KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due November 18, 16:50, 2008

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. [1+2+1p]. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 2 & -1 & 0 \\ 1 & 0 & 1 & -2 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} u \\ y &= (1\ 0\ 0\ 0)x, \end{aligned}$$

where $x = (x_1, x_2, x_3, x_4)^T$.

(a) Is the system observable?

yes.

(b) Compute $\mathcal{V}^*$ and $\mathcal{R}^*$ contained in $\mathcal{V}^*$, and find (parameterize) ALL friends F of $\mathcal{V}^*$.

$R^* = V^* = span\{e_3, e_4\}$.

(c) Is the closed-loop system resulting from (b) observable (no further computation is allowed)?

no.

2. [4p]. Consider

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $x \in R^n,\ u \in R^m,\ y \in R^p$. Determine if each of the following statements is true or false.

(a) If $V^* = \{0\}$, then the original system is observable.

true.

(b) Suppose the system is reachable (controllable), then a necessary condition for having a reachability subspace that is strictly contained in R^n is that $m > 1$.

true.

3. [6p]. Consider

$$\begin{aligned}\dot{x} &= Ax + Bu + Ew \\ y &= Cx,\end{aligned}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \ C = (1\ 2\ 1).$$

(a) Find the minimum constraint on E such that DDP is solvable.
$E \subset V^* = kerC = span\{[1\ 0\ \ -1]^T, [0\ 1\ \ -2]^T\}$**.**

(b) Find $u = Fx$ that solves the DDP problem while makes the closed-loop system stable, i.e. $A + BF$ has only eigenvalues with negative real part.
$u = -x_2 - 4x_3 - (1 + k^2)(x_1 + 2x_2 + x_3)$**.**

(c) Verify that there exists an $E \in V^*$ such that (A, E) is controllable. Explain why even in this case the DDP problem is solvable (namely $w(t)$ will not at all influence the output).
$E = [1\ 0\ \ -1]^T$ **for example. But** $(A + BF, E)$ **is not controllable.**

4. [4p]. Consider

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_3 + u_1 \\ \dot{x}_2 &= -x_2 + x_3 - u_1 \\ \dot{x}_3 &= -x_3 + 2x_4 + u_2 \\ \dot{x}_4 &= x_4 + u_1 \\ y_1 &= x_1 + x_2 \\ y_2 &= x_4\end{aligned}$$

(a) What is the relative degree for the system?
$r = (2, 1)$**.**

(b) Convert the system into the normal form and compute the zero dynamics.
$\xi_{11} = y_1, \xi_{12} = -y_1 + 2x_3, \xi_{21} = y_2$**. There are many choices of** z**. Zero dynamics:** $\dot{z} = -z$**.**