Equivalence classes

A binary relation $\sim$ on a set X is an equivalence relation if it is

Reflexive: $a \sim a$

Symmetric if $a \sim b$, then $b \sim a$

Transitive if $a \sim b$ and $b \sim c$, then $a \sim c$

for all a, b, c in X.

The equivalence class of a under $\sim$ is defined as $[a] = \{ b \in X \mid a \sim b \}$

Ex: Let X be the set of states of a Markov chain

Let the binary relation $\sim$ between two states $a, b \in X$ be defined by if a and b communicates with each other. This defines an equivalence relation.