KTH Matematik

SF2842: Geometric Control Theory

Homework 2

Due February 22, 16:50pm, 2017

---

1. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -2 & 1 & 0\\ 2 & 0 & 0 & 1\\ 0 & 0 & 1 & -1\end{pmatrix} x + \begin{pmatrix}\alpha & 1\\ 2 & 1\\ \alpha & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix}\\ \begin{pmatrix}y_1\\ y_2\end{pmatrix} &= \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} x,\end{aligned}$$

where α is a real constant.

(a) For what values of α is the noninteracting control problem solvable? .....(2p)

(b) What is the (transmission) zero(s) of the system when the noninteracting control problem is solvable? ..... (2p)

(c) Suppose now the first output y_1 is taken away from the system, namely only y_2 is kept. What is the (transmission) zero(s) of the system now if $\alpha = 2$? (3p)

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= -x_1 - 3x_2 - 3x_3 + w_1\\ \dot{w}_1 &= 2w_2\\ \dot{w}_2 &= -2w_1\\ y &= c_1x_1 + c_2x_2 + x_3,\end{aligned}$$

where $c_1,\ c_2$ are real constants and $c_1 - c_2 + 1 \neq 0$.

(a) Compute the invariant subspace $x = \Pi w$. [2p]

(b) For what value(s) of c_1, c_2 is the above system (consisting of x and w) unobservable? Explain why. [2p]

(c) Can we find c_1, c_2 such that $y(t) = w_1(t)$ in the steady state? [2p]

3. Consider a control system subject to disturbance:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -2x_1 - x_2 + x_3 + u + 2w_1 \\
\dot{x}_3 &= \alpha x_3 + u \\
y &= x_1,
\end{aligned}$$

where w_1 is an unknown nonzero constant (disturbance) and α is a real constant.

(a) Is the disturbance decoupling problem (DDP) solvable? ..... (1p)

(b) When $u = 0$, show that if $\alpha < 0$, then for all initial conditions, $y(t) \to w_1$ as $t \to \infty$. ..... (1p)

(c) For what values of α is the error feedback output regulation problem guaranteed to be solvable if we choose $y_r = 0$ as the reference output (you do not need to design the controller)? ..... (2p)

(d) For $\alpha = 2$, solve the the full information output regulation problem for $y_r = 0$. [3p]