Home assignment number 1, autumn 2017, in SF1811 Optimization.

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This assignment should be carried out in groups containing one or two persons. There should be one (short!) report per group, in which you describe "in your own words" how the problem was formulated and solved, and answer the given questions. Limited cooperation ("discussions") with other groups is allowed, but should in that case be mentioned in the beginning of the report. It is never allowed to copy parts of another groups report or matlab code! State your name, personal number and *email* address at the front of the report.

Your report should be uploaded to canvas not later than **15:00, November 22**. Both the report and your matlab code should be handed in as a separate files (e.g., a report as pdf file and code as m-files). It should be possible for David to, without any effort!, run your code and obtain the results you presented in the report. Some students may, partly by random, be selected for individual oral defense of their assignments. Those students will be contacted by email, so check your email regularly.

A correct report (and defense if asked for) gives you **2** bonus points on the exam.

In this home assignment you should formulate a network flow problems as an LP problem, solve it using Matlab, and present the results. You should also modify the problem slightly, solve it again, and present the results.

In the network with 9 nodes and 12 links shown at the top of the next page, one should simultaneously send a units of "red flow" from node 1 to node 9, and b units of "blue flow" from node 4 to node 6. Each link consists of two parallel "channels", one for the red flow and one for the blue flow, so flows of different colours do not mix.

Further, as indicated by arrows in the figure, red flow can only go "south-east" or "south-west" in the channels, and blue flow can only go "north-east" or "south-east".

The *link load* on a given link is defined as the *sum* of the sizes of the "red channel flow" and the "blue channel flow" in the link, and one would like to send the required units through the network in such a way that the maximal *link load* (over all links) becomes as small as possible.

1. *Formulate* the above problem of minimizing the maximal link load as an LP problem!

   Explain and motivate your formulation in details: variables, objective function, and in particular all the needed constraints.

   Hint 1: There should be two non-negative variables associated to each link, one for the red channel flow and one for the blue channel flow (in the permitted directions).

   Hint 2: Study Example 3.7 on page 21 in the course compendium concerning how to formulate a problem of minimizing the maximum of a set of functions by introducing an additional variable w.
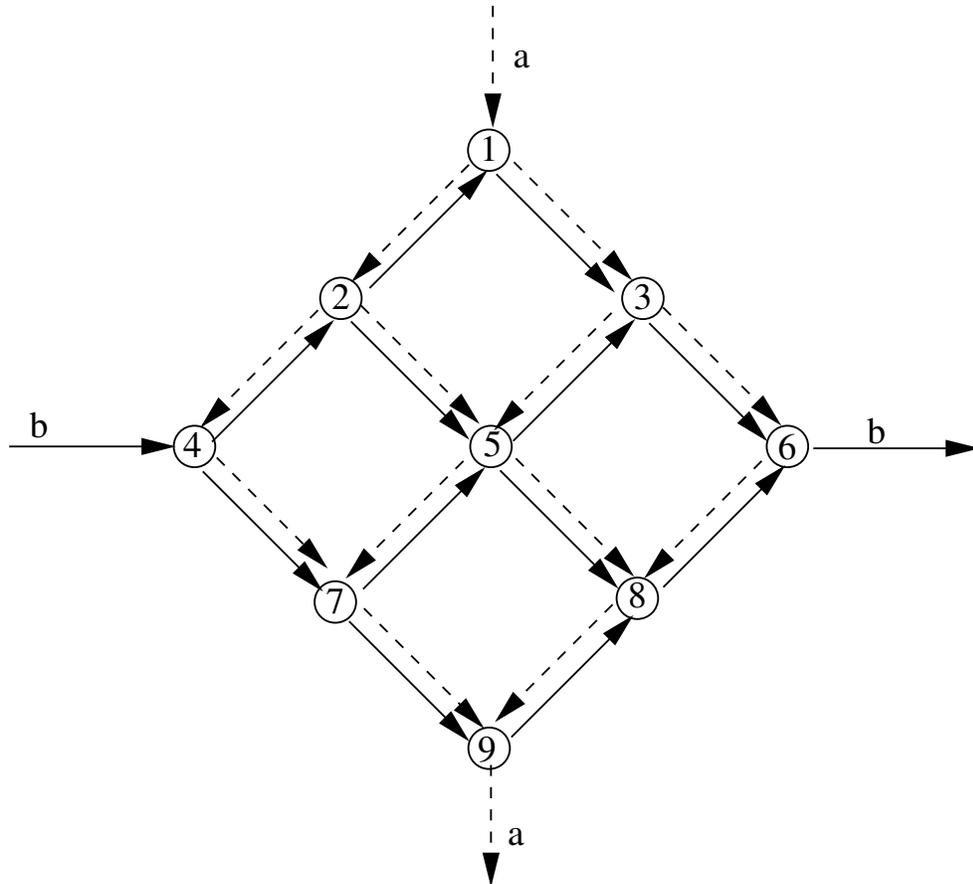
2. Let $a = 36\,(d_1 + d_2 + 5)$ and $b = 24\,(d_1 + d_2 + 5)$,

   where $d_1 \in \{1, \ldots, 31\}$ is the day of the month the first member of your group was born, and $d_2 \in \{1, \ldots, 31\}$ is the corresponding number for

the second member.

If there is no second member of the group then let $d_2 = d_1$.

Use the matlab function `linprog` to solve the problem for these values of a and b.

Illustrate your obtained solution in a figures like the one on the last page. On each link in that figure, write the size of the red flow with a red pen, the size of the blue flow with a blue pen, and the link load with a black pen.

3. Assume now that, due to technical problems, it is no longer possible to send blue flow in two of the channels, namely the blue channel from node 2 to node 1 and the blue channel from node 9 to node 8. (The red channels still work.)

   Describe how your LP formulation is changed by these new restrictions, solve the problem again, and illustrate your obtained solution in a separate figure as the one mentioned above.

   Hint 3: The vector `ub` (sometimes denoted `UB`) in `linprog` may be useful.

The matrices `Redarc` and `Bluearc` below define the directed channels (arcs) for the red flow and the blue flow. In column k of the the matrix `Redarc` you find the numbers of the starting node (row 1) and ending node (row 2) for the k:th red flow channel. Similarly, in column k of the the matrix `Bluearc` you find the numbers of the starting node (row 1) and ending node (row 2) for the k:th blue flow channel. The two matrices can be copied from the homepage of the course, and they can be used for generating the incidence matrices needed in the LP formulation of the problems, see pages 75-76 in course compendium, just use a `for`-loop in matlab!

```
Redarc=[1 1 2 2 3 3 4 5 5 6 7 8    Bluearc=[2 1 4 2 5 3 4 7 5 8 7 9
        2 3 4 5 5 6 7 7 8 8 9 9];            1 3 2 5 3 6 7 5 8 6 9 8];
```