KTH Matematik

Homework 3
Mathematical Systems Theory, SF2832
Fall 2016
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Consider

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x. \end{aligned}$$

(a) Find the values of $k = (k_1 \ k_2)$ such that $A + bk$ has two identical eigenvalues. (2p)

(b) Find the values of h such that $A + bhc$ has two identical eigenvalues. . . . . . (1p)

(c) Can we conclude that for almost all k and h, the two matrices have distinct eigenvalues? . . . . (1p)

2. Consider

$$\begin{aligned} \dot{x} &= Ax + Bu \\ x(0) &= x_0, \end{aligned}$$

where (A, B) is controllable. Given the cost function

$$J(u, t_1) = \int_0^{t_1} u^T u dt,$$

with $x(t_1) = 0$, we know from Chapter 3 that $u^* = -B^T e^{A^T(t_1-t)} W(0, t_1)^{-1} e^{At_1} x_0$ is the optimal control.

(a) Show that u^* can be rewritten as $u^* = -B^T e^{A^T(t_1-t)} W(t, t_1)^{-1} e^{A(t_1-t)} x(t)$ (You are not allowed to argue by Bellman's Principle here). . . . . (3p)

(b) Show that $J(u^*, t_2) < J(u^*, t_1)$ if $t_2 > t_1$. . . . . (3p)

3. Consider

$$\dot{x}_1 = ax_1 + x_2$$
$$\dot{x}_2 = x_1 + u,$$

where a is a constant, and the cost function

$$J = \int_0^\infty (x_2^2 + \epsilon^2 u^2) dt.$$

As $\epsilon \to 0$, the optimal control problem becomes the so-called "cheap control" problem.

Assume $x^*(t)$ is the optimal trajectory for a given initial point $(x_1(0)\ x_2(0))^T$.

(a) For $a > 0$, find $\lim_{\epsilon\to 0} x_1^*(t)$. .... (2p)

(b) For $a < 0$, find $\lim_{\epsilon\to 0} x_1^*(t)$. .... (2p)

(c) Explain what happens to $\lim_{\epsilon\to 0} x_1^*(t)$ when $a = 0$. .... (2p)

4. (a) Suppose (A, B) is controllable and (C, A) is observable, and P is the positive definite solution to

$$AP + PA^T - PC^T(DRD^T)^{-1}CP + B^TB = 0.$$

Show $A - PC^T(DRD^T)^{-1}C$ is a stable matrix. .... (2p)

(b) Consider

$$x(t+1) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} \frac{a^2}{2} \\ a \end{bmatrix} v(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + w(t),$$

where v, w are uncorrelated white noises, with covariances $q,\ r$ respectively. Design Kalman filter for the system. .... (2p)