KTH Matematik

Homework 1
Mathematical Systems Theory, SF2832
Fall 2016
You may use min(5,(your score)/4) as bonus credit on the exam.

1. (*a*) Find the input-output description for $\ddot{y} = -y + u$,
where $y \in R$ is the output and $u \in R$ is the input. ....(2p)

(*b*) Find the state transition matrix for the following system

$$\dot{x}(t) = \begin{bmatrix} 0 & t & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} x(t)$$

.... (2p)

2. Consider the rotational motion of a point x in R^3 with respect to the origin:

$$\dot{x} = \omega \times x,$$

where $\omega = (\omega_1, \omega_2, \omega_3)^T \neq 0$, a constant vector, is the angular velocity, and "×" is the vector cross product.

(*a*) Express the kinematics of $x(t)$ in the form of $\dot{x} = Ax$.....................(1p)

(*b*) Compute e^{At}. ....(4p)

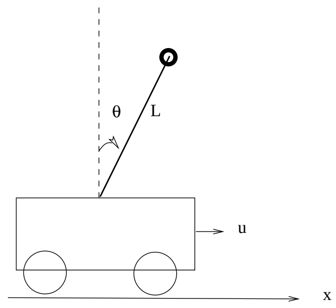
3. Consider

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, A, B and C are constant matrices. Let Ω be the observability matrix as is defined in the compendium. State and prove the necessary and sufficient condition such that for any $x(0) \in ker\ \Omega$, $x(t) \in ker\ \Omega$, $\forall t \geq 0$, no matter what $u(t)$ is used.

.... (3p)

4. Consider the inverted pendulum as we did in the lecture



Figur 1: Inverted pendulum on a cart.

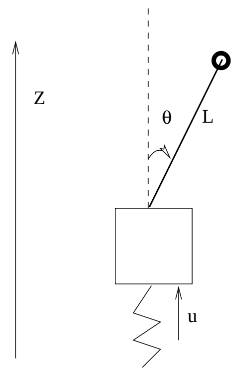
The following equation describes the motion of the pendulum around the equilibrium $\theta = 0$:

$$L\ddot{\theta} - g\sin(\theta) + \ddot{x}\cos(\theta) = 0$$

(a) We consider $\ddot{x}$ as the input u and θ as the output y. Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. Derive the state space model for the linearized system (i.e. let $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$)..........(1p)

(b) Show the model you derive in (a) is both controllable and observable. ... (2p)

5. Now consider an inverted pendulum with oscillatory base



Figur 2: Pendulum with oscillatory base.

The following equation describes the motion of the pendulum around the equilibrium $\theta = 0$:

$$L\ddot{\theta} - g\sin(\theta) - \ddot{z}\sin(\theta) = 0$$

(a) We consider $\ddot{z}$ as the input u and θ as the output y. Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. Derive the state space model for the *linearized* system (i.e. let $\sin(\theta) \approx \theta$).(1p)

(b) Is the model you derive in (a) controllable? .... (1p)

(c) Let $z = 0.1\sin(\omega t)$ (be careful with the variable!). Use Matlab simulation to find out what will happen to the motion (use the original nonlinear model!) of the pendulum near $\theta = 0$ when the oscillation is "slow" and when the oscillation is "fast". Take $L = 1,\ \theta(0) = 0.02$ and $\dot{\theta}(0) = 0$. . . . . (3p)