KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Spring 2012
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Consider

$$\dot{x} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} x.$$

(a) Is the system controllable? .... (1p)

(b) Can a stabilizing feedback controller $u = kx$ (i.e. the closed-loop poles are all placed at the open left half plane) make the system unobservable? .... (2p)

(d) Assume $n = 2$ and the full state is not available. Design an observer-based control that stabilizes the overall system, with the closed-loop poles located at $\{-1, -2\}$ and the observer dynamics having poles at $\{-1, -2\}$. .... (2p)

2. Consider

$$\dot{x} = Ax + Bu$$

where (A, B) is controllable and A does not have any eigenvalue on the imaginary axis. Given the cost function

$$J = \int_0^\infty u^T u dt,$$

we can show that $u = -B^T P x$ is the optimal control, where P is a positive *semi-definite* solution to the corresponding ARE.

(a) What are the eigenvalues of $(A - BB^T P)$? .... (2p)

(b) What will happen to the optimal control problem if A has eigenvalues on the imaginary axis? (It is enough to use an as simple as possible example to explain). (3p)

3. Consider

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u,$$

and the cost function

$$J = \int_0^{t_1} ((x_1 + x_2)^2 + u^2)dt.$$

(a) Find the optimal control. .... (3p)

(b) Let $t_1 = \infty$. What is the optimal control? .... (2p)

4. (a) Suppose (A, B) is controllable and (C, A) is observable, and P is the positive definite solution to

$$A^T P + PA - PBB^T P + C^T C = 0.$$

Show $A - kBB^T P$ is a stable matrix for all $k \geq 1$. .... (3p)

(b) Consider

$$x(t+1) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} \frac{a^2}{2} \\ a \end{bmatrix} v(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + w(t),$$

where v, w are uncorrelated white noises, with covariances q, r respectively. Design Kalman filter for the system. .... (2p)