Marginal allocation algorithm for generating efficient solutions

Assumptions:

f is integer-convex and strictly decreasing in each variable,
g is integer-convex and strictly increasing in each variable.

Let $\mathbf{x}^{(0)} = \mathbf{0}$ (which is an efficient solution)
Then generate efficient solutions $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \ldots$ "from left to right", i.e., each new generated point has a higher value on $g(\mathbf{x})$ but a lower value on $f(\mathbf{x})$ than the previously generated point.
Let $\mathbf{x}^{(k)}$ denotes the k:th generated efficient solution.
Stop when there is no more efficient solution with $g(\mathbf{x}) \leq g^{\max}$.

Step 0:

Generate a table with n columns as follows. For $j = 1, \ldots, n$, fill the j:th column from the top and down with the quotients

$$-\Delta f_j(0)/\Delta g_j(0), \quad -\Delta f_j(1)/\Delta g_j(1), \quad -\Delta f_j(2)/\Delta g_j(2), \cdots$$

(A moderate number of quotients will suffice, additional quotients can be calculated as needed.)
Note that the quotients are positive and strictly decreasing in each column.
Set $k = 0$, $\mathbf{x}^{(0)} = (0, \ldots, 0)^{\mathsf{T}}$, $g(\mathbf{x}^{(0)}) = g(\mathbf{0})$ and $f(\mathbf{x}^{(0)}) = f(\mathbf{0})$.
Let all the quotients in the table be *uncanceled*.

Step 1:

Select the *largest uncanceled* quotient in the table (if there are several equally large, choose one of these arbitrarily). *Cancel* this quotient and let ℓ be the number of the column from which the quotient was canceled.

Step 2:

Let $k := k + 1$. Then let $x_\ell^{(k)} = x_\ell^{(k-1)} + 1$ and $x_j^{(k)} = x_j^{(k-1)}$ for all $j \neq \ell$.
Calculate $f(\mathbf{x}^{(k)}) = f(\mathbf{x}^{(k-1)}) + \Delta f_\ell(x_\ell^{(k-1)})$, $g(\mathbf{x}^{(k)}) = g(\mathbf{x}^{(k-1)}) + \Delta g_\ell(x_\ell^{(k-1)})$.
If $g(\mathbf{x}^{(k)}) \geq g^{\max}$, terminate the algorithm. Otherwise, go to Step 1.

$\mathbf{x}^{(k)}$ differs from the previous solution $\mathbf{x}^{(k-1)}$ in one component.
The name of the algorithm stems from the fact that

$$\frac{-\Delta f_j(x_j)}{\Delta g_j(x_j)} = \frac{\text{decrease in } f(\mathbf{x}) \text{ if } x_j \text{ is increased by 1}}{\text{increase in } g(\mathbf{x}) \text{ if } x_j \text{ is increased by 1}}.$$

We increase the x_j which gives marginally the largest decrease in $f(\mathbf{x})$ per increase in $g(\mathbf{x})$.

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Let $\mathbf{s}^{(0)} = \mathbf{0}$ (which is an efficient solution)

Note that $\Delta f_j(s_j) = \Delta EBO_j(s_j) = -R_j(s_j)$ and $\Delta g_j(s_j) = \Delta c_j s_j = c_j$, so

$$\frac{-\Delta f_j(s_j)}{\Delta g_j(s_j)} = \frac{R_j(s_j)}{c_j}$$

Step 0:

Generate a table with n columns as follows. For $j = 1, \ldots, n$, fill the j:th column from the top and down with the quotients

$j=1$	$j=2$	$\cdots$	$j=n$
$\frac{R_1(0)}{c_1}$	$\frac{R_2(0)}{c_2}$	$\cdots$	$\frac{R_n(0)}{c_n}$
$\frac{R_1(1)}{c_1}$	$\frac{R_2(1)}{c_2}$	$\cdots$	$\frac{R_n(1)}{c_n}$
$\frac{R_1(2)}{c_1}$	$\frac{R_2(2)}{c_2}$	$\cdots$	$\frac{R_n(2)}{c_n}$
$\frac{R_1(3)}{c_1}$	$\frac{R_2(3)}{c_2}$	$\cdots$	$\frac{R_n(3)}{c_n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Note that the quotients are positive and strictly decreasing in each column.
Let $C^{(0)} = 0$ and $EBO^{(0)} = \sum_{j=1}^{n} \lambda_j T_j$.
Let all the quotients in the table be *uncanceled*.

Step 1:

Select the *largest uncanceled* quotient in the table (if there are several equally large, choose one of these arbitrarily). *Cancel* this quotient and let ℓ be the number of the column from which the quotient was canceled.

Step 2:

Let $k := k+1$. Then let $s_\ell^{(k)} = s_\ell^{(k-1)} + 1$ and $s_j^{(k)} = s_j^{(k-1)}$ for all $j \neq \ell$.
Calculate $C^{(k)} = C^{(k-1)} + c_\ell$ and $EBO^{(k)} = EBO^{(k-1)} - R_\ell(s_\ell^{(k-1)})$.
If $C^{(k)} \geq C^{\max}$, terminate the algorithm. Otherwise, go to Step 1.