Robust Control with Classical Methods — QFT
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• Review of the classical Bode-Nichols control problem
• QFT in the basic Single Input Single Output (SISO) case
• Uncertainty and Fundamental Design Limitations
• QFT for non-minimum phase and computer controlled systems
• QFT for cascaded systems, and for a class of non-linear plants
• QFT for Multi-Input Multi-Output (MIMO) plants
• A comparison between QFT and other robust and adaptive control

Fundamental Design Limitations — a simplistic view

• Stability
  - The Nyquist stability criterion
• Phase margin
• Sensitivity and complementary sensitivity
• Bode’s gain-phase relationship
• Bode’s integral theorem
• Unstable poles
• RHP zeros and delay
• Exercise
• Example

The Nyquist stability criterion

Open loop \( L(s) = P(s)G(s) \)

\[
\frac{1}{2\pi} \text{arg} \left( 1 - L(j\omega_c) \right) = P_c - P_o
\]

\( P_c \) = number of closed loop unstable poles
\( P_o \) = number of open loop unstable poles

Phase margin

\( \Rightarrow \) in the cross-over frequency region: \( \text{arg}(L(j\omega)) \geq -150^\circ \)

Sensitivity & complementary sensitivity

\[
Y = SD + FSR - SN \quad , \quad S + T = 1
\]

• Desired \( S \) small and \( T \) small

Resolution: frequency separation
\( S \) small for low frequencies
\( T \) small for high frequencies
Bode’s gain-phase relationship

- Assume that \( L(s) \) has no poles or zeros in the RHP, and \( L(s) \geq 0 \)
- \[ \arg L(j\omega) = \frac{1}{\pi} \log \left( \frac{\omega / A}{\omega} \right) \log \cosh \left( \frac{\pi}{\omega} \right) \]
- So, for desired \( \theta_c \) in crossover frequency region, \( L(s) = L_L(s)^{\theta_c/2} \approx 135^\circ, -30\text{dB/dec} \)
- Limited roll-off

Bode’s integral theorem

- Water bed effect: \( \exists \omega_j, \omega_c \) for which \( \left| L(j\omega) \right| = \text{dB} \)
- Stable controller \( G(s) \) preferable.

- Horowitz: “optimal” sensitivity design, by double resonance/ NMP in \( G(s) \). See e.g Nordin and Gutman, ECC, 1995.
- Active vibration damping.

More about sensitivity and stability

- Recall: \( S = \left| \frac{1}{1+\Delta} \right| \) \( S = \left| \frac{L}{L_L} \right| \)
- Show that \( S = dS / dL \)
- Of greater design interest is \( \theta_S / \theta_L \)
- Let \( L(s) = L_{\text{norm}}(s) + \Delta_L(s) \)
  - Additive unstructured uncertainty \( \Delta_L(s) \)
  - Closed loop stability margin \( \Rightarrow |1+\Delta_L| < \phi(s) \)

Bode’s integral theorem, cont’d

- References:
  - Bode
  - Kwakernaak & Sivan
  - Horowitz, ch. 10

Unstable \( P(s) \)

- ... sensitivity constraint.
- Minimum open loop gain constraint to achieve closed loop stability: \( \omega_c \geq \omega_{\text{crit}}, \text{k} \geq k_{\text{crit}} \)
- Example 1: Root locus
- Example 2: Nichols chart

Unstable \( P(s) \), cont’d
Non-minimum phase plants

- with RHP zero(s)
- delay $e^{-\tau}$
- one RHP zero
- step response
- maximum open loop gain constraint to achieve closed loop stability:
  $\omega_c \leq \omega_{c\text{crit}}$ or $k \leq k_{\text{crit}}$

Non-minimum phase plants, cont’d

Right half plane zero

- Let $P(s) = P_m(s) A(s)$,
- $A(s) = (a-s)/(a+s)$, $|A(s)|=1$, all-pass,
- $P(s)$ stable, minimum phase,
- Roll-off given by $G(s) P_m(s)$, reasonable to have around $\omega_c \approx -20 \text{ dB/dec}$ \( \Rightarrow \arg \approx -90 \text{ deg} \),
- Desired $\phi_m \geq 35 \text{ deg} \Rightarrow \arg(A(j\omega_c)) < 55 \text{ deg} \Rightarrow \omega_c < a/2$

Delay

$i.e. \text{ there is a RHP zero at } (2/\tau). \text{ Hence }$

$$\omega_c < 1/\tau \text{ [rad/s]}$$

Exercise

- Find a stabilizing controller for each one of the plants, respectively, whose pole-zero maps are depicted.

Example

- Spec: $|y(t)| < A/10$ in steady state (1)
- minimal control energy (2)
- $|G(j\omega)|$ as small as possible (3)
- $|G(j\omega)|$ as small as possible (4)
- $|P(j\omega)| < 0.1$ \( \Rightarrow |P(j\omega)G(j\omega)| > 10 \)

- CL active vibration control
- Note: not a notch filter

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