KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2014
You may use min(5,(your score)/5) as bonus credit on the exam.

1. Consider

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= u\\ x(0) &= x_0.\end{aligned}$$

In problem 3c. of the second homework, we have shown that high gain control can lead to unbounded x as the gain tends to infinity. Now let us try a different idea with high gain control.

(*a*) Design a stabilizing feedback control $u = k_1 x + k_2 x_2$ that is also the optimal control to

$$\min_u \int_0^\infty (x_1^2(t) + \frac{1}{h^2}u^2(t))dt,$$

here we can view h as the control gain. .... (3p)

(*b*) What is the optimal cost $V(0)$ as $h \to \infty$? .... (3p)

(*c*) Now let $y = x_1$ and design an observer gain $(l_1\ l_2)^T$ such that the observer functions also as optimal filter in steady state (as $t \to \infty$) if we consider u as noise of covariance $1\delta(t-s)$, and there is also a measurement noise w of covariance $\eta^2\delta(t-s)$. .... (3p)

2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

(*a*) Can we find $b \in R^4$ such that (A, b) is controllable? .... (1p)

(*b*) Find a 4×2 matrix B with as few non-zero elements as possible such that (A, B) is controllable. .... (3p)

(*c*) Suppose $c = (c_1, c_2, c_3, c_4)$, together with A and B you found in (b) is a realization of some transfer matrix $R(s)$. What is the highest MacMillan degree $R(s)$ can have among all real elements of c? .... (2p)

3. Consider a controllable system

$$\dot{x} = Ax + Bu.$$

In the compendium we derive first the optimal control for a fixed end-point problem as

$$u = -B^T P(t)x(t) + B^T \Phi(t_1, t)^T W(t_0, t_1)^{-1}[x_1 - \Phi(t_1, t_0)x_0], \tag{1}$$

then use Bellman's principle to argue that the optimal control can be rewritten as

$$u = -B^T P(t)x(t) + B^T \Phi(t_1, t)^T W(t, t_1)^{-1}[x_1 - \Phi(t_1, t)x(t)]. \tag{2}$$

In this exercise, you are asked to show that (2) is equivalent to (1) without using Bellman's principle. ....(5p)

4. At time $t = 1, 2, 3, \cdots$, an observation $y(t)$ is made of an unknown constant x. The observation error $y(t) - x$ is zero mean white noise with variance σ^2. Our apriori knowledge on x has variance p_0.

(a) Design a Kalman filter for the estimation of x. .... (1p)

(b) Express the covariance matrix $p(t) = E\{(x - \hat{x}(t))^2\}$ in terms of $t,\ \sigma,\ p_0$. (2p)

(c) What is $\hat{x}(t)$ (expressed in terms of $y(1), \cdots, y(t-1)$) if we do not have any apriori knowledge on x? (Hint: what is p_0 in this case?) .... (2p)