# REPETITION: GLOBALT & LOKALT FEL ODE

ODE: $\begin{cases} y'(t) = f(t, y(t)) & t > 0 \\ y(0) = y_0 \end{cases}$

$f, y_0$ givna

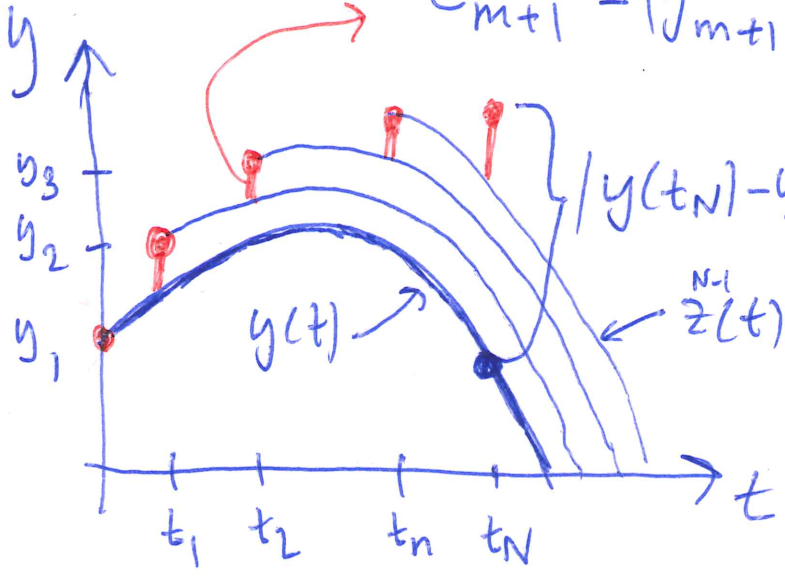
Eulers metod:

1. Indelning: $t_n = n\Delta t$, $n = 0, 1, 2, \ldots$
2. Approximation: $y_n \simeq y(t_n)$

$$\frac{y_{n+1} - y_n}{\Delta t} = f(t_n, y_n) \quad n = 0, 1, 2, \ldots$$

$$\Longleftrightarrow \quad y_{n+1} = y_n + \Delta t\, f(t_n, y_n)$$

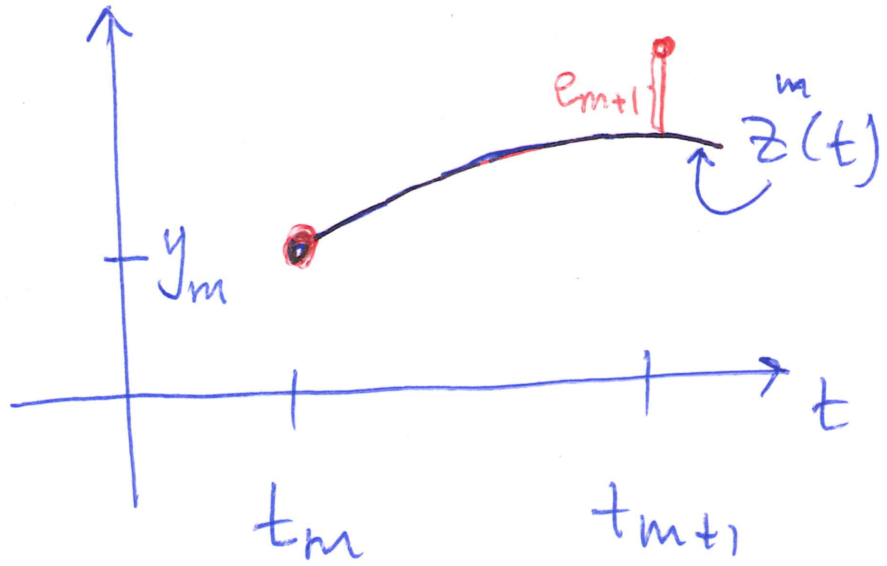
$e_{m+1} = |y_{m+1} - z^{m}(t_{m+1})| =$ Lokalt fel




$|y(t_N) - y_N| =$ Globalt fel

lokalt fel steg $m+1$ =

$= e_{m+1} = |y_{m+1} - z^{m}(t_{m+1})|$




$$\begin{cases} \dfrac{d}{dt} z^{m}(t) = f(t, z^{m}(t)) \\ z^{m}(t_m) = y_m \end{cases}$$

SATS 6.4. Antag $\max_{t,y} \left| \frac{\partial f}{\partial y}(t,y) \right| \leq K$

och lokala felet $e_m \leq C(\Delta t)^2$, alla $m$,

då uppfyller Eulermetodens globala fel vid tiden $t_N$

$$|y(t_N) - y_N| \leq C \Delta t \, \frac{e^{K t_N} - 1}{K}.$$

Bevis sats i två steg 1) Felrepresentation

2) ODE fel i sluttid från fel i starttid

2) sats : Antag $\max_{t,y} \left| \frac{\partial f}{\partial y}(t,y) \right| \le K$

och $y'(t) = f(t, y(t))$

$z'(t) = f(t, z(t))$

då gäller

$$|y(t) - z(t)| \le e^{K(t-s)} |y(s) - z(s)|, \quad t > s.$$

(5)

Steg (1): $|y(t_N) - y_N| =$ [teleskopsumma]

$$= |y(t_N) - z^1(t_N) + z^1(t_N) - z^2(t_N) + \dots z^{N-1}(t_N) - y_N|$$

$$\overset{\text{Sats 2}}{\leq} e_1 e^{K(t_N - \Delta t)} + e_2 e^{K(t_N - 2\Delta t)} + \dots + e_N e^{K(t_N - t_N)}$$

$$\leq \max_n e_n \sum_{n=0}^{N-1} e^{Kn\Delta t} \leq C(\Delta t)^2 \frac{e^{KN\Delta t} - 1}{\underbrace{e^{K\Delta t} - 1}_{\geq K\Delta t}}$$

$$\leq C\Delta t \frac{e^{Kt_N} - 1}{K}, \text{ klart.}$$

<u>Bevis sats 2:</u> Låt $E(t) = y(t) - z(t)$.

Om $E(s) = 0$ då $y(t) = z(t)$ för alla $t$ på grund av entydighet. Antag därför $E(t) > 0$

$$\frac{d}{dt} E(t) = y'(t) - z'(t) = f(t, y(t)) - f(t, z(t))$$

$$\overset{\xi \text{ mellan } y \text{ och } z}{=} \frac{\partial f}{\partial y}(t, \xi)\,(y(t) - z(t))$$

$$\leq K\, E(t)$$

Så

$$\underbrace{\int_s^t \frac{dE}{E}}_{\ln \frac{E(t)}{E(s)}} \leq K \int_s^t d\tau = K(t-s)$$

$$\Longrightarrow \quad \frac{E(t)}{E(s)} \leq e^{K(t-s)}$$

$$\Longleftrightarrow \quad E(t) \leq e^{K(t-s)} E(s) \text{ , klart.}$$