KTH Matematik

SF2842: Geometric Control Theory

Homework 3

Due March 8, 16:50pm, 2017

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot, \cdot]$ is the Lie Bracket.

(a) What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [2p]

(b) Is the distribution $span\{g_1, g_2\}$ involutive? [1p]

(c) Show that the system is locally strongly accessible and controllable. [3p]

2. Determine and justify if each of the following statements is true or false.

(a) Consider the multi-agent system: $\dot{x}_i = u_i,\ x_i \in R,\ u_i \in R,\ i = 1, \cdots, N$. Define the output of the system as $y = x_1$. If $u_i = \sum_{j \in N_i}(x_j - x_i)$ is applied, the system is observable if the interaction graph is connected. [3p]

(b) Consider a smooth nonlinear control system

$$\dot{x} = f(x) + g(x)u,$$

where $f(0) = 0$. If $x = 0$ is exponentially stabilizable by a Lipschitz continuous feedback control, then the system must be exactly linearizable around the origin. [2p]

3. Consider

$$\begin{aligned}
\dot{x}_1 &= \alpha x_1 + x_2 + 2x_1^2 + x_1^3 x_2 \\
\dot{x}_2 &= -x_2 + \beta x_1^2,
\end{aligned}$$

where α and β are constant.

(a) Discuss for what value of α the stability of the origin does not depend on β. [1p]

(b) For the remaining case analyze the stability in terms of β. [2p]

4. Consider in a neighborhood N of the origin

$$\begin{aligned}
\dot{x}_1 &= x_2^3 + e^{x_3} u \\
\dot{x}_2 &= -x_2 + \alpha x_1^3 + x_2^2 \\
\dot{x}_3 &= -x_2^3 + e^{x_3} u \\
y &= x_3,
\end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form. .... (3p)

(b) Design a feedback control to stabilize the nonlinear system for the case when the zero dynamics is asymptotically stable. .... (2p)

(c) Show the system without the output is not exactly linearizable. .... (1p)