SF3810 Convexity and optimization in linear spaces, 2020.

Home assignments, collection number 6.

Due date: May 13, 2020.

Note: You may discuss the problems with other students, but you should write your own solutions, “in your own words”.

1. Consider the problem

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}\int_0^1 u^2(t)\,dt \\ \text{subject to} & \ddot{x}(t) = u(t), \quad t \in [0,1] \\ & x(0) = \dot{x}(0) = 0, \\ & 1 - x(1) \le 0. \end{array}$$

This problem can be written on the form: minimize $f(u)$ subject to $g(u) \le 0$, where f and g are real-valued convex functionals. (Integrate $\ddot{x}(t) = u(t)$ twice.) Formulate the corresponding dual problem (according to section 8.6), and deduce the optimal solutions to the primal and dual problems. Check optimality conditions and that the optimal values are equal.

2. Repeat the above exercise 1, but now with the following additional constraints included in the problem formulation: $u(t) \le 2.5$ for all $t \in [0,1]$.

3. Repeat the above exercise 1, but now with the following additional constraint included in the problem formulation: $\dot{x}(1) \le 1.25$.

4. Repeat the above exercise 1, but now with the following additional constraints included in the problem formulation: $u(t) \ge 0$ for all $t \in [0,1]$ and $\dot{x}(1) \le 1.25$.

5. Repeat the above exercise 1, but now with the following additional constraints included in the problem formulation: $\dot{x}(t) \le 1.25$ for all $t \in [0,1]$.

Good luck!