Our Example:

$$\begin{bmatrix} \text{minimize} & C_{11}y_{11} + C_{12}y_{12} + C_{21}y_{21} + C_{22}y_{22} \\ \text{s.t.} & y_{11}+y_{12} - y_{11}p_{11}(1) - y_{21}p_{21}(1) - y_{12}p_{11}(2) - y_{22}p_{21}(2) = 0 \\ & y_{21}+y_{22} - y_{11}p_{12}(1) - y_{21}p_{22}(1) - y_{12}p_{12}(2) - y_{22}p_{22}(2) = 0 \\ & y_{11}+y_{12}+y_{21}+y_{22} = 1 \\ & y_{11} \geq 0 \quad y_{12} \geq 0 \quad y_{21} \geq 0 \quad y_{22} \geq 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{minimize} & C^T y \\ \text{s.t} & Ay = b \\ & y \geq 0 \end{bmatrix} \quad \text{where} \quad C = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \end{bmatrix} = \begin{bmatrix} -350 \\ -400 \\ -50 \\ -250 \end{bmatrix} \quad y = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix}$$

2 linearly independent rows ⇓ 2 basic variables

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.1 & 0.4 & -0.7 & -0.2 \\ -0.1 & -0.4 & 0.7 & 0.2 \end{bmatrix} = A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1-p_{11}(1) & 1-p_{11}(2) & -p_{21}(1) & -p_{22}(2) \\ -p_{12}(1) & -p_{12}(2) & 1-p_{22}(1) & 1-p_{22}(2) \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \hat{y} = \begin{pmatrix} 2/3 \\ 0 \\ 0 \\ 1/3 \end{pmatrix} \quad \text{i.e.} \quad \hat{y} = \begin{bmatrix} \hat{y}_{11} & \hat{y}_{12} \\ \hat{y}_{21} & \hat{y}_{22} \end{bmatrix} = \begin{bmatrix} 2/3 & 0 \\ 0 & 1/3 \end{bmatrix}$$

Now $\hat{\pi}_1 = \sum_{k=1}^{2} \hat{y}_{1k} = \hat{y}_{11} + \hat{y}_{12} = \frac{2}{3} + 0 = \frac{2}{3}$ $= \pi_1(2)$

$\hat{\pi}_2 = \sum_{k=1}^{2} \hat{y}_{2k} = \hat{y}_{21} + \hat{y}_{22} = 0 + \frac{1}{3} = \frac{1}{3}$ $= \pi_2(2)$

and

$$\hat{D}_{11} = \frac{\hat{y}_{11}}{\hat{\pi}_1} = \frac{2/3}{2/3} = 1 \qquad \hat{D}_{12} = \frac{\hat{y}_{12}}{\hat{\pi}_1} = \frac{0}{2/3} = 0$$

$$\hat{D}_{21} = \frac{\hat{y}_{21}}{\hat{\pi}_2} = \frac{0}{1/3} = 0 \qquad \hat{D}_{22} = \frac{\hat{y}_{22}}{\hat{\pi}_2} = \frac{1/3}{1/3} = 1 \qquad D(R_2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

So the optimal policy is to use decision 1 in state 1 and decision 2 in state 2. As before !

Note: $C^T \hat{y} = -350 \cdot \frac{2}{3} + (-250) \cdot \frac{1}{3} = \underline{-316.7}$ As before