EXAM FOR OPTIMIZATION SF1811/SF1831/SF1841

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Instructions: Calculators are **not** allowed! A formula sheet is provided. Motivate your answers carefully. Write your name on each page of the solutions you hand in and number the pages. Write the solutions to the different questions 1,2,3,4,5,6,7 on separate sheets.

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(1) Consider the following linear programming problem:

$$(LP): \begin{cases} \text{minimize} & 4x_1 + x_2 - x_3 + 2x_4 \\ \text{subject to} & 3x_1 - 3x_2 + x_3 = 3, \\ & 6x_1 - 2x_2 + x_4 = 2, \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0. \end{cases}$$

Solve the problem (LP) using the simplex method. Start by taking x_3 and x_4 as basic variables.

(6 points)

(2) Suppose that $A \in \mathbb{R}^{n\times n}$ is a matrix with the property $A^\top = -A$, that $c \in \mathbb{R}^{n\times 1}$, and that the following linear programming problem has a feasible solution:

$$\begin{cases} \text{minimize} & c^\top x \\ \text{subject to} & Ax \geq -c, \\ & x \geq 0. \end{cases}$$

Conclude that the problem has an optimal solution. What is the optimal objective function value of this problem?

(5 points)

(3) Persons A,B,C,D each own a warehouse and they operate their four warehouses together. The individual capacities of their warehouses are given in the following table:

Warehouse	A	B	C	D
Capacity (tonnes)	150	200	300	350

A,B,C,D have together entered into a contract with an farming company F, which produces a certain agricultural product P, for storing this year's produce of P. The farming company F owns three farms F1, F2, F3, and each of these produces P. In the current year, the amount produced by these three farms is indicated in the table below:

Farm	F1	F2	F3
Amount produced (tonnes)	250	250	500

Because these farms and the warehouses are spread across the country, there are differences in the transportation costs for taking the product P from the farms to the warehouses. The cost of transportation between the various farms and the warehouses is shown in the table below, in units of 1000 SEK/tonne:

	A	B	C	D
F1	10	5	11	11
F2	10	2	7	12
F3	9	1	4	8

The farming company wants to transport all of its produce of P in this year to the warehouses, and it wants to do so in a manner so as to minimize the transportation cost, while satisfying the constraints specified above.

(a) Formulate the above problem as a balanced transportation problem. State the variables and constraints clearly.
(b) Find an initial basic feasible solution to the problem described above, using the northwest corner method.
(c) Find an optimal solution.
(d) Suppose that owing to a road works on the way to the warehouse belonging to person A, the effective cost of transportation to this warehouse from each of the three farms increases by 2 units, that is by 2000 SEK/tonne. Explain why the optimal solution should not change, by using the structure of general balanced transportation problems.

(2+2+4+2=10 points)

(4) A civil engineer is assigned the task of determining the heights above sea level of three hills, H_1, H_2, H_3. He stands at sea level and measures the heights (in meters) of H_1, H_2, H_3 as 1236, 1941, 2417, respectively. Then to check his work, he climbs hill H_1 and measures the height of H_2 above H_1 as 711m, and the height of H_3 above H_1 as 1177m. Noting that these latter measurements are not consistent with those made at sea level, he climbs hill H_2, and measures the height of H_3 to be 474m above H_2. Again he notes the inconsistency of this measurement with those made earlier. As he drives back to his office, he suddenly remembers his days as a student in the Optimization course, and he decides to solve a quadratic optimization problem associated with this problem by considering the problem of mimimizing the least squares error associated with the measurements. Compute the optimal solution for him so that he can keep both hands on the steering wheel.
(You may use without justification the fact that the following system of equations in the unknowns x_1, x_2 and x_3

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -652 \\ 2178 \\ 4068 \end{bmatrix}$$

has the solution $x_1 = 1235.5$, $x_2 = 1943$, $x_3 = 2415.5$.)

(8 points)

(5) Suppose that x_1, x_2, x_3, x_4, x_5 are real numbers such that

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 8, \quad \text{and} \\ x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 &= 16. \end{aligned}$$

We want to determine the largest possible value of x_5. Pose this as an optimization problem, and solve it using the method of Lagrange multipliers. You should explain why your solution is a global maximizer.

(7 points)

(6) Consider the following nonlinear optimization problem:

$$(NP): \begin{cases} \text{minimize} & e^{-(x_1+x_2)} \\ \text{subject to} & e^{x_1} + e^{x_2} \leq 20, \\ & x_1 \geq 0. \end{cases}$$

(a) Show that (NP) is a regular convex optimization problem.
(b) Write the KKT optimality conditions and solve them. Is there a globally optimal solution to the problem (NP)? Justify your answer.

(3+6=9 points)

(7) Use the method of Lagrange relaxation to solve the following optimization problem:

$$\begin{cases} \text{minimize} & x_1 + x_2 \\ \text{subject to} & \dfrac{1}{x_1} + \dfrac{1}{x_2} \leq 1, \\ & x_1 > 0 \\ & x_2 > 0. \end{cases}$$

(5 points)

End of the exam.