(QP) minimera $\frac{1}{2}x^T H x + c^T x$

då $Ax = b$

a) Nollrumsmetoden

b) Lagrangemetoden

$$H = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}, \quad c = \begin{pmatrix} -1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 77/12 \\ 75/4 \end{pmatrix}$$

$$H = LDL^T = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} \begin{pmatrix} 3 & & & 0 \\ & 1 & & \\ & & 2 & \\ 0 & & & 4 \end{pmatrix} \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

a) $d_i > 0 \quad \forall i \Rightarrow H \succ 0 \Rightarrow \bar{H} = Z^T H Z \succ 0$ där kolonnerna i Z bildar bas till $\mathcal{N}(A)$.

Hitta $\bar{x}$, där $A\bar{x} = b$, och Z.

$Ax = b$ ger

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 77/12 \\ 3 & 2 & 1 & 5 & 75/4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 77/12 \\ 0 & 1 & 2 & -2 & 1/2 \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} \beta_1 & \beta_2 & \gamma_1 & \gamma_2 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 77/12 \\ 0 & 1 & 2 & -2 & 1/2 \end{array}\right) \end{array}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -2 \end{pmatrix}}_{=T}$$

Välj $x_3 = x_4 = 0$ ger $\bar{x} = \left(\frac{77}{12}, \frac{1}{2}, 0, 0\right)^T$

$$\dim(\mathcal{N}(A)) = n - \dim(\mathcal{R}(A)) = 4 - 2 = 2$$

Hitta 2 linjärt oberoende vektorer z_i s.a. $Tz_i = 0$

$x_\beta^1 = -U_\gamma e_1 = -\begin{pmatrix} -1 & 3 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad x_\gamma^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ger $x^1 = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$

$x_\beta^2 = -U_\gamma e_2 = +\begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \quad x_\gamma^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ger $x^2 = \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$

$$\Rightarrow Z = \begin{pmatrix} 1 & -3 \\ -2 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\bar H$ 70
(unik) Optimallösning till (QP') ges av

$$\bar H \hat v = -\bar c$$

där
$$\bar H = Z^T H Z = \dots = \begin{matrix} 9 & 13 \end{matrix}$$

$$\bar c = Z^T\left[H\bar x + c\right] = \dots = \begin{pmatrix} 55/4 \\ -193/4 \end{pmatrix}$$

ger $\hat v = \begin{pmatrix} 1 \\ 7/4 \end{pmatrix}$

(unik) Optimallösning till (QP) ges av

$$\hat x = \bar x + Z\hat v = \begin{pmatrix} 77/12 \\ 1/2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ -2 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 7/4 \end{pmatrix} = \begin{pmatrix} 5/3 \\ 2 \\ 1 \\ 7/4 \end{pmatrix}$$

b) Lös

$$\begin{pmatrix} H & -A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} = \begin{pmatrix} -c \\ b \end{pmatrix}$$

dvs

$$\left[\begin{array}{cccc|cc} 3 & & & & -1 & -3 \\ & 1 & & & -1 & -2 \\ & & 2 & & -1 & -1 \\ & & & 4 & -1 & -5 \\ \hline 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 5 & 0 & 0 \end{array}\right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \hline u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ \hline 77/12 \\ 75/4 \end{pmatrix}$$

$$\Rightarrow \hat x = \begin{pmatrix} 5/3 \\ 2 \\ 1 \\ 7/4 \end{pmatrix} \quad \text{och} \quad \hat u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$