

Our example:

$$\left[\begin{array}{l} \text{Minimize } C_{11}y_{11} + C_{12}y_{12} + C_{21}y_{21} + C_{22}y_{22} \\ \text{s.t. } y_{11} + y_{12} - \alpha y_{11}p_{11}(1) - \alpha y_{21}p_{21}(1) - \alpha y_{12}p_{11}(2) - \alpha y_{22}p_{21}(2) = \beta_1 \\ y_{21} + y_{22} - \alpha y_{11}p_{12}(1) - \alpha y_{21}p_{22}(1) - \alpha y_{12}p_{12}(2) - \alpha y_{22}p_{22}(2) = \beta_2 \\ y_{11} \geq 0 \quad y_{12} \geq 0 \quad y_{21} \geq 0 \quad y_{22} \geq 0 \end{array} \right]$$

$$\left[\begin{array}{l} \text{Minimize } C^T y \\ \text{s.t. } Ay = b \\ y \geq 0 \end{array} \right]$$

where $C = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \end{bmatrix} = \begin{bmatrix} -350 \\ -400 \\ -50 \\ -250 \end{bmatrix}$

$$y = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix} \quad b = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.19 & 0.46 & -0.63 & -0.18 \\ -0.09 & -0.36 & 0.73 & 0.28 \end{bmatrix} = A = \begin{bmatrix} 1 - \alpha p_{11}(1) & 1 - \alpha p_{11}(2) & -\alpha p_{21}(1) & -\alpha p_{21}(2) \\ -\alpha p_{12}(1) & -\alpha p_{12}(2) & 1 - \alpha p_{22}(1) & 1 - \alpha p_{22}(2) \end{bmatrix}$$

$$\beta = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \Rightarrow \hat{y} = \begin{pmatrix} 6.22 \\ 0 \\ 0 \\ 3.78 \end{pmatrix} \quad \hat{y} = \begin{bmatrix} 6.22 & 0 \\ 0 & 3.78 \end{bmatrix} \Rightarrow \hat{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: $\sum y_{ik} = 10$
 $= \frac{1}{1-0.9}$

$$\hat{Z} = -3122 = \frac{1}{2} V_1(R_2) + \frac{1}{2} V_2(R_2) = \frac{1}{2}(-3257) + \frac{1}{2}(-2986)$$

↑ from the policy iteration.

In fact, here:

$$\beta = \begin{bmatrix} 1 & 0 \end{bmatrix} \Rightarrow \hat{y} = \begin{pmatrix} 7.57 \\ 0 \\ 0 \\ 2.43 \end{pmatrix} \quad \hat{y} = \begin{bmatrix} 7.57 & 0 \\ 0 & 2.43 \end{bmatrix} \Rightarrow \hat{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{Z} = -3257 = V_1(R_2)$$

$$\beta = \begin{bmatrix} 0 & 1 \end{bmatrix} \Rightarrow \hat{y} = \begin{pmatrix} 4.86 \\ 0 \\ 0 \\ 5.14 \end{pmatrix} \quad \hat{y} = \begin{bmatrix} 4.86 & 0 \\ 0 & 5.14 \end{bmatrix} \Rightarrow \hat{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{Z} = -2986 = V_2(R_2)$$