Analytic interpolation with degree constraint with applications to systems and control and signal processing

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Abstract

Many important problems in circuit theory, robust stabilization and control, signal processing, speech synthesis, and stochastic systems theory can be formulated as analytic interpolation problems. For these problems the (real) interpolants are (i) positive real, i.e., analytic in the unit disc (or its complement) and mapping it into the closed left-half plane, and (ii) rational of a prescribed maximal degree. Removing the degree requirement (ii), this is a classical problem going back to Schur, Carathéodory, Toeplitz, Nevanlinna and Pick. However, the degree constraint is essential in applications, and it considerably alters the mathematical problem. In fact, it has been shown only recently that, if there are $n + 1$ interpolation points, there is a complete parameterization of all interpolants of degree at most n in terms of spectral zeros, which hence become design parameters. Each such solution can be determined by solving a convex optimization problem.

This series of three talks will cover the main aspects of the theory of analytic interpolation with degree constraint. The first talk will give an overview of the subject matter and illustrate it by means of applications from systems and control and signal processing. The second talk will be devoted to the geometry of positive real functions. As a corollary, a proof of the basic parameterization theorem is obtained. In the third talk a convex optimization approach for determining all the interpolants will be presented.

- **Lecture 1:** Why Nevanlinna-Pick interpolation theory is important in systems and control and how it can be modified to be more useful.
- **Lecture 2:** The geometry of positive real functions.
- **Lecture 3:** A convex optimization approach to analytic interpolation with degree constraint.

Why Nevanlinna-Pick interpolation theory is important in systems and control and how it can be modified to be more useful

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This first talk will be devoted to an overview of the theory of analytic interpolation with degree constraint, which will be motivated in terms of applications from systems and control. We will begin by considering a special case, the covariance extension problem, in the context of speech processing. We will conclude with the more general Nevanlinna-Pick problem with degree constraint, which will be illustrated in terms of sensitivity minimization in robust control, maximal power transfer, and spectral estimation. In particular, a new approach to spectral estimation will be presented, which is based on the use of filter banks as a means of obtaining spectral interpolation data. By suitable choices of filter-bank poles and spectral zeros the estimator can be tuned to exhibit high resolution in targeted regions of the spectrum.

The geometry of positive real functions

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Abstract

Positive real, rational functions (i.e., rational Carathéodory functions) play a central role in both deterministic and stochastic linear systems theory, arising in circuit synthesis, filtering, interpolation, spectral analysis, speech processing, stability theory, stochastic realization theory and systems identification – to name just a few. For this reason, results about positive real transfer functions and their realizations typically have many applications and manifestations.

In this second talk, certain manifolds and submanifolds of positive real transfer functions will be studied, and they will be motivated in terms of both analytic interpolation and a fast algorithm for Kalman filtering, viewed as a nonlinear dynamical system on the space of positive real transfer functions. In particular, a fundamental geometric duality between filtering and analytic interpolation will be described. This duality, while interesting in its own right, has several corollaries which provide solutions and insight into some very interesting and intensely researched problems. One of these is the rational covariance extension problem with degree constraint, which was formulated by Kalman; another is the Nevanlinna-Pick problem with degree constraint. For both, the duality theorem yields complete solutions.

A convex optimization approach to analytic interpolation with degree constraint

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Abstract

In the second talk we proved that the space of all solutions to an analytic interpolation problem with $n+1$ interpolating conditions and the degree of interpolants bounded by n is completely parameterized by the spectral zeros. The problem of actually computing these interpolants is a difficult nonlinear problem.

In this third talk, it will be shown that each interpolant can be obtained as the solution to a problem of maximizing a generalized entropy gain. These optimization problems have duals which are convex optimization problems in a finite-dimensional space. This leads to an efficient computational procedure.

The question of minimal-degree interpolants will also be addressed. In particular, the differences between deterministic and stochastic partial realization theory will be considered. Moreover, some existence and robustness results concerning the degrees of minimal stochastic partial realizations will be presented. As a corollary to these results, we note that, in sharp contrast with the deterministic case, there is no generic value of the degree of a minimal stochastic realization of partial covariance sequences of fixed length. This has important ramifications for the popular "subspace identification" methods.