Robust Control with Classical Methods – QFT

Per-Olof Gutman

- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Uncertainty and Fundamental Design Limitations
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

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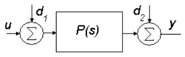
Review of the classical Bode-Nichols control problem

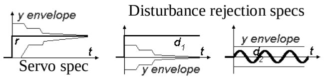
- The control problem
- Design in the open loop: the Bode-Nichols design problem
- Alternatives to 1 degree-of-freedom feedback control
- Closed loop transfer functions
- Bode and Nichols diagrams
- Translation of specifications
  - Test signals and specifications envelopes
  - Specifications for the transient
  - Steady state state specifications
- The loop shaping problem
- Compensation networks
- A design example
- The robust control problem

The control problem

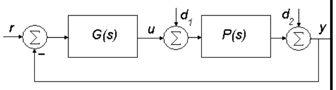
Given

1. Plant with disturbances
2. Closed loop specifications





Find: Regulator $G(s)$ such that the closed loop satisfies the specifications

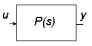


The Bode-Nichols design problem

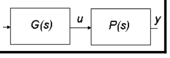
- The problem with ”Plant with disturbances” & ”Closed loop specifications” is transformed to

Given

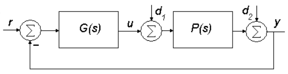
1. Plant $P(s)$
2. Open loop specifications $\varphi_m, A_m, \omega_c, e_0, e_1, \ldots, c_0, c_1, \ldots$



Find: Regulator $G(s)$ such that the open loop $G(s)P(s)$ satisfies the specifications

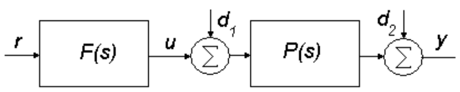


- Hopefully, the closed loop satisfies the original specifications:



Alternatives to 1 d-o-f design

- **Open loop control**. Can be used when
  - d_1, d_2 ”small”, and
  - $P(s)$ ”well known”



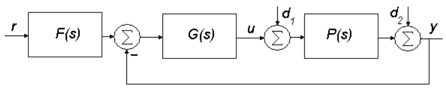
Alternatives to 1 d-o-f design

- **2 d-o-f closed loop control**: the 2 d-o-f closed loop t.f.

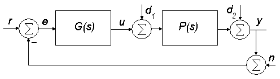
$$T(s)=\frac{Y(s)}{R(s)}=\frac{F(s)P(s)G(s)}{1+P(s)G(s)}$$

is ”decoupled” from the disturbance rejection functions

$$S(s)=\frac{Y(s)}{D_2(s)}=\frac{1}{1+P(s)G(s)} \quad \text{and} \quad \frac{Y(s)}{D_1(s)}=\frac{P(s)}{1+P(s)G(s)}$$

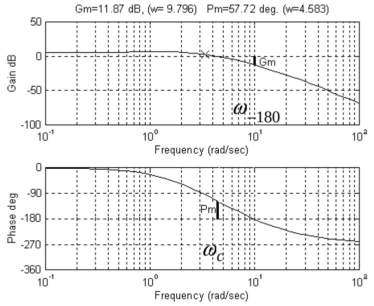


Closed loop transfer functions

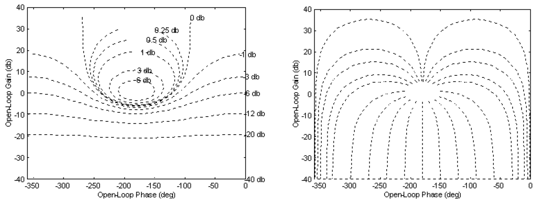


- Open loop $L(s)=P(s)G(s)$
- Complementary sensitivity $\bar{S}(s)=\frac{Y(s)}{R(s)}=\frac{P(s)G(s)}{1+P(s)G(s)}$
- Sensitivity $S(s)=\frac{E(s)}{R(s)}=\frac{Y(s)}{D_2(s)}=\frac{1}{1+P(s)G(s)}$
- Plant input disturbance rejection $\frac{Y(s)}{D_1(s)}=\frac{P(s)}{1+P(s)G(s)}$
- ”Cost of feedback” t.f. $\frac{U(s)}{N(s)}=\frac{-G(s)}{1+P(s)G(s)}$
- Note: $S(s)+\bar{S}(s)=1$

Bode diagram with margins



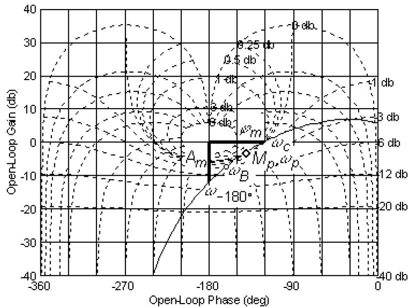
Nichols diagram with closed loop loci



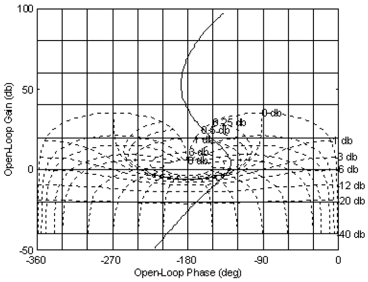
|*L*/(1+*L*)|=const arg(*L*/(1+*L*))=const

where *L*=*PG* is the open loop frequency function

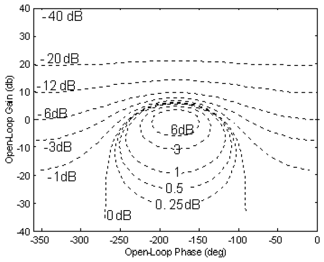
Nichols diagram with margins



Example: conditional stability



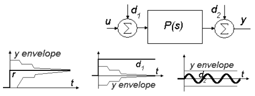
Nichols diagram with closed loop sensitivity loci



Exercise: Prove that the sensitivity loci |1/(1+*L*)|=const are the mirror images w.r.t the 0-db axis of the M-locus |*L*/(1+*L*)|=const!

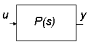
Recall: Design in the open loop

- Plant with disturbances
- Closed loop servo and disturbance rejection specs



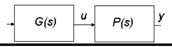
Translate: closed loop time domain specificationss → open loop frequency domain specifications

1. Plant $P(s)$



2. Open loop specifications $\varphi_m, A_m, \omega_c, e_0, e_1, \ldots, c_0, c_1, \ldots$

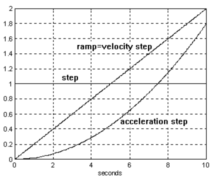
Find: Regulator $G(s)$ such that the open loop $G(s)P(s)$ satisfies the open loop specifications

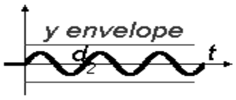


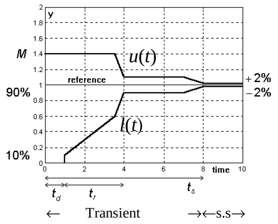
Translation of specifications

- Test signals in the time domain:
  step, ramp, sinus, ...
- Specification envelopes
- Transient
- Steady state

Test signals and specification envelopes







t_d=delay time; t_r=delay time

t_s=settling time; M= overshoot

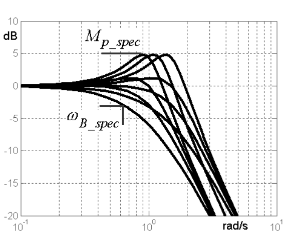
Translation of a transient specification

1. **Dominant pole assumption**: Model the as yet unknown closed loop by e.g. a 2nd or 3rd order transfer function.
2. **Find all such models that satisfy the transient** of the considered closed loop time domain specification. Example:

$$Q=\left\{(\varsigma,\omega_0)\,\middle|\, l(t)\le L^{-1}\left\{\frac{\omega_0^2}{s^2+2\varsigma\omega_0 s+\omega_0^2}\cdot\frac{1}{s}\right\}\le u(t)\right\}$$

Translation of a transient specification, cont’d

3. Display the Bode plots of the acceptable models and determine some **closed loop frequency domain specifications**. Example:

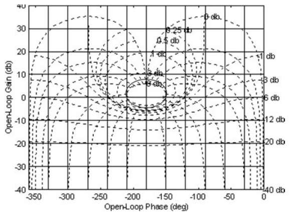


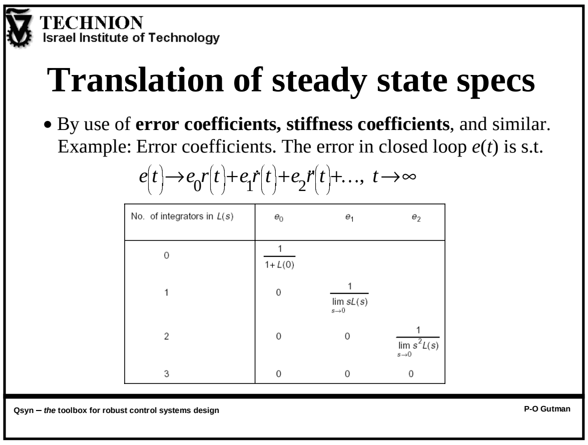
Translation of a transient specification, cont’d

4. Get **open loop frequency domain specifications** by
   - Rule of thumb. Example: $\omega_{c\_spec}\approx 0.8\omega_{B\_spec}$
   - Use of M-loci in Nichols diagram.

M_{p_spec} yields

$\varphi_{m\_\mathrm{spec1}}$ and A_{m_spec1}



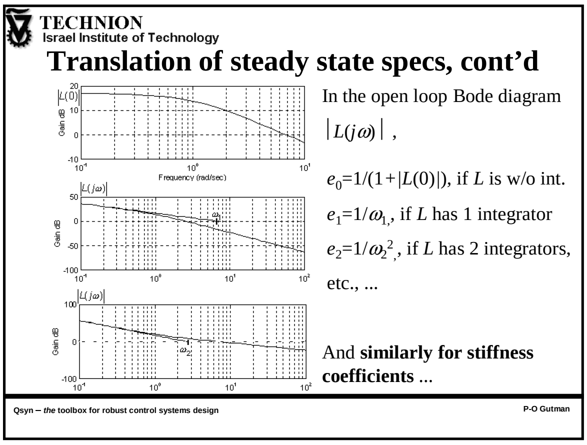


Translation of steady state specs

- By use of **error coefficients, stiffness coefficients**, and similar. Example: Error coefficients. The error in closed loop $e(t)$ is s.t.

$$e(t)\rightarrow e_0 r(t)+e_1\dot{r}(t)+e_2\ddot{r}(t)+\ldots,\ t\rightarrow\infty$$

No. of integrators in $L(s)$	e_0	e_1	e_2
0	$\frac{1}{1+L(0)}$		
1	0	$\frac{1}{\lim_{s\to 0} sL(s)}$	
2	0	0	$\frac{1}{\lim_{s\to 0} s^2L(s)}$
3	0	0	0



Translation of steady state specs, cont'd

In the open loop Bode diagram $|L(j\omega)|$,

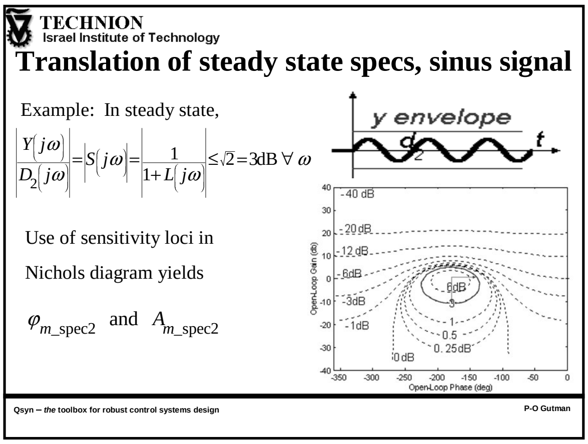
e_0=1/(1+|L(0)|), if L is w/o int.

e_1=1/ω_1, if L has 1 integrator

e_2=1/ω_2^2, if L has 2 integrators,

etc., ...

And **similarly for stiffness coefficients** ...



Translation of steady state specs, sinus signal

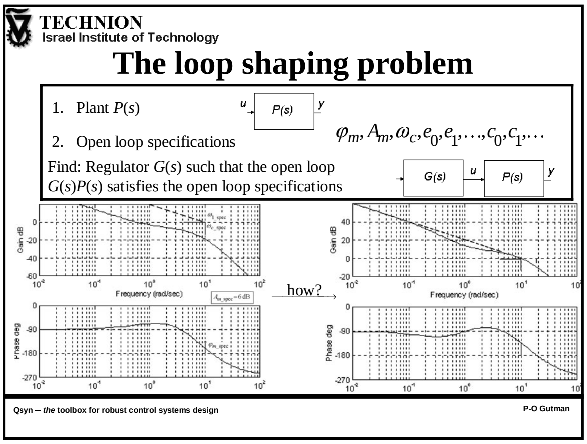
Example: In steady state,

$$\left|\frac{Y(j\omega)}{D_2(j\omega)}\right|=\left|S(j\omega)\right|=\left|\frac{1}{1+L(j\omega)}\right|\leq\sqrt{2}=3\text{dB}\ \forall\ \omega$$

Use of sensitivity loci in

Nichols diagram yields

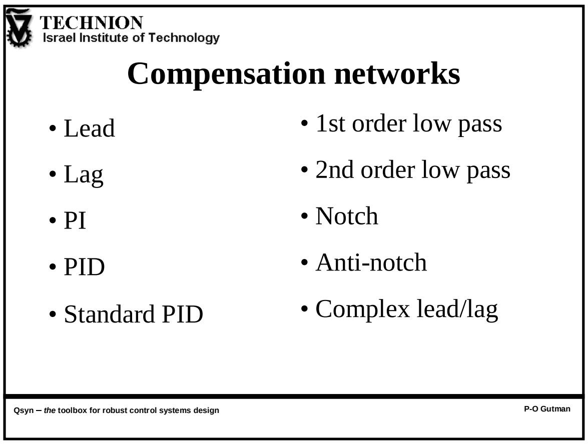
φ_{m_spec2} and A_{m_spec2}



The loop shaping problem

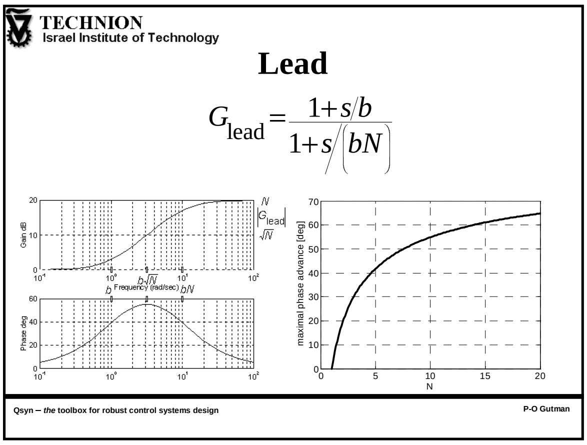
1. Plant $P(s)$
2. Open loop specifications $\varphi_m, A_m, \omega_c, e_0, e_1, \ldots, c_0, c_1, \ldots$

Find: Regulator $G(s)$ such that the open loop $G(s)P(s)$ satisfies the open loop specifications



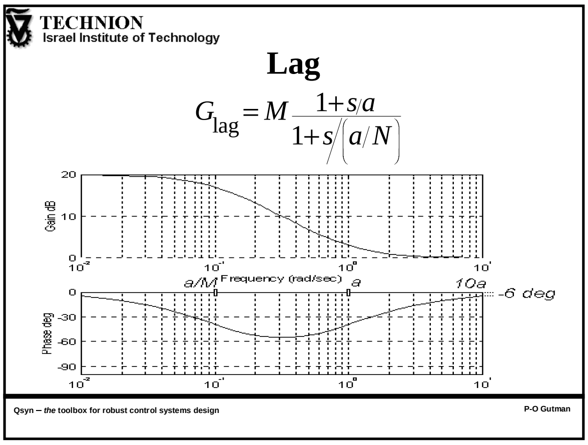
Compensation networks

- Lead
- Lag
- PI
- PID
- Standard PID
- 1st order low pass
- 2nd order low pass
- Notch
- Anti-notch
- Complex lead/lag



Lead

$$G_{\text{lead}}=\frac{1+s/b}{1+s/(bN)}$$



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Lag

$$G_{\text{lag}} = M\frac{1+s/a}{1+s/(a/N)}$$

Qsyn – *the* toolbox for robust control systems design

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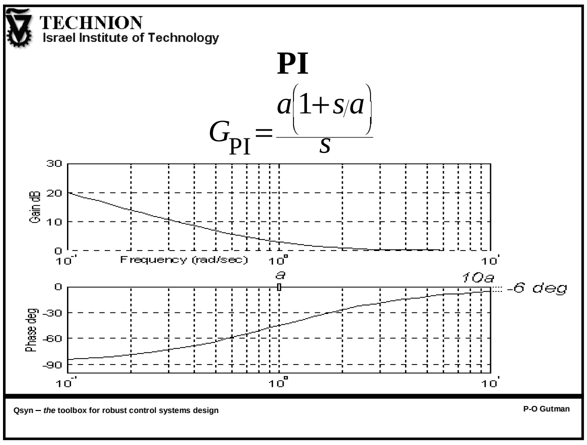

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PI

$$G_{\text{PI}} = \frac{a(1+s/a)}{s}$$

Qsyn – *the* toolbox for robust control systems design

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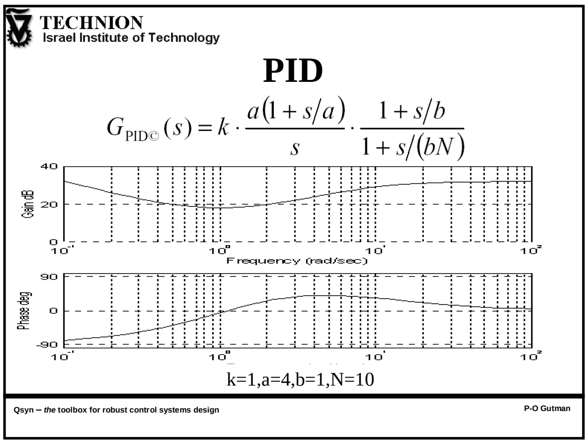

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PID

$$G_{\text{PID©}}(s) = k\cdot\frac{a(1+s/a)}{s}\cdot\frac{1+s/b}{1+s/(bN)}$$

k=1,a=4,b=1,N=10

Qsyn – *the* toolbox for robust control systems design

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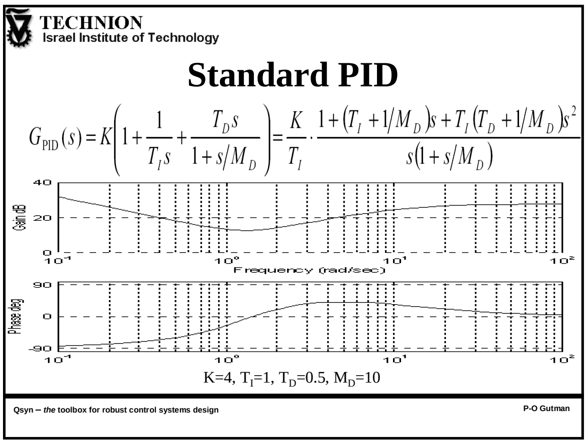

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Standard PID

$$G_{\text{PID}}(s)=K\left(1+\frac{1}{T_I s}+\frac{T_D s}{1+s/M_D}\right)=\frac{K}{T_I}\cdot\frac{1+(T_I+1/M_D)s+T_I(T_D+1/M_D)s^2}{s(1+s/M_D)}$$

$K=4$, $T_I=1$, $T_D=0.5$, $M_D=10$

Qsyn – *the* toolbox for robust control systems design

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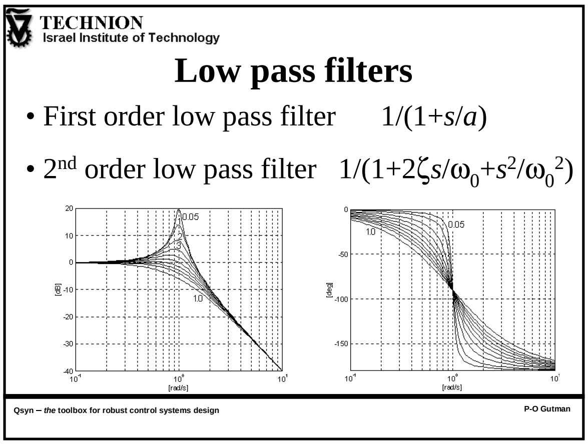

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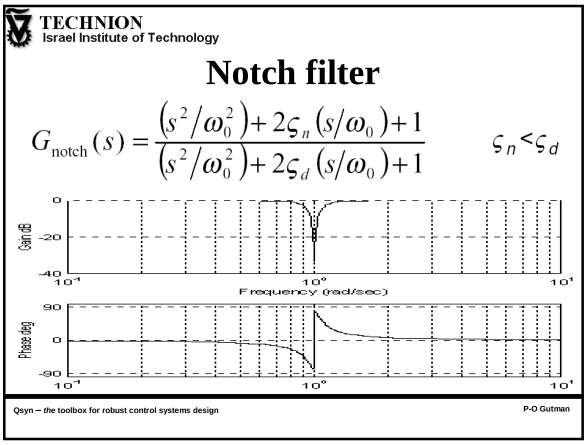
Low pass filters

- First order low pass filter $1/(1+s/a)$
- 2nd order low pass filter $1/(1+2\zeta s/\omega_0+s^2/\omega_0^2)$

Qsyn – *the* toolbox for robust control systems design

P-O Gutman



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Notch filter

$$G_{\text{notch}}(s)=\frac{(s^2/\omega_0^2)+2\varsigma_n(s/\omega_0)+1}{(s^2/\omega_0^2)+2\varsigma_d(s/\omega_0)+1} \qquad \varsigma_n<\varsigma_d$$

Qsyn – *the* toolbox for robust control systems design

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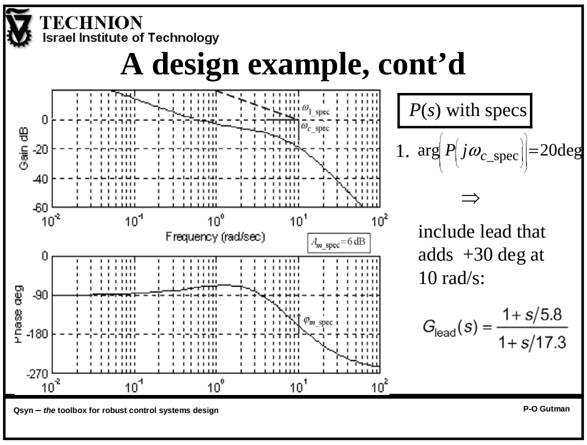
A design example

- Plant $P(s)=\frac{250(s+1)}{s(s+5)(s+10)^2}$

- Specifications in the open loop frequency domain $\begin{cases}\omega_c = 10 \quad \text{rad/s}\\ \varphi_m = 44\,\text{deg}\\ e_0 = 0, e_1 \le 0.1\end{cases}$



A design example, cont'd

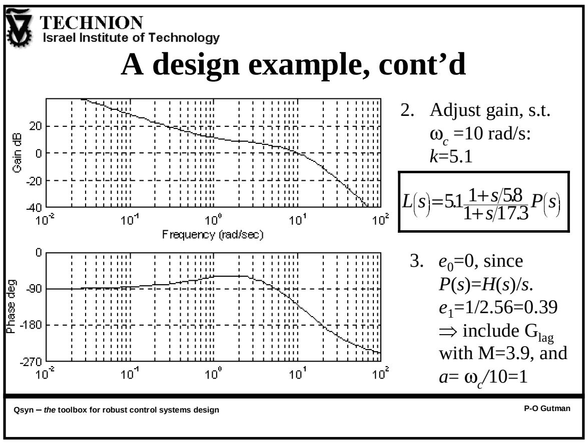
$P(s)$ with specs

1. $\arg\left(P\left(j\omega_{c\_spec}\right)\right)$=20deg

$\Rightarrow$

include lead that adds +30 deg at 10 rad/s:

$G_{\text{lead}}(s)=\frac{1+s/5.8}{1+s/17.3}$

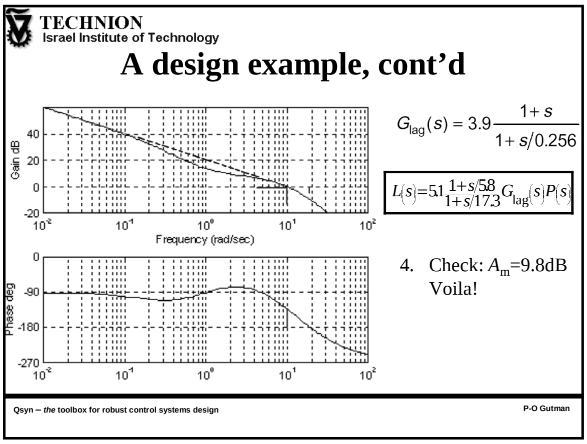


A design example, cont'd

2. Adjust gain, s.t. ω_c =10 rad/s: k=5.1

$L(s)=5.1\frac{1+s/5.8}{1+s/17.3}P(s)$

3. e_0=0, since $P(s)=H(s)/s$. e_1=1/2.56=0.39 $\Rightarrow$ include G_{lag} with M=3.9, and $a=\omega_c/10$=1

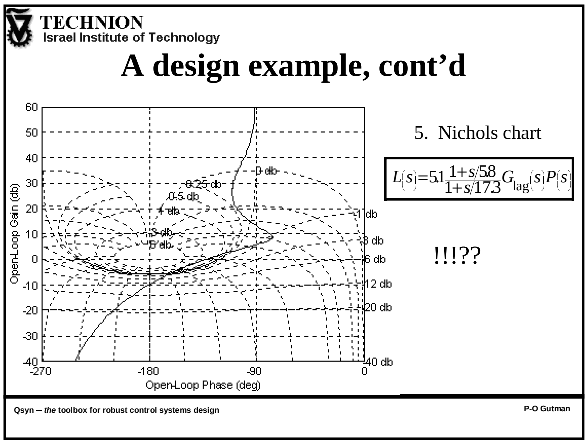


A design example, cont'd

$G_{\text{lag}}(s)=3.9\frac{1+s}{1+s/0.256}$

$L(s)=5.1\frac{1+s/5.8}{1+s/17.3}G_{\text{lag}}(s)P(s)$

4. Check: A_m=9.8dB Voila!

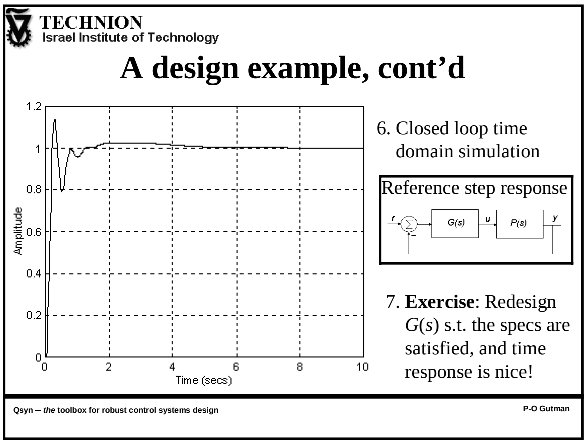


A design example, cont'd

5. Nichols chart

$L(s)=5.1\frac{1+s/5.8}{1+s/17.3}G_{\text{lag}}(s)P(s)$

!!!??



A design example, cont'd

6. Closed loop time domain simulation

Reference step response

7. **Exercise**: Redesign $G(s)$ s.t. the specs are satisfied, and time response is nice!

The robust control problem

- Given a set of plants, $P(s) \in \{P_i(s)\}$
- Design **one** feedback compensator $G(s)$ (and **one** prefilter $F(s)$) such that the specifications are satisfied for each $P_i(s)$.
- Difficult if individual transfer functions are retained in design calculations.

A robust control problem example

- **Exercise**:

Consider the following double integrator plant with uncertain gain and delay

$$P(s) = \frac{k}{s^2}e^{-\tau s}, k = \{1, 10\}, \tau = \{0, 1\}.$$

Design a feedback controller, $G(s)$, such that the modulus of the sensitivity function

$$\left|\frac{1}{1+P(s)G(s)}\right| \le 6 \text{ dB},$$

and that the crossover frequency

$\omega_c > 0.07$ rad/s.

A robust control problem example, cont'd