KTH Matematik

SF2842: Geometric Control Theory

# Homework 1

Due February 11, 16:50pm, 2016

You may use min(5,(your score)/4) as bonus credit on the exam

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**1.** Consider the system

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -2 & 0 & 0 & -1 \\ 0 & -2 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 1 \\ 1 & 1 \end{pmatrix} u \\
y &= (1\ 1\ 0\ 0)x.
\end{aligned}
$$

(a) Compute $\mathcal{V}^*$ and express all friends $F$ of $\mathcal{V}^*$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (2p)

(b) Compute $\mathcal{R}^*$ that is contained in $ker\ C$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (2p)

(c) Can we find a friend $F$ of $\mathcal{V}^*$ such that $(A + BF)$ has all eigenvalues with negative real parts? . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (3p)

**2.** Consider

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx,
\end{aligned}
$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$.

(a) Show the controllable subspace is (A+BF)-invariant for any $F$. . . . . . . . . . . (2p)

(b) Assume further that $CA^kB \neq 0$, for some $k < n$, and $(C, A)$ is not observable. Show the unobservable subspace $ker\ \Omega$ is not (A+BF)-invariant for all $F$. (3p)

(c) Suppose $(C, A)$ is observable and the dimension of $\mathcal{V}^*$ is greater or equal to one. Show it is not possible to express a friend $F$ of $\mathcal{V}^*$ as $F = LC$, namely it is not possible to use output feedback to make $\mathcal{V}^*$ invariant. . . . . . . . . . . . . . . . . . . . . (2p)

**3.** Consider

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 + x_3 + x_4 \\
\dot{x}_2 &= -x_1 - \alpha u \\
\dot{x}_3 &= -x_2 - 2x_3 + u \\
\dot{x}_4 &= x_2 - u \\
y &= x_3 + x_4,
\end{aligned}
$$

where $\alpha$ is a constant.

(a) Convert the system into the normal form and compute the zero dynamics. (2p)

(b) Computer $\mathcal{V}^*$ and $\mathcal{R}^*$ in $ker\ C$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (2p)

(c) For what $\alpha$ we can find a friend $f$ of $\mathcal{V}^*$ such that $(A + bf)$ is a stable matrix? (2p)