Convergence, restart, breakdown of Arnoldi method for eigenvalue problems

SF2524 - Matrix Computations for Large-scale Systems

2014-11-11

Agenda lecture 3

- Convergence Arnoldi method for eigenvalue problems
  - Computer demo
  - Angle estimators
  - Rule-of-thumbs
- Restart / breakdown
  - Explicit restart
  - Breakdown / numerical instability
  - Implicit restart
- Lanczos $\Leftrightarrow$ Arnoldi for symmetric matrices

* Illustrate convergence with matlab demo *

Example of convergence theory of the Arnoldi method for eigenvalue problems:

Theorem (Jia, SIAM J. Matrix. Anal. Appl. 1995)

Let Q_n and H_n be generated by the Arnoldi method and suppose $\lambda_i^{(n)}$ is an eigenvalue of H_n. Assume that $\ell_i = 1$ and the associated value $\|(I - Q_n Q_n^T)x_i\|$ is sufficiently small. Let $P_i^{(n)}$ be the spectral projector associated with $\lambda_i^{(n)}$. Then,

$$|\lambda_i^{(n)} - \lambda_i| \leq \|P_i^{(n)}\|\gamma_n \frac{\|(I - Q_n Q_n^T)x_i\|}{\|Q_n Q_n^T x_i\|} + \mathcal{O}\left(\frac{\|(I - Q_n Q_n^T)x_i\|^2}{\|Q_n Q_n^T x_i\|^2}\right)$$

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The theorem is not a part of the course. In this course we will gain qualitative understanding by bounding

$$\|(I - Q_n Q_n^T)x_i\|.$$