Prototype example: A manufacturing unit has two states describing its condition:

State 1: The unit is working well

State 2: The unit is working poorly.

When in state 1, it generates an income of 400 \$ /week.
— " — 2, — " — 250 \$/week.

For decision policy R let

$$d_i(R) = \begin{cases} 1 & \text{if maintenance is done when in state } i \\ 2 & \text{if maintenance is } \underline{\text{not}} \text{ done when in state } i \end{cases}$$

There are four different policies

$$d(R_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad d(R_2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad d(R_3) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad d(R_4) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

The corresponding transition matrices are

$$P(R_1) = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}7 & 0{,}3 \end{bmatrix} \quad P(R_2) = \begin{bmatrix} 0{,}9 & 0{,}1 \\ 0{,}2 & 0{,}8 \end{bmatrix} \quad P(R_3) = \begin{bmatrix} 0{,}6 & 0{,}4 \\ 0{,}7 & 0{,}3 \end{bmatrix} \quad P(R_4) = \begin{bmatrix} 0{,}6 & 0{,}4 \\ 0{,}2 & 0{,}8 \end{bmatrix}$$

and stationary distributions

$$\pi(R_1) = \left[\tfrac{7}{8} \quad \tfrac{1}{8} \right] \quad \pi(R_2) = \left[\tfrac{2}{3} \quad \tfrac{1}{3} \right] \quad \pi(R_3) = \left[\tfrac{7}{11} \quad \tfrac{4}{11} \right] \quad \pi(R_4) = \left[\tfrac{1}{3} \quad \tfrac{2}{3} \right]$$

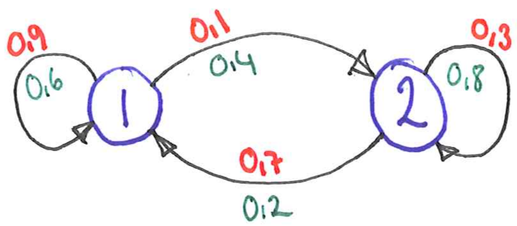
The cost of maintenance in state 1 is 50 \$ and in state 2 it is 200 \$

$$\Rightarrow \quad C_{11} = -350 \text{ \$} \qquad C_{12} = -400 \text{ \$} \qquad C_{21} = -50 \text{ \$} \qquad C_{22} = -250 \text{ \$}$$

Stationary expected cost (per week) is $F(R_k) = \sum_{i=1}^{2} C_{i\,d_i(R_k)} \pi_i(k)$

$$\Rightarrow \quad F(R_1) = -312{,}5 \text{ \$} \qquad \underline{F(R_2) = -316{,}7 \text{ \$}} \qquad F(R_3) = -272{,}7 \text{ \$} \qquad F(R_4) = -300 \text{ \$}$$

optimal.



if $k=1$, i.e. maintenance

if $k=2$, i.e. no maintenance