KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Spring 2011
You may use min(5,(your score)/5) as bonus credit on the exam.

1. Determine a state feedback K such that the eigenvalues of the closed-loop system $\dot{x} = (A + BK)x$ are located in $\{-1, -2, -3, -4\}$, for the case where

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

.... (4p)

2. Consider

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x.$$

(a) Is the system observable? .... (1p)

(b) Design a feedback controller $u = kx$ such that the closed-loop poles are placed at $\{-1, -2, -3\}$. .... (2p)

(c) Explain what happens to the observability of the closed-loop system...... (2p)

(d) Assume now that the full state is not available. Can we always design an observer-based control that stabilizes the overall system, with the closed-loop poles located at $\{-1, -2, -3\}$ and the observer dynamics having poles at $\{-1, -2, -3\}$? (1p)

3. Consider

$$\dot{x} = Ax + Bu$$

where (A, B) is controllable and A does not have any eigenvalue on the imaginary axis. Given the cost function

$$J = \int_0^\infty u^T u dt,$$

we can show that $u = -B^T Px$ is the optimal control, where P is a positive *semi-definite* solution to the corresponding ARE.

(a) What are the eigenvalues of $(A - BB^T P)$ if A is a stable matrix? . . . . (1p)

(b) What are the eigenvalues of $(A - BB^T P)$ if $-A$ is a stable matrix? . . . . . . (4p)

Hint: an optimal control is necessarily a feasible control.

4. Consider again

$$\dot{x} = Ax + Bu,$$

where (A, B) is controllable. Given the cost function

$$J = \int_0^{t_1} u^T u dt + x(t_1)^T S x(t_1),$$

where S is positive definite, and let $u = -B^T P(t_1 - t)x$ denote the optimal control.

(a) Compute $\lim_{t_1 - t \to \infty} P(t_1 - t)$ for the case A is a stable matrix. . . . . (2p)

(b) Compute $\lim_{t_1 - t \to \infty} P(t_1 - t)$ for the case $-A$ is a stable matrix. . . . . (3p)

(c) Comparing your results with those P in Problem 3, what conclusion can you draw? . . . . (2p)

Hint: to determine P is the same as determining P^{-1} if P is invertible.

5. Consider a second order random walk model

$$x(t+1) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} \frac{a^2}{2} \\ a \end{bmatrix} v(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + w(t),$$

where v, w are uncorrelated white noises, with covariances $q,\ r$ respectively.

Design Kalman filter for the system. . . . . (3p)