KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Fall 2016
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Consider a time-invariant system

$$\dot{x} = Ax,$$

where $x \in R^n$, $A \neq 0$ and $tr(A) = 0$, $tr(\cdot)$ denotes the trace of a matrix.

Show

(a) The system is not asymptotically stable (around $x = 0$). .... (2p)

(b) The system is not even (critically) stable if A is also symmetric. ....(1p)

(c) The system is (critically) stable if A is also skew symmetric ($A^T = -A$). . (1p)

2. Given the following system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & c_2 \cdots & c_n \end{bmatrix} x.$$

(a) We say (C, A) is detectable if $Ce^{At}x_0 = 0,\ \forall t \geq 0$ implies $\lim_{t\to\infty} e^{At}x_0 = 0$. For the case $c_2 = 1,\ c_i = 0,\ i \geq 3$, discuss condition on c_1 such that the system being detectable but *not* observable is possible. .... (3p)

(b) Use Kalman decomposition to show that Theorem 4.3.4 in the compendium can be modified as: Assume (C, A) is detectable. Then A is a stable matrix iff $A^TP + PA + C^TC = 0$ has a positive semi-definite solution P such that $x^TPx > 0\ \forall x \notin ker\ \Omega$. .... (3p)

3. (a) Consider the system in Problem 2 and let $n = 3$, $c_3 = 1$. Find the feedback control $u = kx$ that makes $y(t) = 0\ \forall t \geq 0$ (this implies that we assume $y(0) = 0$). .... (2p)

(b) Show that for all solutions $x(t)$ of the closed-loop system in 3(a) such that $cx(t) = 0\ \forall t \geq 0$, $\lim_{t\to\infty} x(t) = 0$ iff $c_1 > 0$ and $c_2 > 0$. .... (2p)

4. Consider

$$R(s) = \begin{bmatrix} \dfrac{k}{s+1} & \dfrac{1}{s+2} \\ \dfrac{1}{s+1} & \dfrac{1}{s+1} \end{bmatrix},$$

where k is a constant.

(a) Determine the standard reachable realization of $R(s)$. .... (2p)

(b) Determine the standard observable realization of $R(s)$. .... (2p)

(c) What is the McMillan degree of $R(s)$? .... (2p)