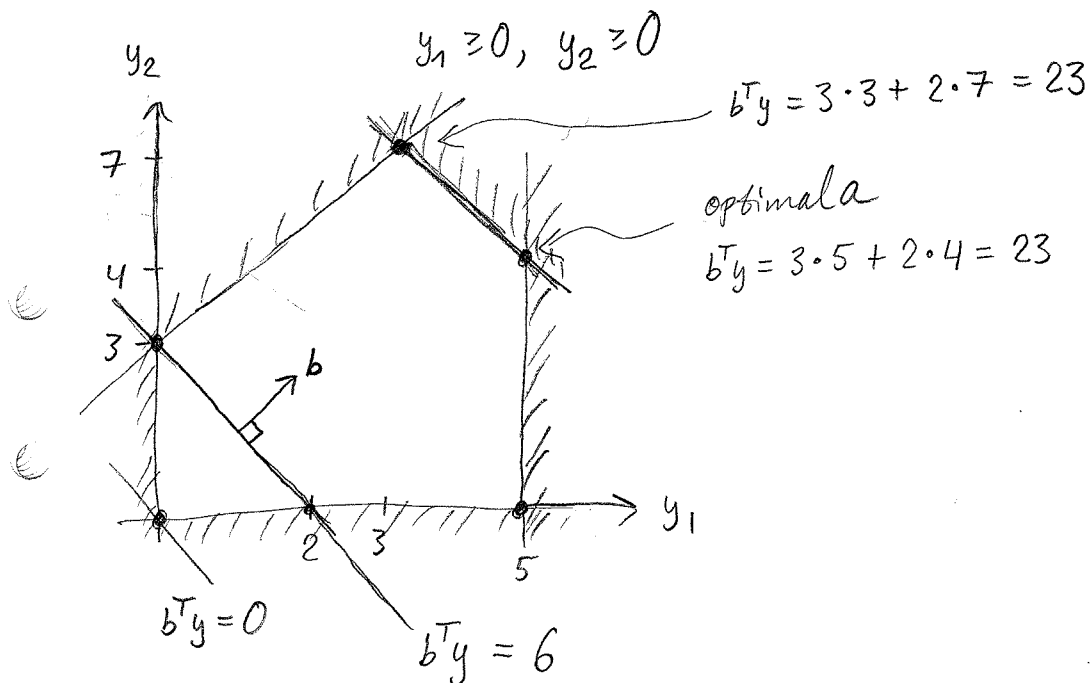


a) Lös (D) grafisch, das
maximiere $3y_1 + 2y_2$

$$(D) \quad \begin{aligned} \text{d.h.} \quad & \frac{3}{2}y_1 + y_2 \leq 23/2 \\ & -\frac{4}{3}y_1 + y_2 \leq 3 \\ & y_1 \leq 5 \end{aligned}$$



b) Formuliere das primale LP problem.

$$(D) \quad \begin{aligned} \text{maximiere} \quad & b_1^T \bar{y}_1 \\ \text{d.h.} \quad & A_{11}^T \bar{y}_1 \leq c_1 \\ & \bar{y}_1 \geq 0 \end{aligned}$$

$$\begin{aligned} b_1^T &= \begin{pmatrix} 3 & 2 \end{pmatrix} \\ A_{11}^T &= \begin{pmatrix} 3 & 2 \\ -\frac{4}{3} & 1 \\ 1 & 0 \end{pmatrix}, \quad \bar{y}_1 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad c_1 = \begin{pmatrix} 23 \\ 3 \\ 5 \end{pmatrix} \end{aligned}$$

$$(P) \quad \begin{array}{ll} \text{minimera} & C_1^T \bar{x}_1 \\ \text{då} & A_{11} \bar{x}_1 \geq b_1 \\ & \bar{x}_1 \geq 0 \end{array}$$

dar $C_1^T = (23, 3, 5)$

$$A_{11} = \begin{pmatrix} 3 & -4/3 & 1 \\ 2 & 1 & 0 \end{pmatrix}, \quad \bar{x}_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$(P) \quad \begin{array}{ll} \text{minimera} & 23x_1 + 3x_2 + 5x_3 \\ \text{då} & 3x_1 - \frac{4}{3}x_2 + x_3 \geq 3 \\ & 2x_1 + x_2 \geq 2 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{array}$$

$$\left(\begin{array}{ll} \text{minimera} & c^T x \\ \text{då} & Ax \geq b \\ & x \geq 0 \end{array} \right)$$

c) Givet att $y^* = (3, 7)^T$, beräkna x^* .

Komplementaritetsatsen ger

$$\begin{cases} x^* \text{ optimal till (P)} \\ y^* \text{ optimal till (D)} \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} x^* \geq 0, \quad s = Ax^* - b \geq 0 \\ y^* \geq 0, \quad r = C - A^T y^* \geq 0 \\ x_j r_j = 0 \quad \forall j, \quad y_i s_i = 0 \quad \forall i \end{cases}$$

vi har lösning till (D) dar

$$y^* = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \geq 0, \quad r = \begin{pmatrix} 23 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ -4/3 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$x_j r_j = 0 \quad \forall j \Rightarrow x_3 = 0$$

$$y_i s_i = 0 \quad \forall i \Rightarrow s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Primal tillåtenhet och $s=0$ ger

$$s = Ax^* - b = 0$$

$$\begin{pmatrix} 3 & -\frac{4}{3} & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \text{ger} \quad \begin{cases} x_1 = 1 \\ x_2 = 0 = x_3 \end{cases}$$

$$c^T x^* = 23 = b^T y^* \quad (\text{ok.})$$

d) Givet att $y^* = (5, 4)^T$, beräkna x^* .

$$y^* = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \geq 0, \quad r = \begin{pmatrix} 23 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ -4/3 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 17/3 \\ 0 \end{pmatrix}$$

$$x_j r_j = 0 \quad \forall j \Rightarrow x_2 = 0$$

$$y_i s_i = 0 \quad \forall i \Rightarrow s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$s = Ax^* - b = 0$$

$$\begin{pmatrix} 3 & -\frac{4}{3} & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ 0 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$c^T x^* = 23 = b^T y^* \quad (\text{ok.})$$