Homework 3, DN2230

Due December 13, 2013

If a correct solution is handed in before the deadline two bonus points will be awarded to the final written exam. If a solution that is handed in before that date is not correct, it has to be redone, but the second time without yielding bonus points for the exam.

Solutions must be clearly written, and easy to follow. If not, they will not generate bonus points, and must be redone.

Together with your solution, I want you to send me your code by e-mail.

1. Implement the Arnoldi method (TB algorithm 33.1) and apply it to compute the eigenvalues of the matrix generated by

   ```
   A=gallery('poisson',10);
   ```

   Also carry out Arnoldi's method for A^{-1}. Implement it without explicitly forming an inverse. To which eigenvalues of A do they converge?

2. Implement the Generalized Minimum Residual method (GMRES). Apply your implementation to the matrix and vector generated by

   ```
   alpha=5;
   rand('state',5);
   A = sprand(100,100,0.5);
   A = A + alpha*speye(100); A=A/norm(A,1);
   b = randn(length(A),1);
   ```

   (a) Verify your code by showing that the Arnoldi relation is satisfied (up to machine precision),

   $$AQ_n = Q_{n+1}\tilde{H}_n.$$

   and that Q_n is satisfied up to machine precision

   $$Q_n^T Q_n = I.$$

   (b) Plot the error as a function of iteration as well as the residual. Experiment with other values of α and relate it to the convergence theory of GMRES, in particular how the convergence changes when α is large. The matrix is sufficiently small such that you can use `plot(full(eig(A)))` for theoretical purposes.

(c) Relate your experiments in (b) numerically and theoretically with the following code:

```
n=30; K=zeros(100,n);  K(:,1)=b/norm(b);
for k=2:n
  K(:,k)=A*K(:,k-1);
end
clf;
for k=1:n
  x=K(:,1:k)*((A*K(:,1:k))\b);
  semilogy(k,norm(x-A\b),'*');   hold on
end
```

Explain warnings and error messages from matlab.

3. Exercise 38.5 in "Numerical Linear Algebra" (NLA).

Good luck!