(1) (a) Let $\beta = (2, 3)$. We show that the given solution is a basic feasible solution corresponding to $\beta$. We have $A_\beta = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ and so $\beta$ is a basic tuple, since $A_\beta$ is invertible. Moreover,

$$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 14/5 \\ 7/5 \end{bmatrix} = \begin{bmatrix} \frac{14+21}{5} \\ \frac{28+7}{5} \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix} = b.$$

So the given $x$ is a basic solution. It is also feasible, since $x \geq 0$. Finally we check optimality by calculating the reduced costs for the nonbasic variables. The simplex multiplier vector $y$ is given by $A_\beta^T y = c_\beta$ i.e.,

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} y = \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \text{ and so } y = \begin{bmatrix} 12/5 \\ 8/10 \end{bmatrix}.$$

The reduced costs for the nonbasic variables are given by $r_\nu$, where $r_\nu = c_\nu - A_\nu^T y$, i.e.,

$$r_\nu = \begin{bmatrix} 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 12/5 \\ 8/10 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 12/5 \\ 4/5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 28/5 \\ 24/5 \end{bmatrix} = \begin{bmatrix} 7/5 \\ 6/5 \end{bmatrix} \geq 0.$$

So the given solution is optimal.

(1) (b). The dual (D) to the primal problem (P) in standard form

$$(P): \begin{cases} \min. & c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{cases}$$

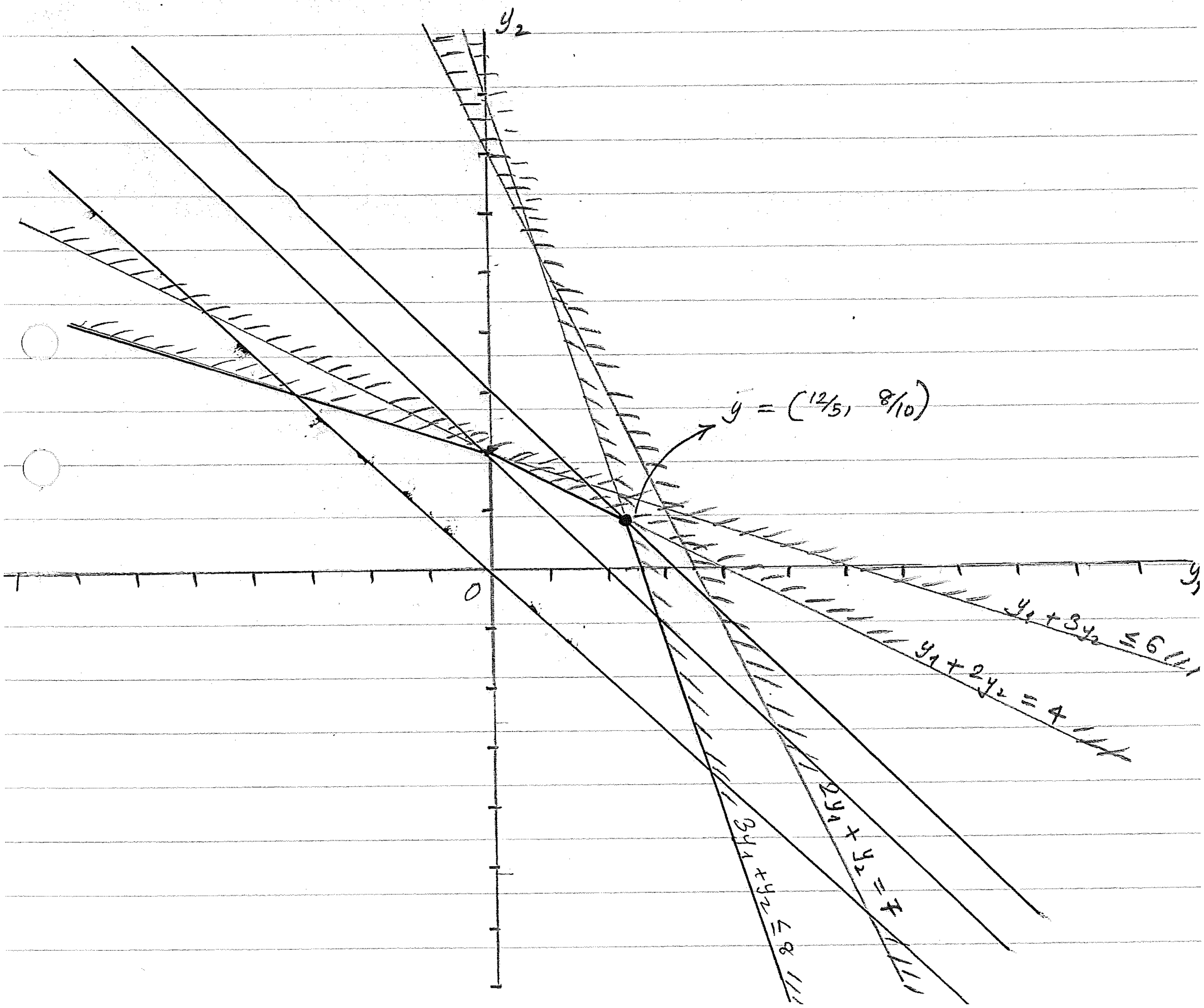
is given by

$$(D): \begin{cases} \max. & b^T y \\ \text{s.t.} & A^T y \leq c. \end{cases}$$

In our case, we obtain

$$(D): \begin{cases} \text{max.} & 7y_1 + 7y_2 \\ \text{s.t.} & 2y_1 + y_2 \le 7 \\ & 1y_1 + 2y_2 \le 4 \\ & 3y_1 + y_2 \le 8 \\ & y_1 + 3y_2 \le 6. \end{cases}$$

(1) (c). Graphical solution to (D): The feasible region is shown below. We have also shown the level curves of the objective function, i.e., $\{(y_1, y_2) : 7y_1 + 7y_2 = k\}$ for various values of k.



The unique optimal solution $y$ is given by:

$$\begin{cases} y_1 + 2y_2 = 4 \\ 3y_1 + y_2 = 8 \end{cases}$$

and so $y_1 = 12/5$

$y_2 = 8/10$.

This is the simplex multiplier vector corresponding to the optimal basic feasible solution $x$ of the primal problem.

(d) The given $x$ is still a basic feasible solution since $A$ or $b$ have not changed. The simplex multipliers vector $y$ is now given by $A_\beta^T y = \tilde{c}_\beta$, i.e.,

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} y = \begin{bmatrix} 4+\delta \\ 8+\delta \end{bmatrix}$$

and so $y = \frac{1}{1-6} \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4+\delta \\ 8+\delta \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 4+\delta-16-2\delta \\ -12-3\delta+8+\delta \end{bmatrix}$

$$= -\frac{1}{5} \begin{bmatrix} -\delta-12 \\ -2\delta-4 \end{bmatrix} = \begin{bmatrix} \frac{\delta+12}{5} \\ \frac{2\delta+4}{5} \end{bmatrix}.$$

So the reduced costs of the nonbasic variables are $r_\nu = \tilde{c}_\nu - A_\nu^T y$

$$= \begin{bmatrix} 7+\delta \\ 6+\delta \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \frac{\delta+12}{5} \\ \frac{2\delta+4}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 7+\delta \\ 6+\delta \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 2\delta+24+2\delta+4 \\ \delta+12+6\delta+12 \end{bmatrix} = \begin{bmatrix} 7+\delta \\ 6+\delta \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 4\delta+28 \\ 7\delta+24 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 35+5\delta-4\delta-28 \\ 30+5\delta-7\delta-24 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 7+\delta \\ 6-2\delta \end{bmatrix}.$$

The solution is optimal iff $r_\nu \geq 0$, i.e., ($7+\delta \geq 0$ and $6-2\delta \geq 0$), i.e., ($\delta \geq -7$ and $3 \geq \delta$), i.e., $\delta \in [-7, 3]$.

(2)(a) The incidence matrix is given by:

| | | edge (1,2) | (1,5) | (2,3) | (2,5) | (3,4) | (5,3) | (5,4) |
|---|---|---|---|---|---|---|---|---|
| A = | node ① | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | ② | −1 | 0 | +1 | 1 | 0 | 0 | 0 |
| | ③ | 0 | 0 | −1 | 0 | 1 | −1 | 0 |
| | ④ | 0 | 0 | 0 | 0 | −1 | 0 | −1 |
| | ⑤ | 0 | −1 | 0 | −1 | 0 | 1 | 1 |

The constraints are given by:

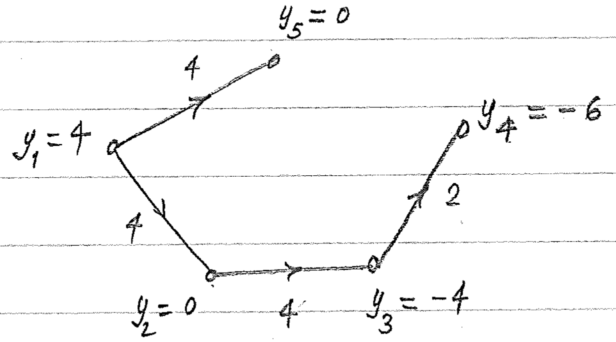
$$\begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

where $x = [x_{12} \ x_{15} \ x_{23} \ x_{25} \ x_{34} \ x_{53} \ x_{54}]^T$ and

$b = [+5 \ \ +5 \ \ -4 \ \ -3 \ \ -3]^T$.

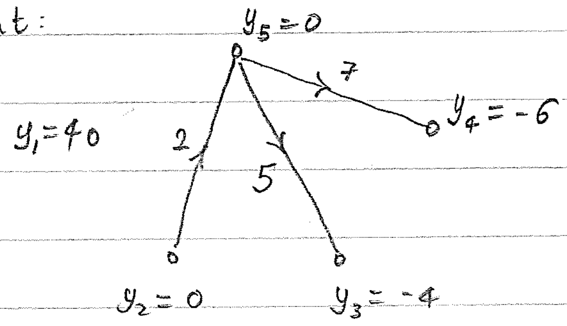
2(b). The given solution satisfies the flow balance at each node, the flows in each edge is $\geq 0$ and the nonzero flows are in edges which form a tree. So it is a basic feasible solution.

The simplex multipliers vector $y$ can be determined using $c_{ij} = y_i - y_j$ for tree edges $(i,j)$




The reduced costs for the nonbasic variables can be found out:

using $r_{ij} = c_{ij} - (y_i - y_j)$ for nonbree edges $(i,j)$.




$r_{54} = c_{54} - (y_5 - y_4)$
$= 7 - (0 - (-6)) = 1 \geq$

$r_{53} = c_{53} - (y_5 - y_3)$
$= 5 - (0 - (-4)) = 1 \geq$

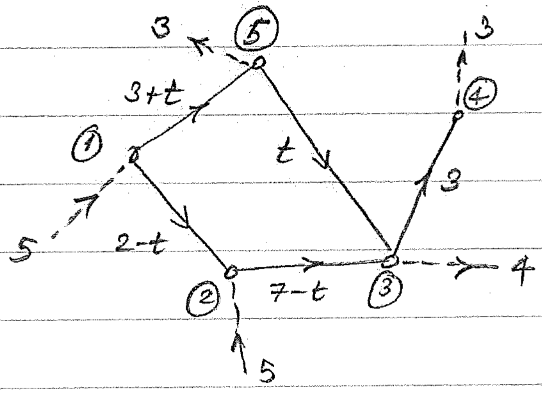
$r_{25} = c_{25} - (y_2 - y_5)$
$= 2 - (0 - 0) = 2 \geq 0$

Since all $r_{ij} \geq 0$, we conclude this (basic feasible) solution is optimal.

(c) Since only the cost vector has changed, the solution is still a basic feasible solution. Also the cost of a <u>nontree</u> edge has changed, and so the simplex multipliers vector is the same ( $A_\beta^T y = c_\beta$ ; $A_\beta$, $c_\beta$ are the same!). Also the reduced costs $r_{54}$, $r_{25}$ are the same as before. We have now

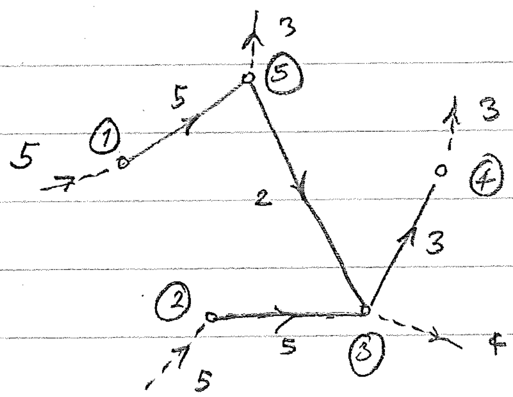
$$r_{53} = \tilde{c}_{53} - (y_5 - y_3) = \boxed{3} - (0 - (-4)) = -1 < 0 .$$

So the solution is not optimal. We let $x_{53} = t$ and let $t$ increase from 0.

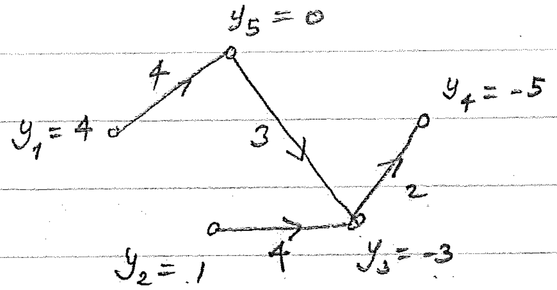



So $t$ can increase to a maximum of 2.

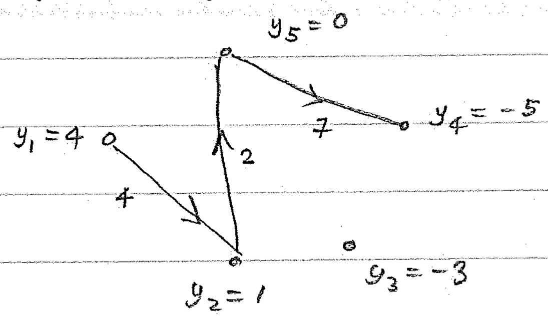
The new basic feasible solution is



The simplex multipliers vector $y$ can be determined using $c_{ij} = y_i - y_j$ for tree edges $(i,j)$:




The reduced costs for the nonbasic variables are given by $r_{ij} = c_{ij} - (y_i - y_j)$ for the nontree edges $(i,j)$:




$r_{12} = c_{12} - (y_1 - y_2) = 4 - (4 - 1) = 1 \geq 0$

$r_{25} = c_{25} - (y_2 - y_5) = 2 - (1 - 0) = 1 \geq 0$

$r_{54} = c_{54} - (y_5 - y_4) = 7 - (0 - (-5)) = 2 \geq 0$.

Since all $r_{ij} \geq 0$, the new basic feasible solution is optimal

(3)(a). $f : C \to \mathbb{R}$ is said to be a <u>convex function</u> if $\forall x_1, x_2 \in C$ and $\forall t \in (0,1)$

$$f(tx_1 + (1-t)x_2) \le t f(x_1) + (1-t) f(x_2).$$

(3)(b) We have

$$\frac{\partial f}{\partial x_1}(x) = 2x_1 + 2x_2,$$

$$\frac{\partial f}{\partial x_2}(x) = 10x_2 + 2x_1 + 4x_3 + 4x_2^3, \quad \text{and}$$

$$\frac{\partial f}{\partial x_3}(x) = 2ax_3 + 4x_2.$$

Hence

$$\frac{\partial^2 f}{\partial x_1^2}(x) = 2, \quad \frac{\partial^2 f}{\partial x_2 \partial x_1}(x) = 2, \quad \frac{\partial^2 f}{\partial x_3 \partial x_1} = 0,$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2}(x) = 2, \quad \frac{\partial^2 f}{\partial x_2^2}(x) = 10 + 12x_2^2, \quad \frac{\partial^2 f}{\partial x_3 \partial x_2} = 4,$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_3}(x) = 0, \quad \frac{\partial^2 f}{\partial x_2 \partial x_3}(x) = 4, \quad \frac{\partial^2 f}{\partial x_3^2}(x) = 2a.$$

So the Hessian $F(x)$ of $f$ at $x$ is given by

$$F(x) = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 10 + 12x_2^2 & 4 \\ 0 & 4 & 2a \end{bmatrix}.$$

The interior of $C := \mathbb{R}^3$ is not empty, and so we know that $f$ is convex iff $F(x)$ is positive semidefinite at all $x \in C = \mathbb{R}^3$.

For some elementary matrix $E_1$ we have

$$E_1 F(x) E_1^T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 + 12x_2^2 & 4 \\ 0 & 4 & 2a \end{bmatrix}.$$

As $x_2^2 \geq 0$, we have $8+12x_2^2 > 0$. So for an appropriate elementary matrix $E_2$ we have

$$E_2E_1F(x)E_1^TE_2^T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8+12x_2^2 & 0 \\ 0 & 0 & 2a - \frac{4}{2+3x_2^2} \end{bmatrix}.$$

So we see that $f$ is convex iff $2a - \frac{4}{2+3x_2^2} \geq 0 \quad \forall x_2 \in \mathbb{R}$ (since $2>0$ and $8+12x_2^2 > 0$ already).

We claim that $f$ is convex iff $a \geq 1$.

<u>If</u>: $a \geq 1$ and $1 \geq \frac{2}{2+3x_2^2}$ gives $a \geq \frac{2}{2+3x_2^2}$

and so $2a \geq \frac{4}{2+3x_2^2}$ i.e., $2a - \frac{4}{2+3x_2^2} \geq 0$.

Hence $f$ is convex.

<u>Only if</u>: If $f$ is convex, then $2a - \frac{4}{2+3x_2^2} \geq 0 \quad \forall x_2 \in \mathbb{R}$

In particular, take $x_2 = 0$. So $2a \geq \frac{4}{2+0}$ i.e., $2a \geq 2$ i.e., $a \geq 1$

So <u>$f$ is convex iff $a \geq 1$.</u>

(3)(c) We first calculate candidates for local minimizers

$$\nabla g(x) = [\, 4x_1^3 - 12x_2 \quad -12x_1 + 4x_2^3 \,].$$

Thus $\nabla g(x) = 0$ iff $\begin{Bmatrix} 4x_1^3 = 12x_2 \\ 4x_2^3 = 12x_1 \end{Bmatrix}$

iff $\left(x_1^3 = 3x_2 \text{ and } x_1 = \frac{1}{3}x_2^3\right)$ (*)

If (*) holds, $\frac{1}{3^3}x_2^9 = 3x_2$ i.e., $x_2(x_2^8 - 3^4) = 0$.

So $x_2 = 0$ or $x_2^2 = 3$ i.e., $x_2 \in \{0, \sqrt{3}, -\sqrt{3}\}$

If $x_2 = 0$, $x_1 = 0$; if $x_2 = \sqrt{3}$, $x_1 = \sqrt{3}$; if $x_2 = -\sqrt{3}$, $x_1 = -\sqrt{3}$

So if (*) holds, then $x \in \{(0,0), (\sqrt{3},\sqrt{3}), (-\sqrt{3},-\sqrt{3})\}$.

Conversely if $x \in \{(0,0), (\sqrt{3},\sqrt{3}), (-\sqrt{3},-\sqrt{3})\}$, then (*) holds.

We have $g(0,0) = 0$ and $g(\sqrt{3},\sqrt{3}) = g(-\sqrt{3},-\sqrt{3}) = 18 - 12\cdot 3$
$= -18$.

Since we are interested in global minimizers, we discard $(0,0)$. Also, the Hessian $G(x)$ of $g$ at $x$ is given by

$$G(x) = \begin{bmatrix} 12x_1^2 & -12 \\ -12 & 12x_2^2 \end{bmatrix} = 12\begin{bmatrix} x_1^2 & -1 \\ -1 & x_2^2 \end{bmatrix}.$$

Thus $G(\sqrt{3},\sqrt{3}) = G(-\sqrt{3},-\sqrt{3}) = 12\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$, which is positive definite.

(For a suitable elementary matrix $E_1$, $E_1\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}E_1^T = \begin{bmatrix} 3 & 0 \\ 0 & 3-\frac{1}{3} \end{bmatrix}$ and $3>0$, $3-\frac{1}{3}>0$.)

So $(\sqrt{3},\sqrt{3})$, $(-\sqrt{3},-\sqrt{3})$ are both local minimizers with the value of $g$ being the same at these points.

We have

$$g(x) = x_1^4 + x_2^4 - 12x_1x_2 \geq 2(x_1^2+x_2^2)^2 - 12\left(\frac{x_1^2+x_2^2}{2}\right)$$
$$= 2\|x\|_2^4 - 6\|x\|_2^2$$
$$\longrightarrow \infty \quad \text{as } \|x\|_2 \longrightarrow \infty.$$

Thus $\exists\, R>0$ s.t. $\forall\, \|x\|_2 \geq R$, $g(x) > -18 = g(\sqrt{3},\sqrt{3})$
$= g(-\sqrt{3},-\sqrt{3})$

In the compact set $K := \{x \in \mathbb{R}^2 \mid \|x\|_2 \leq R\}$, $g|_K$ has a global minimizer. But then these have to be the points $(\sqrt{3},\sqrt{3})$, $(-\sqrt{3},-\sqrt{3})$. So they now serve as global minimizers for $g$ in $\mathbb{R}^2$.

( $g(\sqrt{3},\sqrt{3}) = g(-\sqrt{3},-\sqrt{3}) \leq g(x)$ for $x \in K$ as well as
$g(\sqrt{3},\sqrt{3}) = g(-\sqrt{3},-\sqrt{3}) = -18 \leq g(x)$ for $x \notin K$.)

(7) (a). The problem is :

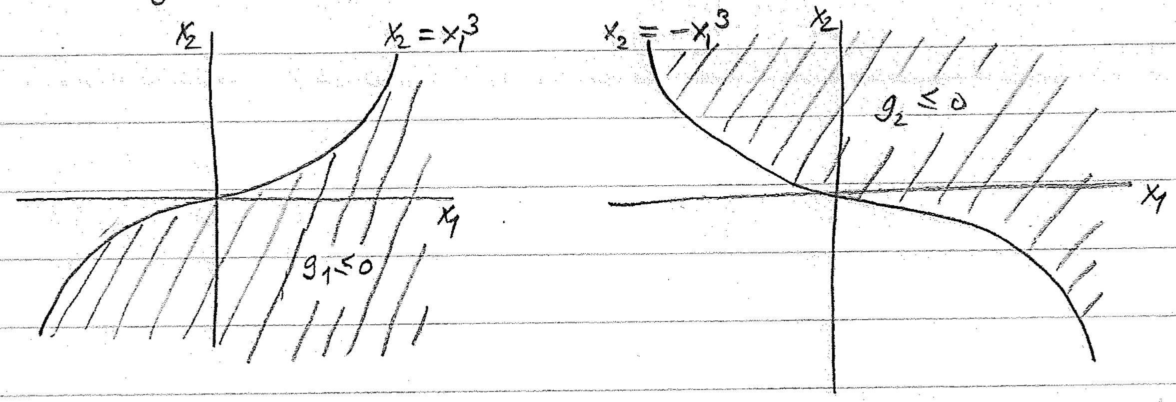
$$(NP): \begin{cases} \text{minimize} & f(x_1, x_2) \\ \text{subject to} & g_1(x_1, x_2) \leq 0 \\ & g_2(x_1, x_2) \leq 0 \end{cases}$$

where $f(x_1, x_2) := x_1$,

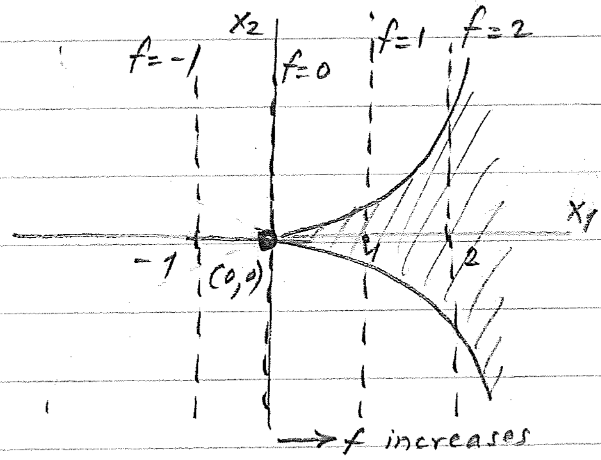
$g_1(x_1, x_2) := x_2 - x_1^3$

$g_2(x_1, x_2) := -x_2 - x_1^3$.

The regions $g_1 \leq 0$ and $g_2 \leq 0$ look like this:




Thus the feasible region looks like this:




The level curves of $f$ are shown by dotted lines. From the figure above we see that $(0,0)$ is a global minimizer.

(4).(b) The KKT-conditions are given by

(KKT-1) $\nabla f(x) + y^T \nabla g(x) = 0$.

In our case $\nabla f(x) = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$$\nabla g(x) = \begin{bmatrix} \nabla g_1(x) \\ \nabla g_2(x) \end{bmatrix} = \begin{bmatrix} -3x_1^2 & 1 \\ -3x_1^2 & -1 \end{bmatrix}.$$

So we have

$$\begin{bmatrix} 1 & 0 \end{bmatrix} + \begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} -3x_1^2 & 1 \\ -3x_1^2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

i.e. $\begin{cases} 1 - 3x_1^2 y_1 - 3x_1^2 y_2 = 0 \\ y_1 - y_2 = 0 \end{cases}$.

(KKT-2) $g(x) \le 0$

In our case: $\begin{cases} x_2 - x_1^3 \le 0 \\ -x_2 - x_1^3 \le 0 \end{cases}$.

(KKT-3) $y \geq 0$.

In our case: $\begin{cases} y_1 \geq 0 \\ y_2 \geq 0 \end{cases}$

(KKT-4) $y_i g_i(x) = 0$ for $i = 1, \ldots, m$.

In our case: $y_1(x_2 - x_1^3) = 0$

$y_2(-x_2 - x_1^3) = 0$.

For the point $(0,0)$, (KKT-2) and (KKT-4) are clearly satisfied. However (KKT-1) can never be satisfied since

$$1 - 3x_1^2 y_1 - 3x_1^2 y_2 \Big|_{(x_1,x_2)=(0,0)} = 1 \neq 0.$$

So there is no $y \in \mathbb{R}^2$ for which (KKT-1) – (KKT-4) are satisfied at $(0,0)$.

We know that the KKT-conditions are necessary for a local (and hence global) optimal solution $x$ <u>under the condition that $x$ is a regular point</u>.

However, we have: $I_a(0,0) = \{1, 2\}$ since $g_1(0,0) = 0$ and $g_2(0,0) = 0$.

Moreover, $\nabla g_1(0) = [0 \quad 1]$,

$\nabla g_2(0) = [0 \quad -1]$,

and we have with $v_1 := 1 \geq 0$

$v_2 := 1 \geq 0$

that $\sum_{i \in I_a(0,0)} v_i = 1 + 1 = 2 > 0$ and

$$\sum_{i \in I_a(0,0)} v_i \nabla g_i(0,0) = v_1 \cdot \nabla g_1(0) + v_2 \cdot \nabla g_2(0)$$
$$= 1 \cdot [0 \quad 1] + 1 \cdot [0 \quad -1]$$
$$= 0.$$

So $(0,0)$ is <u>not</u> a regular point.

(5.) (a) The problem is:

$$\begin{cases} \min. & \frac{1}{2} x^T H x + c^T x + c_0 \\ \text{s.t.} & A x = b \end{cases}$$

where $c_0 = 0$, $c = 0 \in \mathbb{R}^2$, $H = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$,

$A = [1 \quad -1]$ $b = [1]$. $H$ is positive definite on $\mathbb{R}^2$. So the problem is convex.

The kernel of $A$ is:

$$\ker A = \{ x \in \mathbb{R}^2 \mid Ax = 0 \} = \{ x \in \mathbb{R}^2 \mid x_1 = x_2 \}$$
$$= \text{span} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Hence $Z = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

As $\bar{x}$ we can take any solution to $Ax = b$.

Let us take $\bar{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

We find $\hat{v}$ using $Z^T H Z \hat{v} = -Z^T(H\bar{x} + c)$.

$$Z^T H Z = [1 \quad 1] \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [1 \quad 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2.$$

$$H\bar{x} + c = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

$$-Z^T(H\bar{x} + c) = -[1 \quad 1] \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -1.$$

Hence $\hat{v} = -\frac{1}{2}$.

Finally, $\hat{x} = \bar{x} + Z\hat{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\left(-\frac{1}{2}\right) = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}.$

Since the problem is convex, $\hat{x} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$ is an optimal solution.

5. (b). We relax the constraint $x_1 - x_2 \leq 1$. So $X = \mathbb{R}^2$, and the Lagrangian is

$$L(x, y) = x_1^2 - x_1 x_2 + x_2^2 + y(x_1 - x_2 - 1), \quad x \in \mathbb{R}^2, y \in \mathbb{R}$$

For $y \geq 0$, the relaxed Lagrange problem $(PR_y)$ is:

$$(PR_y): \begin{cases} \min. & x_1^2 - x_1 x_2 + x_2^2 + y(x_1 - x_2 - 1) \\ s.t. & x \in \mathbb{R}^2. \end{cases}$$

This is an unconstrained quadratic optimization problem, and since the corresponding $H$ is given by

$$H = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix},$$

which is positive definite, we know there is a unique global minimizer, given by $\hat{x}(y) = -H^{-1}c$

i.e.,

$$\hat{x}(y) = -\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} y \\ -y \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} y \\ -y \end{bmatrix} = \frac{y}{3}\begin{bmatrix} -1 \\ +1 \end{bmatrix}.$$

The dual objective function is:

$$\varphi(y) = \frac{y^2}{9} + \frac{y^2}{9} + \frac{y^2}{9} + y\left(-\frac{2y}{3} - 1\right)$$

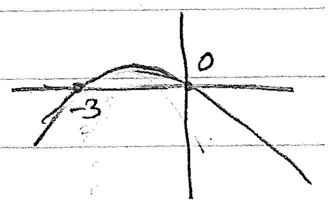
$$= \frac{y^2}{3} - \frac{2y^2}{3} - y = -\frac{1}{3}y^2 - y.$$

The dual problem is

$$(D): \begin{cases} \max & \varphi(y) \\ s.t. & y \geq 0 \end{cases}$$

i.e.,

$$(D): \begin{cases} \max. & -\frac{1}{3}y^2 - y = -y\left(\frac{1}{3}y + 1\right) \\ s.t. & y \geq 0. \end{cases}$$




The optimal solution to (D) is $\hat{y} = 0$.

The pair $(\hat{x}, \hat{y}) = \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0\right)$ satisfy the global optimality conditions associated with (QP') since

(1) $L(\hat{x}(y),y) = \min_{x \in X} L(x,y)$ for all $y \geq 0$

and in particular when $y = \hat{y} = 0$, we have $\hat{x}(0) = \hat{x}$

and $L(\hat{x},\hat{y}) = \min_{x \in X} L(x,\hat{y})$.

(2) $g(\hat{x}) = 0 - 0 - 1 = -1 \leq 0$

(3) $\hat{y} = 0 \geq 0$.

(4) $\hat{y}^T g(\hat{x}) = 0 \cdot (-1) = 0$.

So $\hat{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is optimal for (QP').