

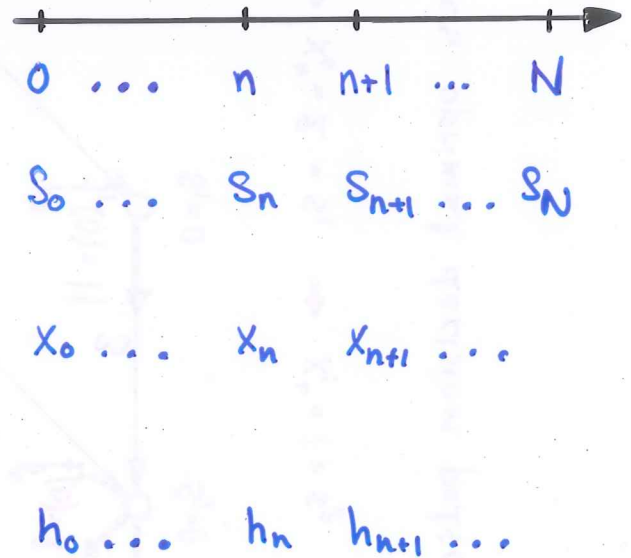
Deterministic Dynamic Programming

Consider a multistage problem

at each stage k the state s_k describes the current status

at each stage k a policy decision x_k is taken ($x_k \in \tilde{F}_k(s_k)$ feasibility)

which determines the next state
 $s_{k+1} = h_k(s_k, x_k)$



Consider the optimization problem:

$$(P) \left[\begin{array}{ll} \text{Minimize} & F(s_0, \dots, s_N, x_0, \dots, x_{N-1}) \\ \text{s.t.} & \left\{ \begin{array}{ll} s_{k+1} = h_k(s_k, x_k) & k=0, 1, \dots, N-1 \\ x_k \in \tilde{F}_k(s_k) & k=0, 1, \dots, N-1 \\ s_0 = z_0 \end{array} \right. \end{array} \right]$$

We assume that F can be decomposed as

$$F(s_0, \dots, s_N, x_0, \dots, x_{N-1}) = \sum_{k=0}^{N-1} G_k(s_k, x_k)$$

or

$$F(s_0, \dots, s_N, x_0, \dots, x_{N-1}) = \prod_{k=0}^{N-1} G_k(s_k, x_k) \quad \text{where } G_k \geq 0.$$

Let

(P_n)

$$\left[\begin{array}{ll} \text{Minimize} & F_n(s_n, \dots, s_N, x_n, \dots, x_{N-1}) \\ \text{s.t.} & \begin{cases} s_{k+1} = h_k(s_k, x_k) & k=n, \dots, N-1 \\ x_k \in F_k(s_k) & k=n, \dots, N-1 \\ s_n = s \end{cases} \end{array} \right]$$

Let $f_n^*(s)$ denote the optimal value of (P_n) with $s_n = s$.

$$\text{or } F_n = \sum_{k=n}^{N-1} G_k$$

$$F_n = \prod_{k=n}^{N-1} G_k$$

Let

(P_n^z)

$$\left[\begin{array}{ll} \text{Minimize} & F_n(s_n, \dots, s_N, x_n, \dots, x_{N-1}) \\ \text{s.t.} & \begin{cases} s_{k+1} = h_k(s_k, x_k) & k=n, \dots, N-1 \\ x_k \in F_k(s_k) & k=n, \dots, N-1 \\ s_n = s, \quad \underline{x_n = z} \end{cases} \end{array} \right]$$

Let $f_n^*(s, z)$ denote the optimal value of (P_n^z) with $s_n = s$ and $x_n = z$

The optimal values are related by $f_n^*(s) = \min_{z \in F_n(s)} \{ f_n^*(s, z) \} = f_n^*(s, x_n^*)$

a recursive equation is given by

$$f_n^*(s, z) = G_n(s, z) \overset{+ \text{ or } \times}{*} f_{n+1}^*(\overset{= s_{n+1}}{h_n(s, z)})$$

Thus

$$f_n^*(s) = \min_{z \in F_n(s)} \{ G_n(s, z) * f_{n+1}^*(h_n(s, z)) \}$$

often $f_N^*(s)$ is easy to determine, recursive relationship gives

$f_{N-1}^*(s), \dots, f_0^*(s)$, then $f_0^*(s)$ gives optimal value of (P)