KTH Matematik

SF2842: Geometric Control Theory

Homework 3

Due March 10, 16:50pm, 2016

You may use min(5,(your score)/4) as bonus credit on the exam

---

1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot, \cdot]$ is the Lie Bracket.

- What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [2p]
- Is the distribution $span\{g_1, g_2\}$ involutive? [1p]
- Show that the system is locally strongly accessible and controllable. [3p]

2. Determine and justify if each of the following statements is true or false.

- Consider the consensus control problem in R^2 with N agents: $\dot{x}_i = u_i,\ x_i \in R^2,\ u_i \in R^2,\ i = 1, \cdots, N$. If the initial positions of the agents are contained in a disc, then as the consensus control $u_i = \sum_{j \neq i}(x_j - x_i)$ is applied, no agent can ever move outside the disc. [2p]
- Consider a smooth nonlinear control system

$$\dot{x} = f(x) + g(x)u,$$

where $f(0) = 0$. If $x = 0$ is not exponentially stabilizable by a Lipschitz continuous feedback control, then the system is not exactly linearizable around the origin either. [2p]

3. Consider

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 + 2x_1^4 - x_1^3 x_2 \\ \dot{x}_2 &= 2x_1 - x_2 - \beta x_1^2,\end{aligned}$$

where α and β are constant.

- Discuss for what value of α the stability of the origin does not depend on β. [1p]
- For the remaining case analyze the stability in terms of β. [3p]

4. Consider in a neighborhood N of the origin

$$\begin{aligned}\dot{x}_1 &= x_3 - x_1^5 \\ \dot{x}_2 &= x_1 - (e^{x_2}\cos(x_3) - 1)^3 + \sin(x_3)u \\ \dot{x}_3 &= \cos(x_3)u \\ y &= x_1.\end{aligned}$$

- Convert the system locally into the normal form. [3p]
- Can the system be stabilized locally around the origin? [1p]
- Is the system exactly linearizable (without considering the output) around the origin? [2p]