KTH Matematik

Homework 1
Mathematical Systems Theory, SF2832
Fall 2015
You may use min(5,(your score)/4) as bonus credit on the exam.

(Your homework should be handed in to Yuecheng Yang before the deadline)

1. Find the state transition matrix $\Phi(t,s)$ for the following systems

(a) $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x(t)$

.... (2p)

(b) $\dot{x}(t) = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} x(t).$

.... (3p)

2. (a) Suppose an $n \times n$ matrix $A(t)$ satisfies $\dot{A} = KA - AK$, $A(0) = A_0$, where K is a constant $n \times n$ matrix. Show

$$A(t) = e^{Kt} A_0 e^{-Kt},$$

and the eigenvalues of A are independent of t. .... (2p)

(b) For the same A in (a), consider

$$\dot{x} = A(t)x.$$

Show that a fundamental matrix is $\Psi(t) = e^{Kt} e^{(A_0 - K)t}$. .... (3p)

3. Consider

$$\dot{x} = Ax + bu,$$

where

$$A = \begin{bmatrix} 0 & -a_1 & a_2 \\ a_1 & 0 & -a_3 \\ -a_2 & a_3 & 0 \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) Show that $\forall x(0)$, $\|e^{At}x(0)\| = \|x(0)\|$. .... (2p)

(b) Show that if $\|a\| \neq 0$, then there exists b such that (A,b) is controllable, where $a = (a_1 \; a_2 \; a_3)^T$. .... (3p)

4. The following is linearized model of an inverted pendulum

$$\dot{x} = Ax + Bu$$

where g is the acceleration of gravity, and

$$A = \begin{bmatrix} 0 & 1 \\ g & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

(a) Check controllability for this system. Can we find control $u = k_1 x_1 + k_2 x_2$ such that the closed-loop system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ g - k_1 & -k_2 \end{bmatrix} x$$

is asymptotically stable? ....(3p)

(b) Can we find a scalar output $y = cx$ such that the system is observable? ..(2p)