

Gram-Schmidt Variants

Problem:

Given orthonormalized vectors

$$q_1, q_2, \dots, q_k \in \mathbb{R}^m \text{ and } w \in \mathbb{R}^m$$

with $w \notin \text{span}(q_1, \dots, q_k)$

Compute $h_1, h_2, \dots, h_k, \beta \in \mathbb{R}$

and $q_{k+1} \in \mathbb{R}^m$ such that

$\|q_{k+1}\| = 1$ and q_{k+1} orthogonal

against q_1, \dots, q_k and

$$w = h_1 q_1 + \dots + h_k q_k + \beta q_{k+1}$$

Classic Gram-Schmidt CGS
"single GS"

$$h = Q^T w$$

$$y = w - \alpha h$$

$$\beta = \text{norm}(y)$$

$$q_{k+1} = y/\beta$$

Modified Gram-Schmidt MGS

$$y = w$$

for $i = 1, \dots, k$

$$h_i = q_i^T y$$

$$y = y - q_i h_i$$

End

Version 1

Double Gram-Schmidt $\checkmark =$

Repeated Gram-Schmidt

$$h = Q^T w$$

$$y = w - Qh$$

$$g = Q^T y$$

$$y = y - Qg$$

$$h = h + g$$

$$\beta = \|y\|$$

$$q_{k+1} = y / \beta$$