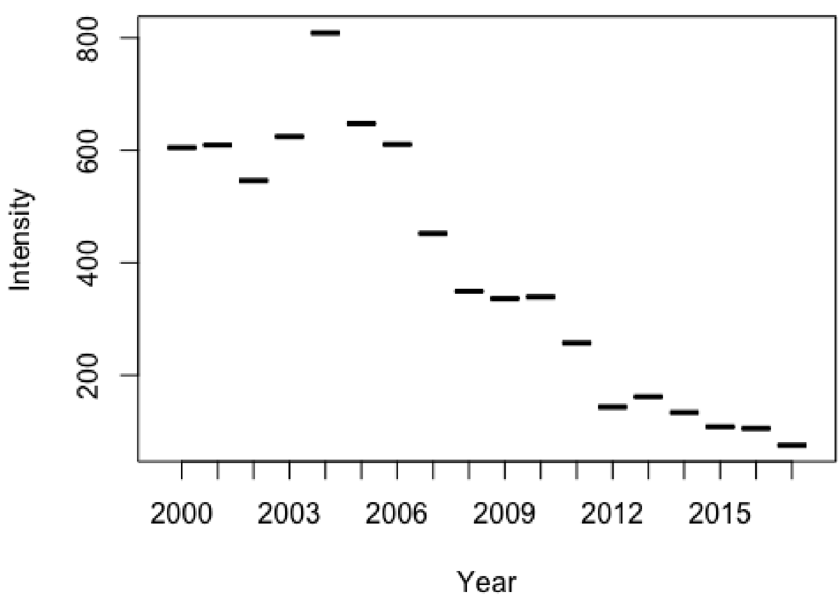


Figure 9: Poisson intensity for occurrence of claims per year.

7.3 Hazard Rates

The piecewise constant hazard rates are estimated using survival analysis, see Section 4.5 for detailed description.

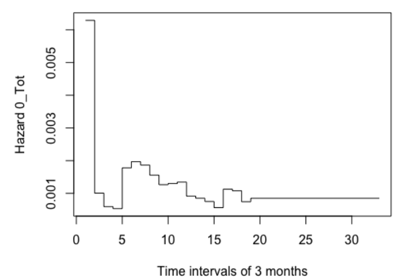
By gathering the waiting times $W_{a,b,i}^{Cen}$ and the censoring indicators $\delta(W_{a,b,i}^{Cen})$ from data, the 9 different hazard rate MLE's are estimated.

For the estimations of the piecewise constant hazard rates the intervals are set as follows

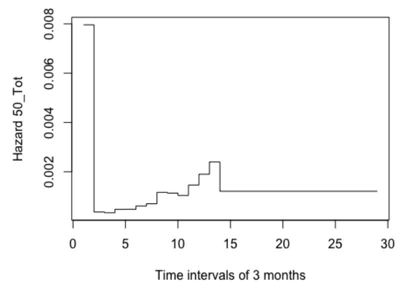
- The hazard rates $h_{0,b}(t)$ are constant on intervals $[0, 3)$ months, $[3, 6)$months,..., $[51, 54)$months and ≥ 54 months.
- The hazard rates $h_{50,b}(t)$ are constant on intervals $[0, 3)$months, $[3, 6)$months,...,$[36, 39)$months and ≥ 39 months.
- The hazard rates $h_{75,b}(t)$ are constant on intervals $[0, 6)$months, $[0, 6)$months...,$[36, 42)$months, and ≥ 42 months.

From investigating the available claim data specifically regarding the state change decisions, we identify a strange feature. Namely that there are no decisions made before 2007. This feature would have a significant impact on the hazard rate estimations, as it entails a substantial lag for the first decisions for claims originating from before 2007. The source of this behaviour is uncertain, and could perhaps be an affect of some change in policy or database transformation. However, investigations of data also show that the lag pattern is not representative for first decisions related to more recent claims. Instead first decisions appear on a more continuous basis for these claims. With this in mind the observed lag is considered to be undesirable for the hazard estimations. As the lag only affects the first decisions, we use jump data from claims who have occurred in 2007 or later for the estimations of all hazard rates from *Reported*, $h_{0,b}(t)$. As first decisions data is the most frequent of all jump observation, this cutoff still generates reasonable data amounts. Furthermore, using the subset of first decision data as of 2007, the hazard estimates reflect the jump behaviour for more recent observations. As the proportion of unobserved development increases with time so should also the required simulations. Therefore using estimations based on more recent data can be justified.

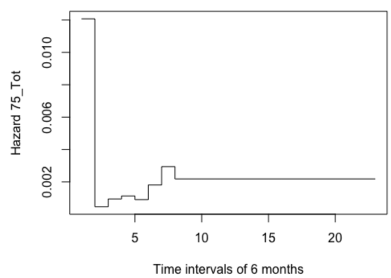
Figure 10 displays the MLE's of the total hazard rates of survival in states $\{Reported, 50\%, 75\%\}$, i.e $h_{0,Tot}(t)$, $h_{50,Tot}(t)$ and $h_{75,Tot}(t)$ respectively. Remember that $h_{a,Tot}$ represents the total intensity of leaving state a. Illustrations of the 9 individual hazard rate estimations are displayed in the Appendix.



(a) Hazard rates from *Reported*.



(b) Hazard rates from 50%.



(c) Hazard rates from 75%.

Figure 10: Total hazard rates for leaving states $\{Reported, 50\%, 75\%\}$.

In the construction of intervals for the piecewise constant estimation we try to set boundaries such that each interval captures enough observed data to produce relatively smooth estimates. Thus, the intervals are set quite wide. However, the MLE's still produce some relatively unstable hazard estimates, as can be seen specially for $h_{50,75}(y)$, $h_{50,100}(y)$ and $h_{75,100}(y)$. The simple explanation would be lack of data. The portfolio of economic disability is long tailed and slow developed, which contributes to a limited amount of data for jump decisions, even though the observational period is as long as 18 years.

The lack of data related to later decisions i.e. jumps from state 50% and 75% could be explained by the portfolio characteristics as well as the behaviour of observed data from the first decision, i.e. hazard rates from *Reported*. The event of an individual being exposed to an accident with consequences that would enable them to be eligible for economic disability compensation is rare. Therefore the inflow of claims to the portfolio is limited. Furthermore, the amount of claims arriving at the portfolio seems to be considerably larger than the amount that are actually assigned a disability level worthy of compensation($> 0\%$). From the data of claims that have been assigned a first decision approximately 71%

have been settled at 0%. Thus, a majority of claims get assigned a first decision of 0% disability, and are never eligible for later decisions. This clearly affects the amount of data available for the later decisions.

The pattern of settling at the current disability level, rather then jumping to a higher level, seems to persist for states 50% and 75% as well. In state 50%, of the amount of claims which have made a jump 74 % settles at the current level. For state 75% the corresponding proportion is 84 %. The reason for this behaviour could at least partly be explained by extensive investigations which are conducted before a decision is made. The severity of payments associated with economic disability is of great impact both to companies and individuals. Therefore decisions are thoroughly analyzed before determined, which should increase the probability that the first decision is the "correct" one. This behaviour of decision making could thus further explain the lack of data for hazard estimates of later jumps.

The time window of observation is another important factor to the amount and quality of available data. The window we have used might seem quite big, as it is constituted by 18 years of observations. However, with a long tailed portfolio, the right time censoring still becomes influential. The parts of data which are most affected by time censoring are of course the later decisions. If these jumps are to be observed, previous elements such as reporting delays and earlier decisions, are not allowed to be too time consuming, subject to the occurrence time. The obvious remedy for sparse data would be to increase the window of observation. However, using data from too far back in the past could instead create other problems, mainly of quality. In the judgment of which historic years to include in the analysis one must incorporate factors regarding business development, society trends and changes in policy. Looking back too far, the portfolio under consideration was not constructed in the same way with similar levels of disability. Further, decision policies have been different historically. Thus, including claim data from a different era would affect hazard estimates to not represent the behaviour of the practical decision policies that are used today. Determining how large a time window of observation to use could be seen as a trade-off problem between amount and quality.

With our model setup as presented in Figure 1, the observation window from 2000 to 2017 is deemed appropriate to maximize the amount of data subject to quality. The quality factor is based upon presence of relevant decision policies as well as implementation of a similar state model.

As presented, the last intervals of the various hazard rates are set with an open right boundary. The right censoring prevents us from observing the full state change development for some claims in the data. With an open last interval we let the tail of the survival in states be constant for all of the most extreme survival times. Since this last piece of the estimates should represent the survival patterns as time goes towards infinity, we want to set these intervals such that we capture enough data to make the tail representative of the earlier piecewise levels. Typically, it would be preferable if the last interval does not shift to an extremely high level relative to hazard rates in earlier intervals.

8 Simulations

As mentioned in Section 4.1 there are three different claim types under consideration: settled, RBNS and IBNR. For settled claims the full developments are available and therefore no simulations are needed. For the second claim type, RBNS, the developments are partly known, i.e. developments up to end time of observation τ. However, the future developments of the claims need to be simulated. For the third claim type, IBNR, no parts of the development are available and the full development of the claim needs to be simulated. Furthermore, the number of IBNR claims per accident year needs to be simulated as well.

An individual claim simulation, either RBNS or IBNR, ends when the state *Closed* is reached. From our model construction the maximum number of state jumps is 3, i.e $Reported->50\%->75\%->100\%$ or $Reported->50\%->75\%->Closed$. Note that claims are assumed to close immediately when reaching the state 100%.

8.1 Incurred But Not Reported

The simulation of IBNR claims include several steps. Firstly the number of IBNR claims for a given accident year must be simulated. Further simulations are required for the elements of reporting delay and state changes. In this section we present the simulation scheme that are implemented for IBNR claims.

8.1.1 Number of IBNR Claims for a given Period

From the presented framework the number of IBNR claims follows a Poisson process with intensity:

$$w(t)\lambda(t)(1-P_{U|t}(\tau-t)),$$

where $\lambda(t)$ and $w(t)$ are piecewise constant on yearly intervals. $N_{IBNR}(y)$, the number of IBNR claims occurring in interval $[d_y, d_{y+1})$, year y, has distribution:

$$N_{IBNR}(y) \sim Poisson\big(w_y\lambda_y \int_{d_y}^{d_{y+1}} (1-P_{U|t}(\tau-t))dt\big). \tag{33}$$

The simulation of $N_{IBNR}(y)$ is performed by drawing from (33), where the parameters i.e. the intensity $\lambda(t)$ and the reporting delay distribution $P_{U|t}(t)$ have been estimated previously.

8.1.2 Accident Date

The occurrence date of a claim, occurring in year y, is assumed to be uniformly distributed on $[d_y, d_{y+1})$. Hence, the occurrence date is generated from a uniform distribution over the days of year y.

8.1.3 Reporting Date

Next, we need to simulate the reporting date of the IBNR claims occurring in accident year y. This is done by simulating the reporting delay. The reporting delay for an IBNR claim is generated by the estimated reporting delay distribution (32). The constraint we have to consider is that the reporting delay must exceed the time difference between the accident date and the cutoff date of observation τ.

$$P(U \leq u | U > \tau - t) = \frac{P(\tau - t < U \leq u)}{1 - P(U \leq \tau - t)}.$$

A simulated reporting delay $\hat{u}$ is attained by drawing $q \sim U(0,1)$, and using this q to invert the probability above.

$$\frac{P(\tau - t < U \leq u)}{1 - P(U \leq \tau - t)} = q,$$

$$P(\tau - t < U \leq u) = q(1 - P(U \leq \tau - t)),$$

$$P(U \leq u) - P(\tau - t \leq U) = q(1 - P(U \leq \tau - t)),$$

$$P(U \leq u) = q(1 - P(U \leq \tau - t)) + P(\tau - t \leq U),$$

$$\hat{u} = F_U^{-1}(q(1 - P(U \leq \tau - t)) + P(\tau - t \leq U)).$$

Note that our F_U follows mixture of the empirical distribution and a Weibull tail distribution as presented in (32). By using the estimated CDF representing the reporting delay we numerically solve for $\hat{u}$.

8.1.4 Time to next Jump

After the simulation of reporting delay, the next step is to simulate state changes. In this process we begin with the time of jumps.

For IBNR claims there are no constraints required for the time to next jump simulation. Therefore, given that the current state is a, such a time can is obtained using the probability

$$P(V \leq v_{next}) = 1 - e^{-\int_0^{v_{next}} h_{a,Tot}(t)dt}.$$

Similar to in previous step we invert the probabilty by drawing $q \sim U(0,1)$ and solving for $\hat{v}_{next}$

$$exp(-\int_0^{v_{next}} h_{a,Tot}(t)dt) = 1 - q,$$

$$\int_0^{v_{next}} h_{a,Tot}(t)dt = -ln(1-q).$$

With our piecewise constant specifications of $h_{a,Tot}(t)$ the above expression is solved numerically to obtain $\hat{v}_{next}$

8.1.5 Next State

When a time of next jump have been simulated, we need to determine which state the claim jumped to. The next state is determined by the jump time and the individual hazard rate estimates corresponding to the current state a. If the current state is *Reported*, which it always will be initially for a IBNR claim, the next state is:

$$next\_state : \begin{cases} 50\%, & \text{if: } 0 \leq q < \frac{h_{0,50}(t)}{h_{0,Tot}(t)}, \\ 75\%, & \text{if: } \frac{h_{0,50}(t)}{h_{0,Tot}(t)} \leq q < \frac{h_{0,50}(t)+h_{0,75}(t)}{h_{0,Tot}(t)}, \\ 100\%, & \text{if: } \frac{h_{0,50}(t)+h_{0,75}(t)}{h_{0,Tot}(t)} \leq q < \frac{h_{0,50}(t)+h_{0,75}(t)+h_{0,100}(t)}{h_{0,Tot}(t)}, \\ Close, & \text{if: } \frac{h_{0,50}(t)+h_{0,75}(t)+h_{0,100}(t)}{h_{0,Tot}(t)} \leq q \leq 1, \end{cases}$$

where $q \sim U(0,1)$.

A similar scheme is used when the current state is either $\{50\%, 75\%\}$ although the alternatives for next state are reduced.

I.e given the time of next jump $\hat{v}_{next}$ and the current state a, the probability of going to state b is:

$$\frac{h_{a,b}(t)}{h_{a,Tot}(t)}.$$

If the simulated next state is $\{50\%, 75\%\}$ the algorithm returns to 8.1.4. Else if the simulated next state is $\{100\%, Close\}$ the development of the claim is done. Thus, a settlement date can be set and another IBNR claim can be simulated.

8.2 Reported But Not Settled

RBNS simulations include times and types of state changes after the censoring date τ. Here the simulation of the first event after censoring is different than for later events. In this section we present the simulation scheme used for RBNS claims.

8.2.1 Time to First Jump After Censoring

For RBNS claims, parts of the development are known. At minimum accident date and reporting date. For RBNS, the time to next jump must satisfy $v_{next} > c$ for $c = \tau - t_i^{last}$, where t_i^{last} is the last recorded event date for claim i. Therefore, to simulate the next jump time we have to consider the conditional probability $P(V < v_{next}|V > c)$.

$$P(V \leq v_{next}|V > c) = \frac{P(c < V \leq v_{next})}{1 - P(V \leq c)}.$$

To simulate the time of next jump $\hat{v}_{next}$ we again use $q \sim U(0, 1)$ and invert the probability above as (again, the simulation depends on the current state a)

$$\frac{P(c < V \leq v_{next})}{1 - P(V \leq c)} = q,$$

$$P(c < V \leq v_{next}) = q(1 - P(V \leq c)),$$

$$P(V \leq v_{next}) - P(c \leq V) = q(1 - P(V \leq c)),$$

$$P(V \leq v_{next}) = q(1 - P(V \leq c)) + P(c \leq V).$$

Inserting the CDF of survival times expressed with the hazard rates:

$$P(V \leq v_{next}) = 1 - e^{-\int_0^{v_{next}} h_{a,Tot}(t)dt},$$

gives:

$$1 - e^{-\int_0^{v_{next}} h_{a,Tot}(t)dt} = q(1 - P(V \leq c)) + P(c \leq V),$$

$$e^{-\int_0^{v_{next}} h_{a,Tot}(t)dt} = 1 - q(1 - P(V \leq c)) + P(c \leq V),$$

$$\int_0^{v_{next}} h_{a,Tot}(t)dt = -ln(1 - q(1 - P(V \leq c)) + P(c \leq V)).$$

With our piecewise constant specifications of $h_{a,Tot}(t)$ the above expression is solved numerically to obtain $\hat{v}_{next}$.

8.2.2 Next State

The simulation of next state is the same as in the IBNR case. I.e given the time to next jump $\hat{v}_{next}$ and the current state a, the probability of going to state b is:

$$\frac{h_{a,b}(t)}{h_{a,Tot}(t)}.$$

If the simulated next state is $\{50\%, 75\%\}$ the algorithm returns to simulate the time of the next event, but this time by the same procedure as for an IBNR claim 8.1.4 , since there are no restrictions on $\hat{v}_{next}$. Else if the simulated next state is $\{100\%, Close\}$ the development of the claim is done. Thus, a settlement date can be set and another RBNS claim can be simulated.

9 Results Simulation

9.1 Claim Level

Table 15 shows an illustration of results from simulation on claim level. The first three claims are of type RBNS while the last three are of type IBNR. Note that these are not chosen randomly, but rather chosen to display the development of claims. Choosing a subset randomly would mainly include claims settled without any decisions $> 0\%$, since that particular development is the most common in the data and therefore has that impact on the estimated hazard rates.

ID	Acc. date	Rep. date	Set. date	D1	Date D1	D2	Date D2	D3	Date D3
1R	16-12-02	17-01-13	18-06-24	50%	18-01-30	-	-	-	-
2R	13-07-21	15-04-13	19-01-27	50%	16-03-23	50%	19-01-27	-	-
3R	13-03-05	15-10-16	22-06-25	50%	22-05-08	-	-	-	-
1I	15-05-17	20-01-11	22-03-18	50%	20-04-12	50%	22-03-18	-	-
2I	14-05-02	21-04-22	27-10-20	50%	24-12-29	50%	27-10-20	-	-
3I	08-01-17	19-07-01	33-01-25	50%	33-01-05	-	-	-	-

Table 15: Example of claim developments. Red dates and decisions indicate simulated data.

9.2 Portfolio Level

In this section we present the results of our simulations on portfolio level. Table 16 displays the distribution of claims settling in each state, based on 200 simulations. Note that this is the end distribution for the total portfolio, i.e. settled, IBNR and RBNS accumulated. From the simulation results it can be concluded that a majority of the claims never reach a state with a disability classification, but rather settles directly from state *Reported*. This pattern is in line with the behavior observed from the estimated hazard rates. In every state the jump intensity is substantially larger for jumps to state *Closed*.

For the approximately 26% of claims with an end state separate from 0%, the end state 100% is the most common with a proportion of 14.47%. In addition to the average number of claims, empirical quantiles for each end state are displayed in Table 16. The differences between the $2,5\%$- and $97,5\%$-quantiles are quite small ($< 1\%$). This could be an indication of stability, i.e. the outcomes are similar between simulations.

End state	# of claims	% of total claim	Quantile 2,5%	Quantile 97,5%
0%	5115.45	73.99%	73.66%	74.28%
50%	659.91	9.55%	9.25%	9.82%
75%	138.38	2.00%	1.87%	2.17%
100%	1000.23	14.47%	14.15%	14.80%

Table 16: Display of means and quantile for 200 simulations.

In Table 17 the average times to settlement from the 200 simulations are displayed. Time to settlement is defined as the time difference between the registration date and the settlement date. The indication that time to settlement on average is the shortest for settlement from state *Reported* seems reasonable. Firstly, this is motivated by the hazard rate patterns of jumping to *Closed* with the highest intensity, which from state *Reported* partly could depend on time requirement of decisions to reject disability compensations vs substantial compensation levels of the other states. Furthermore, the event of settling at 0% only requires the time of one jump/decision, while settlements in other states require at least two. The exception is settlement in state 100% which could be determined through one decision only, since we have assumed that claims settle immediately when reaching 100%. However, settling in state 100% could also be achieved through 2 or 3 intermediate decisions.

Settling in state 75% seems to have quite a short average time requirement relative to the other states. Specially since that event requires either 2 or 3 intermediate decisions. However, the proportion of claims that has 75% as end state is quite small, which affects the stability of the average time to settlement measure.

End state	Time to settlement	Quantile 2,5%	Quantile 97,5%
0%	1025.7	0	4872
50%	2009.3	5	5560.2
75%	1773.1	5	4958
100%	1833.5	5	5300.7

Table 17: Time to settlement in days.

9.2.1 IBNR Claims

In this section we examine the results of the simulations of the decomposition of the portfolio corresponding to IBNR claims. As presented previously the simulation procedure for this particular subclass is divided into two main parts, number of claims and individual claim development.

9.2.1.1 Number of IBNR Claims

The average amounts of IBNR claims per accident year are displayed in Table 18. The presented pattern is not what you would expect. In a text book example, with constant claim occurrence intensity, the number of IBNR claims should steadily increase with time, i.e. more recent accident years should have a larger amount of IBNR claims. This is due to the effect of the reporting delay and the decreasing window of observations as the accident years approaches the evaluation date τ. However, in excess of the reporting delay, number of IBNR claims also depends on the claim occurrence intensity $\lambda(t)w(t)$. The exposure measure of earned premiums is as presented in Figure 3 on an increasing trend, which ceteris paribus (all else fixed) would motivate an increasing trend in total amount of claims per accident year, and thus the same behaviour of the amount of IBNR claims. But the occurrence intensity measure has another parameter, $\lambda(t)$, estimated from the pattern of observed claim occurrences (RBNS & settled) in data. As presented in Figure 8b the amount of reported claims in data does not follow a smooth decreasing trend over the accident years. The volatile pattern of the amount of reported claims in data is affecting the estimated $\lambda(t)$

to counter the increasing trend of the occurrence intensity, implied by the exposure $w(t)$. Thus, it also impacts the simulations of number of IBNR claims not to follow a text book increasing trend over the accident years.

Accident year	# of IBNR claims
2000	2.69
2001	4.3
2002	6.1
2003	10.52
2004	20.6
2005	24.9
2006	33.7
2007	35.3
2008	36.2
2009	44.4
2010	54.6
2011	47.6
2012	31.7
2013	42.2
2014	43.4
2015	42.9
2016	53.5
2017	52.2
Total	586.8

Table 18: Average number of IBNR claims per accident years for 200 simulations.

9.2.1.2 Distribution of the Development for IBNR Claims

Table 19 displays the (IBNR) end state distribution for 200 simulations. Comparing these values with the end state distribution of settled claims, displayed in Table 7, a notable difference is identified. Larger proportions of the settled claims have the end states 50% or 100%, while for IBNR claims, the end state 0% dominates even more distinct. The difference between the distributions of IBNR and settled claims is noteworthy. As the results of pure simulations (IBNR) are based completely on the estimated parameters, the intuition is that the end state distribution would be similar between simulations and observed data. However, in the comparison there is one additional factor we have to account for. That is the cutoff in data, applied for the hazard rate estimations of first decision jumps. This modification of the estimation data could well contribute to the difference of IBNR and settled distribution, if the jump pattern varies over the total observation period (i.e before vs after 2007).

End state	# of claims	% of total claim	Quantile 2,5%	Quantile 97,5%
0%	511.6	87.35 %	84.85%	90.14%
50%	24.0	4.10%	2.62%	5.72%
75%	8.2	1.41%	0.51%	2.45%
100%	41.9	7.15%	5.19%	9.19%

Table 19: End state distribution of IBNR for 200 simulations.

To investigate the effect of the cutoff in hazard rate data (first decision only), Table 20 displays the end state distribution of the subset of settled claims originated from dates past 2006-12-31. This is the subset of data which constitutes the basis of the particular hazard rate estimations. The subset distribution indicates that a larger proportion of claims settles directly in 0%, as is the case for the IBNR simulation. This pattern explains the difference in end state distribution of IBNR and settled claims over the full observational period. Worth nothing is that the subset distribution of settled claims (accident year > 2006) is not expected to match the IBNR distribution completely, since it is only the first decision hazard rates that are solely based on jump data from claims that occurred after 2006-12-31.

End state	# of claims	% of total claims
0%	1560	89.3 %
50%	59	3.4 %
75%	17	1.0 %
100%	111	6.3%

Table 20: End state distribution of subset of settled claims, such that the occurrence date happened later than 2006-12-31.

In Table 21 the average times to settlement for IBNR claims are displayed. The measured times follow the same pattern as for the case of settled claims. I.e the state 0% has the shortest time to settlement on average. Further comparison with the case of settled claims indicates that the IBNR category has a lot shorter development times in general. Again, this is due to the lag pattern in first decision data that was presented and dealt with in the hazard rate estimation. The volatile behaviour of time requirements in the portfolio is reflected by the wide spread between the quantiles of the two tails of the simulated distribution.

End state	Time to settlement	Quantile 2,5%	Quantile 97,5%
0%	595.9	4.1	3250.6
50%	991.5	23.1	4303.6
75%	936.4	25.9	4821.9
100%	743.5	4.2	3718.1

Table 21: Time to settlement in days for IBNR claims.

9.2.2 RBNS Claims

In this section we examine the decomposition of the portfolio related to the RBNS claims and their development.

9.2.2.1 Distribution of the Development of RBNS Claims

In Table 22 the distribution of end states for RBNS claims are displayed. Comparing to the end state distribution of IBNR- and settled claims, Tables 19 and 7, the end state 0% is less dominant for RBNS. Furthermore the share in end state 50% is notably larger for RBNS relative to the other subclasses. This pattern is logical since the RBNS claims has a known development part at the evaluation date τ. As presented in Table 10 of the current state distribution of the RBNS-claims there is a proportion of approximately 20% of open claims stationed in the states 50% and 75%. By model construction these claims can never settle at a level of 0% disability, why the shift towards higher levels in the end distribution for RBNS claims is natural.

As an additional remark, the simulation of jumps, both destinations and times, are shifted more to the right tails of the hazard rates for the RBNS category. This is due to the restrictions of the next jump time, which by definition of RBNS can not occur before the evaluation date τ.

End state	# of claims	% of total claim	Quantile 2,5%	Quantile 97,5%
0%	598.6	69.61%	67.33 %	71.51%
50%	109.8	12.77%	10.47 %	14.77%
75%	23.7	2.76%	1.63%	3.72%
100%	127.8	14.87 %	12.67%	18.33%

Table 22: End state distribution of RBNS for 200 simulations.

The average times to settlement for RBNS claims are displayed in Table 23. In comparison to settled and IBNR, the development times are substantially longer across all end states for RBNS. This is explained by the fact that the next jump time, for all RBNS claims, are constrained to be larger than the time remaining to τ =2018-01-01. As the observed RBNS claims range from 2000-2017, this of course yields large times to settlement. Another aspect from the comparison is that the end state 0% does not have the distinct decrease in time requirement as observed for settled and IBNR. This is also affected by the time remaining to τ. As 0% is the most common end state even for RBNS claims, a large proportion of those claims originated from early accident years will settle at 0%. Therefore, the average time to settlement in 0% becomes longer relative to the other end states.

End state	Time to settlement	Quantile 2,5%	Quantile 97,5%
0%	2360.5	220.3	6789.8
50%	2480.5	579.3	6270.2
75%	2237.7	248.6	4487.9
100%	2470.3	412.7	6334.9

Table 23: Time to settlement in days for RBNS claims.

9.3 Introduction of Payments & Comparison to Chain-Ladder

The micro-level reserving model constructed in this thesis treats the claim development of an economic disability portfolio. Up to this point, the actual payments related to the claim developments have been disregarded. Instead the modelling has been focused on the disability levels of states, which are the drivers of the sizes of the payments. With the purpose of evaluating the performance and usability of the considered micro model we now introduce a completely hypothetical insurance amount $A = 1000000$ units. By the introduction of A, the state development can be converted to payments, and the predictions of future development can more clearly be related to an outstanding liability (reserve) of the portfolio. With this approach we can also apply an aggregate reserving technique such as Chain-Ladder and compare the two methods.

With the introduced insurance amount we assume payments in the portfolio to follow the scheme

$$payment = \begin{cases} A * 0.5, & \text{if } State = 50\%, \\ A * 0.75, & \text{if } State = 75\%, \\ A * 1, & \text{if } State = 100\%. \end{cases}$$

The payments corresponding to state changes are incremental, i.e. a claim who first gets a disability level of 50% generates a payment of $0.5A$. If the same claim later changes disability level to 75% the new payment is trivially $0.25A$, such that the total payments made matches the total disability level of the claim. Furthermore, the payments are assumed to occur at the same time as the corresponding state change.

Ideally the performance evaluation should be built upon data from the portfolio up to some past date. In essence, it would be preferable to shift τ back in time and re-estimate all parameters of the model based on that subset of data. Then the simulated estimations of outstanding liabilities, either from the micro model or some alternative methods, could also be compared to the true development shown from the later data set, kept out of the modelling. However, due to a sparse set of data this procedure can not be applied. If we were to cut the observational data a few years back, the parameter estimations would not have sufficient amounts of data for satisfactory approximations. Specially the hazard rate estimations would become unusable. This problem with data arises as an effect of the long tailed and slow developed characteristics of the economic disability portfolio.

9.3.1 Comparison with Mack Chain-Ladder

9.3.1.1 Modifications

As an investigation of usability and performance of our estimated micro model we chose to compare it to the Mack Chain-Ladder method. Mack's Chain-Ladder incorporates a standard error to the point estimate of the reserve (Mack, 1993).

In order to compare the prediction of our micro model with the Mack Chain-Ladder some modifications were performed. Firstly, since the portfolio of economic disability is characterized by slow development and a long tail, there is a large delay between injury date and first decision date. As a consequence, some accident years have no payment development in their first or second development year. This is problematic for the Chain-Ladder technique, since development factors become either undefined or unrealistic. To handle this slow development, we join the first three development periods for each accident year. Thus the first development period is the payment development originated from the accident year plus two additional years. This adjustment makes us unable to use Chain-Ladder on accident years 2016-17. Furthermore, due to no payment development in any of the of the first three years after accident we are also unable to use accident years 2000-04 in the Chain-Ladder.

As a consequence of us only being able to use data for the accident years 2005-2015, with development data up to 2017, the Chain-Ladder can only predict the development 12 years into the future. Therefore, the development of each claim has to be cut after 12 years in the micro model simulation as well, for the methods to be comparable. As a consequence the estimated outstanding liability is interpreted as a 12 year reserve, rather than a total ultimate of the portfolio.

The payment triangle, derived from the data for 2005-2017, used in the reserve prediction with Mack Chain-Ladder is displayed in the Appendix.

9.3.1.2 Reserve Estimates

In Table 24 the results from 1000 simulations with our micro model are displayed, together with the Mack's Chain-Ladder estimate. From comparison of the estimates two conclusion can be drawn. The micro model predicts a substantially lower mean of the outstanding liability 12 years forward. Furthermore, the standard error is much larger for the Mack Chain-Ladder method, approximately with a factor of 10. With no available data representing the true value of

	Micro Model	Mack's Model
Paid out	435 750 000	435 750 000
Ultimate*	490 011 250	776 855 723
Outstanding liability	54 261 250	341 105 722
S.E	6 130 720	57 702 177

Table 24: Results of Micro model simulations & The Mack Chain-Ladder model (Paid out represents the observed payments for Settled and RBNS, Outstanding liability are the predicted payments for RBNS and IBNR).

Accident year	Paid	Micro Model	Mack's Model
2005	134 250 000	2 717 250	0
2006	132 000 000	8 193 000	14 757 764
2007	57 250 000	4 827 250	17 750 907
2008	17 250 000	4 382 500	9 078 616
2009	18 750 000	5 578 000	15 265 773
2010	17 500 000	7 088 500	20 106 083
2011	14 750 000	5 556 250	25 954 543
2012	16 750 000	3 204 500	41 496 866
2013	15 250 000	4 312 750	61 403 508
2014	8 500 000	4 200 250	64 805 404
2015	3 500 000	4 201 000	70 486 259

Table 25: Outstanding liability of Micro model simulations & The Mack Chain-Ladder Model expressed per accident year.

the 12 year outstanding liability no conclusion on accuracy can be drawn from the comparison.

Chain-Ladder methods are best suited for portfolios with a substantial proportion of early development. Further, similar development patterns across various accident periods are preferred, for stability of the development factors. The characteristics of the particular portfolio of economic disability are however quite the opposite. The claim development in the portfolio is rather volatile and slow developed. Therefore the substantial uncertainty of the Mack Chain-Ladder estimate is expected. However, the lesser standard error from the micro model simulations does not prove that method to be more accurate.

The distributions of paid and outstanding liabilities from the two models, decomposed by accident years, are displayed in Table 25. From the values in the table it can be concluded that the difference in prediction of the 12 year outstanding liability between the micro model and Mack's model is mainly explained by the predictions for most recent accident years. This could be due to that accident years 2005 and 2006 seem to follow a different development pattern, see Figure 11. As the development factors are based on all accident years, if 2005 and 2006 follows a more rapid development pattern, Mack's chain-ladder will overestimate the development for the later accident years. Specially

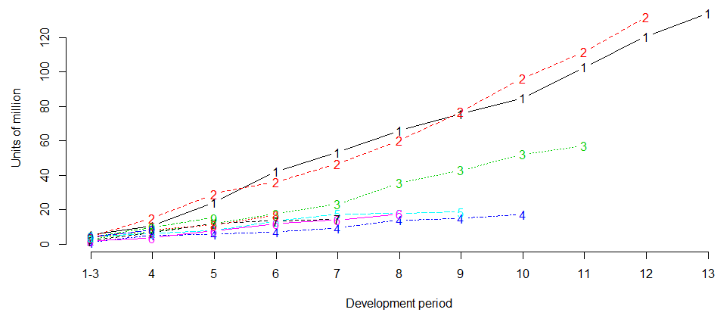


Figure 11: The claim cost development for accident years 2005-2015.

late development periods are affected, as those development factors are mainly or fully based on the development of the accident years 2005 and 2006. From Figure 11, this scenario seems reasonable and could be part of the explanation of the large deviation between the models.

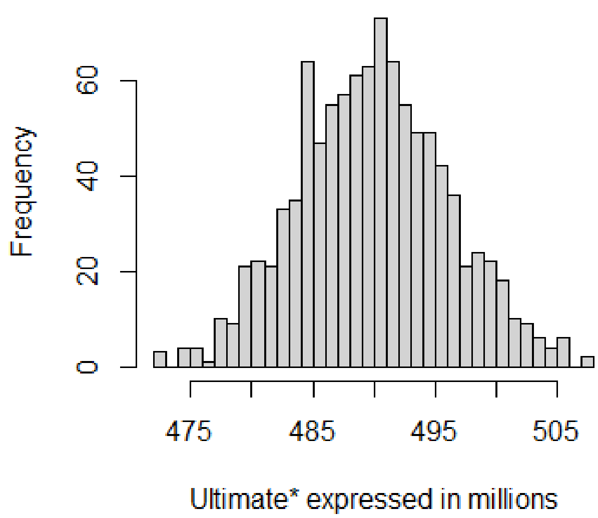


Figure 12: Histogram over 1000 simulations of Ultimate* (x-axis expressed in units of 10^6).

In Figure 12 a histogram of the distribution of ultimate's for the micro model simulations are illustrated. Ultimate* indicates that the ultimate represents total losses for a period of 12 years after the accident.

As an illustration of the effect of cutting the micro model simulations at 12 years, Figure 13 displays the ultimate distribution for full simulation developments. The presented results are still based upon simulations of accident years 2005-2015. As the histogram illustrates the ultimate distribution is shifted approximately 50 million units to right, for the full development relative the cut development. However, the full development predicted by the micro model is still considerable smaller than the ultimate* estimate produced by the Mack Chain-Ladder method.

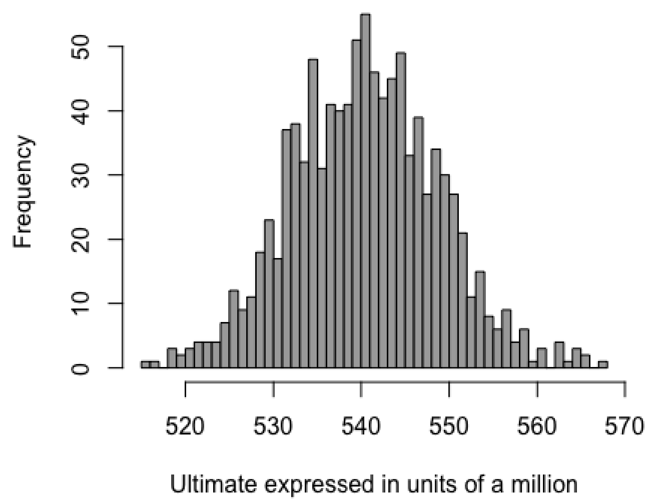


Figure 13: Histogram of 1000 simulations of Ultimate (x-axis expressed in units of 10^6).

10 Conclusions & Discussion

In this section we list the main conclusions arising from our results, followed by discussions around these conclusions as well as possible subjects for future research and model extensions.

- The outcomes from the predictions of the 12 year liability are widely different comparing the micro model to the Mack Chain-Ladder. Both with regards to the expected outcome as well as the variability.
- The micro model predicts a considerably lower 12 year ultimate than Mack Chain-Ladder.
- The standard error from the Mack Chain-Ladder predictions is considerably larger compared to the simulations of the micro model.
- No conclusion can be drawn regarding accuracy from the comparison of the two reserving models.
- End state distributions and times to settlement from the simulations of the portfolio follows patterns that are representative of the reported data.
- The distinct decreasing trend in number of reported claims has a significant impact on the occurrence intensity for recent accident years. Further this impacts simulations of number of IBNR claims as well as the prediction of the outstanding liability.
- The data of collected yearly premiums indicates increasing exposure to claim occurrence with time. This effect is countered by the observed patterns from the number of reported claims and the reporting delay.
- The adjustment of using a subset of the data for estimation of first decision hazard rates impacts the outcomes of the simulations to differ from the distributional patterns of end states and time to settlement relative the reported data from the entire observational period.

The difference, of the 12 year ultimate estimates, from the micro model compared to Mack Chain-Ladder method is noteworthy. One reason for the low prediction of outstanding liability produced by the micro model is derived to the low estimate of the claim occurrence intensity for later accident years. This parameter estimation is affected by the amount of reported claims as well as the reporting delay. As presented in Section 6 the parametric distribution fit, based on observed data only, increases the risk of underestimation relative to the true distribution of reporting delays. In the event of the fitted distribution being underestimated, the consequences would be that the occurrence intensity estimates would decrease and therefore also the forecasted outstanding liability. Thus, not having the problem of underestimation would possibly increase the ultimate prediction. However, it is very unlikely that the entire prediction gap between the methods is explained by the reporting delay distribution fit. Instead a part of the extensive difference is also derived to the extremely high predictions produced by the Mack Chain-Ladder method. Chain-Ladder methods are known to be unsuitable for volatile portfolios with a low degree of early development, as are the characteristics of our particular portfolio. The consequences are that ultimate predictions becomes very sensitive to small changes in early development periods. Furthermore, our economic disability portfolio shows signs of different development patterns across accident years. In particular, the payment developments are much steeper for the early accident years 2005-06, see Figure 11. This feature also impacts the Chain-Ladder prediction, as late development factors are mainly or fully based upon these patterns.

Even though a distinct difference between the micro model and the Mack Chain-Ladder method is evident, we are unable to draw any conclusion regarding which of them is most accurate. For the purpose of such a comparison it would be preferable to have some out of sample data to validate the predictions with the true development, similar to the approach used in Antonio & Plat (2014). As mentioned, this possibility is however restricted by the amount of data for hazard rate estimations. A remedy could be to cut the data set only one or a couple of years back, and use the small set of later data for validation. Then the observations of jumps would not be as limited. However, with that approach the problem would rather appear with the validation data. With only a few years of observed claim developments, the observed future would only constitute a small part of the final ultimate reserve. Consequently, comparison of model predictions and the true value would be meaningless with regards to the model performance of estimating the total outstanding liability. However, the focus in this thesis has been to construct a micro model for economic disability following the insurance policy. Efforts has been made to estimate the reporting delay-, claim occurrence intensity- and hazard rate- parameters as good as possible. Therefore, out-of sample validation has been disregarded. Despite that the comparison with Mack Chain-Ladder does not prove anything with regards to prediction accuracy, it does bring light upon the difference between the methods. Specially in the case of a volatile and slow developed portfolio.

As of the performance of the simulations from the estimated micro model, the results seem to follow the expected patterns. The end state distributions of the simulated decompositions, IBNR and RBNS, are logical in relation to the end state (current state) distribution of the reported data. For example, the RBNS distribution is more shifted towards the states with disability levels worthy of compensation. However, the expected pattern evaluation is based on the historical behaviour of the portfolio. If any element of the claim development would change over time then the estimated parameters would be outdated. For example, if state jump intensity patterns or reporting delay patterns were to change, then the micro model would not be representative of the current setting.

As earlier discussed the exposure data of collected premiums indicates the opposite pattern of claim occurrence than the data of reported claims combined with the estimated reporting delay distribution. The estimated $\lambda(t)$ reflects a decreasing trend for claim occurrence for later accident years. However, the data of collected premiums is clearly increasing with accident years. Thus, it could be argued that collected premiums are not a a good proxy for exposure of the particular portfolio of economic disability. This could well be due to changes in product mix. Specific insurance classes are often grouped together and supplied as packages. Therefore premiums reflect the revenue generated by the product packages, rather than individual insurance classes. Thus, changes in collected premiums could be due to increasing quantities of sold insurances but could just as well arise from changes in product mix, i.e changes in package setup. However, the mismatch of occurrence intensity and exposure measure does not influence the simulations of the micro model, as the additional intensity measure $\lambda(t)$ is implemented. The consequence is instead that the collected premiums data becomes unusable as indication of the claim occurrence intensity.

The decreasing trend of claim occurrence intensity is based on the low number of reported claims in data for recent years, as well as the estimated distribution of reporting delays. Data of reported claims is just the reality, and not something that can be manipulated. However, the estimated reporting delay distribution is managed in the modelling. As mentioned previously, the possible underestimation due to the censored observations of reporting delays could impact the intensity to decrease relative to the "true" levels. Furthermore, we can relate the distribution fit of reporting delays to the problem of time varying patterns. Suppose that the behaviour of reporting delays changes for later accident years, such that it becomes even more time consuming. Then underestimation could also arise as an affect of using data from the full observational period, 2000-17, resulting in lower occurrence intensity estimates than what would be representative of the reporting delay behaviour for recent years. Consequently, the time varying behaviour would be an interesting extension in the micro modelling. Specially in the modelling of the reporting delay, as it could possibly explain or remedy the decreasing trend in claim occurrence intensity.

Micro models require individual claim data, and are therefore sensitive to both quality and quantity of data on the individual level in particular. To be able to use the micro framework with satisfactory estimations of occurrence intensity, reporting delay and hazard rates, the demand on data is high. As mentioned through out the report, there are some disturbances in our dataset, which perhaps might compromise the precision of our model estimates in particular. However, the purpose of this thesis was not necessarily to develop a fully functional reserving model that could be implemented in practice directly, but rather to explore the opportunities of micro modelling in the setting defined by the considered portfolio. We find the opportunities to be promising. The simulation results follows the expected patterns and can be explained by the parameter estimates. Furthermore, the micro model is not restricted to a certain stopping time of the future liabilities, in contrast to Chain-Ladder, and simulates the claims until the full portfolio is settled. Finally, the micro model seems to better capture the effects of variability in claim occurrence across time.

The presented opportunities of the micro modelling are however complemented by some concerns. The demand on amount of data for parameter estimations constrain us to use a very wide time window of observation. As the behaviour of claim- and reserve development is largely affected by business setup, a wide observation window increases the risk of data not being representative of the current policy setting. Thereby, grouping data over the entire time period for parameter estimations might not generate the most accurate predictions of the outstanding liabilities. The wide observation window is a consequence of the trade-off between quantity relative to the up to date quality of data. Finally we must emphasize that our results and conclusions are only based upon findings from the investigation of our particular dataset.

The field of micro-level loss reserving is still quite unexplored. One reason might be the high demand on data, which in practice is hard to satisfy. Specially as the need for the micro modelling characteristics is perhaps greatest for volatile and slow developed portfolios where data naturally becomes more sparse. However, as the technology has developed over the last decade the importance of data has been recognized. Therefore, one might suspect that both the quality and quantity of the insurance companies data will increase. Making micro modelling more viable and thereby more adaptable within loss reserving. In that spirit we identify several potential subjects for further research. First and foremost some implementation of performance validation of our (or similar) micro model would be desirable. That could be attained by revisiting the model in a few years, when sufficient validation data are available. However, in the time perspective it might be even more desirable to construct a similar model based on some other dataset where the validation data is available in the present. For future modelling of state based micro models it might also be interesting with some further deep going investigations regarding the specific parameter estimations. More explicitly, studies focusing on the modelling and estimations of hazard rates or reporting delay distribution, treating the particular problems and obstacles discussed in this thesis. Furthermore investigations on model extensions could possibly focus on the inclusion of factors such as sex, age and/or initial case estimates and their respective impact on model parameters. Extending the modelling to include real life payments and insurance amounts are together with some treatment of factors such as inflation all natural possibilities of future research.

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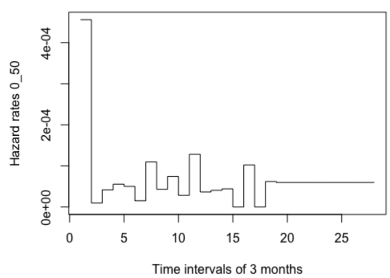
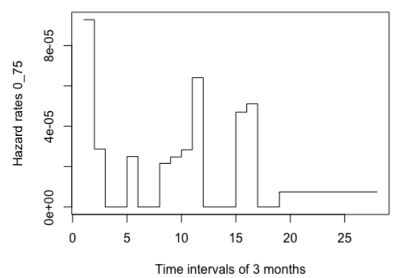
Appendix

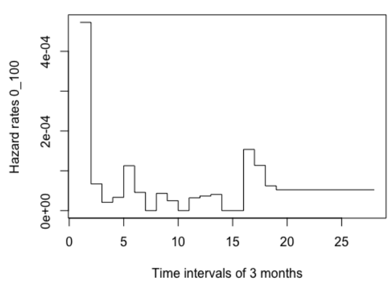
Figures displays the nine different piecewise constant estimations of the hazard rates used in the model.



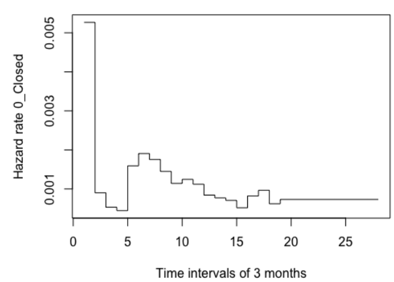
(a) Hazard rates from *Reported* to 50%



(b) Hazard rates from *Reported* to 75%

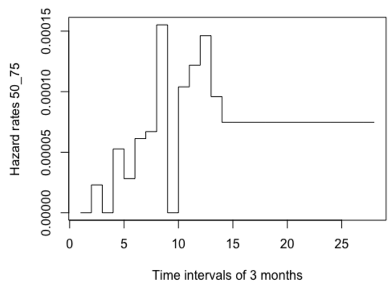


(c) Hazard rates from *Reported* to 100%

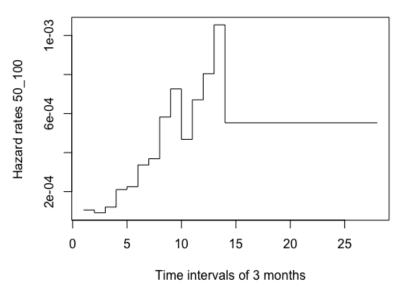


(d) Hazard rates from *Reported* to *Closed*

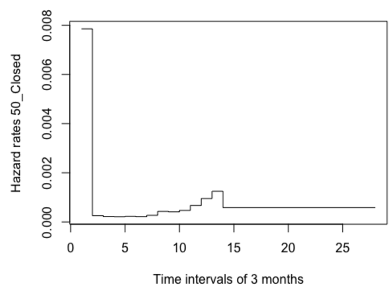
Figure 14: The four different estimations of the piecewise constant hazard rates from state *Reported*.



(a) Hazard rates from 50% to 75%.

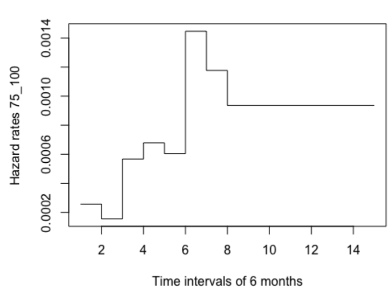


(b) Hazard rates from 50% to 100%.

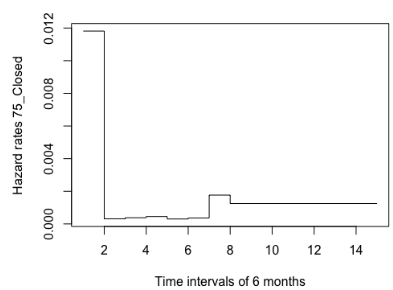


(c) Hazard rates from 50% to *Closed*.

Figure 15: The three different estimations of the piecewise constant hazard rates from state 50%.



(a) Hazard rates from 75% to 100%.



(b) Hazard rates from 75% to *Closed*.

Figure 16: The two different estimations of the piecewise constant hazard rates from state 75%.

Acc per / Dev per	1-3	4	5	6	7	8	9	10	11	12	13
2005	5250000	10750000	24250000	42250000	53500000	66000000	75750000	84750000	102750000	120750000	134250000
2006	4250000	15000000	29250000	36250000	46750000	60250000	77000000	96250000	111750000	132000000	
2007	3000000	7250000	12000000	17750000	23250000	35750000	43000000	52250000	57250000		
2008	1000000	5250000	5750000	7250000	9500000	14000000	15000000	17250000			
2009	4500000	6250000	8000000	13250000	17500000	17750000	18750000				
2010	1750000	3750000	7750000	12000000	14000000	17500000					
2011	2000000	6500000	12000000	13500000	14750000						
2012	3500000	8500000	11000000	16750000							
2013	4000000	9750000	15250000								
2014	4000000	8500000									
2015	3500000										

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