

Periodicity

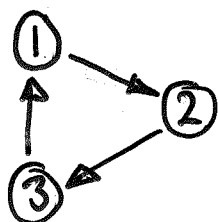
$$\text{Let } N(i) = \{ n \geq 1 \mid P_{ii}^n > 0 \}$$

The period of state i is defined by $d(i) = \begin{cases} 0 & \text{if } N(i) = \emptyset \\ \gcd(N(i)) & \end{cases}$

A state is aperiodic if $d(i) = 1$

If states i and j communicate, then $d(i) = d(j)$

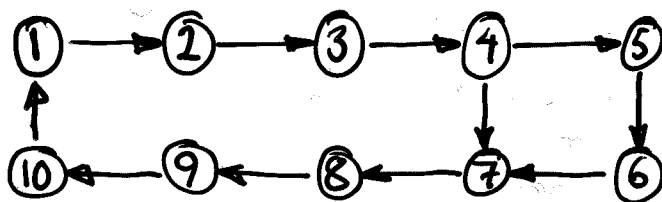
Ex:



$$N(1) = \{ 3, 6, 9, 12, \dots \}$$

$$d(1) = \gcd(N(1)) = 3$$

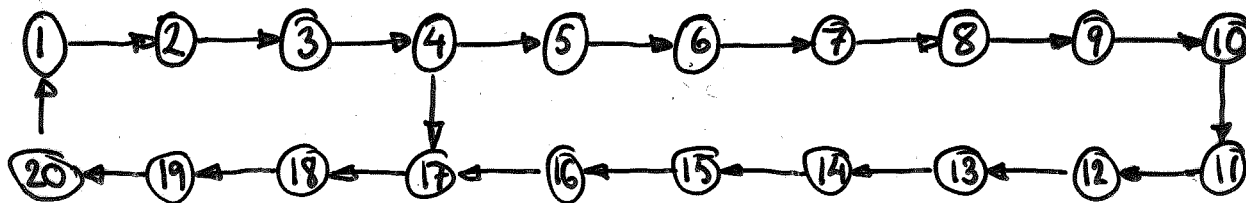
Ex:



$$N(1) = \{ 8, 10, 16, 18, 20, 24, 26, 28, 30, \dots \}$$

$$d(1) = \gcd(N(1)) = 2.$$

Ex



$$N(1) = \{ 8, 16, 20, 24, 28, 32, 36, 40, \dots \}$$

$$d(1) = \gcd(N(1)) = 4.$$

Note: Starting at ① the first next visit could be after 8 steps (> 4) but at steady state it is 4 time steps between possible visits.