Theorem: (Perron - Frobenius)

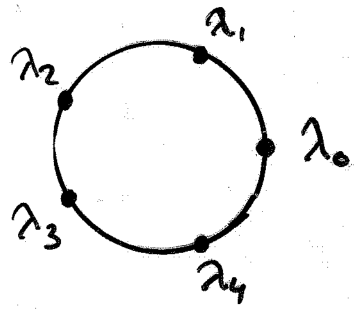
If P is the transition matrix of a finite irreducible chain with period d, then

a) $\lambda_0 = 1$ is an eigenvalue of P

b) the d complex roots of unity
$\lambda_0 = 1 \quad \lambda_1 = w \quad \dots \quad \lambda_{d-1} = w^{d-1}$ where $w = e^{2\pi i/d}$
are eigenvalues of P.

c) the remaining eigenvalues $\lambda_d, \dots, \lambda_m$ satisfy $|\lambda_k| < 1$.

Roots of unity $\lambda_k^d = 1$.



Eigenvalue decomposition of P: $\quad VP = \Lambda V \qquad \Lambda = \mathrm{diag}(\lambda_0 \dots \lambda_m)$

$\Rightarrow P = V^{-1}\Lambda V \qquad \Rightarrow P^2 = V^{-1}\Lambda V V^{-1}\Lambda V = V^{-1}\Lambda^2 V$

$$P^n = V^{-1}\begin{bmatrix} \lambda_0^n & & 0 \\ & \ddots & \\ 0 & & \lambda_m^n \end{bmatrix} V \longrightarrow V^{-1}\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & 0 & \\ 0 & & & \end{bmatrix} V = \bar{P}$$

if $\lambda_0 = 1$ and $|\lambda_k| < 1$ for $k = 1 \dots m$.

Then $\bar{P}$ has rank 1, and will be constant along its columns