



KTH Mathematics

Exam in SF2863 Systems Engineering
Thursday June 9, 2011, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!

25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.

The order of the exercises does not correspond to their difficulty level.

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1. (a) Frasse is in financial trouble. He has M dollars and n days from now he has to pay a loan-shark K dollars. His plan to solve the situation is to play an online game on internet. He can play once a day, and if he plays the cost of playing is 1 dollar. He can also chose not to play, and then he does not have to pay the playing fee. When he plays he can bet 2 or 3 dollars, provided that he has that amount (after paying the playing fee). If he plays, with probability 0.6 he gets back what he betted plus the same amount and with probability 0.4 he loses what he has betted. Frasse wants to find the playing strategy that maximizes his chance of beeing able to pay back the loan after n days.
Define the optimal value function and derive the DynP equation, including boundary conditions, for the given problem. (4p)
 - (b) Solve the problem, *i.e.*, compute the optimal value function and determine the optimal control strategy/strategies explicitly, when $M = 1, 2, 3, 4, 5, 6$, $n = 2$ and $K = 6$ (4p)
 2. (a) Frasse has started a company importing caviar. He has a russian supplier that only deals with him in person, so each time he buys the caviar he has to go there with his car to pick up the delivery. The travel costs are 20 kSEK per trip. The price he pays for the caviar is 4 kSEK per kg and the price he charges is 6 kSEK per kg. Frasse has a group of customers that together consumes a quantity of 1 kg per week, and they will continue to do so as long as there is no shortage in the delivery. To preserve the quality of the caviar it has to be

stored in an odour free and cool environment, and to keep it guarded and safe he has to pay a weekly cost of 0.1 kSEK per week and kg.

What is the optimal order quantity that Frasse should order to minimize the cost per week of running his business? How often should he then travel to russia? (Frasse's brother-in-law Heathcliff is of course running the business when he is away) (4p)

- (b) After one year of running the business, the russian supplier offers Frasse to buy the caviar for half the price if he orders at least 50 kg. That is, if he buys less than 50 kg he pays 4 kSEK per kg and if he buys 50 kg or more he pays 2 kSEK for all the caviar he buys at that occasion.

Will this change the optimal order quantity that Frasse will adapt? and what would it then be? (4p)

3. Frasse has started a factory that is manufacturing aluminium bike frames. His brother-in-law Heathcliff is producing aluminium pipes in his own factory and as soon as the pipes for one bike is ready he delivers the parts to the factory of Frasse. We can assume that the time between the deliveries of parts is exponentially distributed with mean value 20 minutes.

When the bike parts arrive they are placed in one queue and Frasse has employed two welders that puts the parts together to frames. The welders build the frames at the same average speed, the time for welding one frame is assumed exponentially distributed with mean value 30 minutes, and they work alone on each frame.

Once a frame is welded, Frasse comes to inspect it; with probability 0.1 he sends it back to the start of the welding queue again, where it is rewelded - which is assumed to be as much work as the initial welding, and with probability 0.9 he accepts it and sends it to the painting station.

Frasse has employed one painter that paints the frames which are picked in the order that they arrive. The time for painting one frame is assumed to be exponentially distributed with mean value 15 minutes.

Once a frame is painted, Frasse comes to inspect it; with probability 0.2 he sends it back to the start of the painting queue again, where it is repainted - which is assumed to be as much work as the initial painting, and with probability 0.8 he accepts it and sends it to the sales department.

- (a) Determine the probability that there are no parts or frames in queue for welding or painting.

Determine the average queue lengths to the welding station and to the painting station (7p)

- (b) Determine the average time it takes from that a set of parts arrive to the factory to that the corresponding painted frame arrives at the sales department. (3p)

4. Let $f_k(x_k) = 10k - 3x_k$ and $g_k(x_k) = k \sum_{\ell=1}^{x_k} \ell^2$ for $k = 1, 2, 3$ and $x_k = 1, 2, 3, \dots$. Define $f(x) = f_1(x_1) + f_2(x_2) + f_3(x_3)$ and $g(x) = g_1(x_1) + g_2(x_2) + g_3(x_3)$, where $x = (x_1, x_2, x_3)$.

- (a) Does f and g satisfy the conditions necessary to use Marginal allocation.
..... (3p)
- (b) Determine all efficient points x for the simultaneous minimization of f and g such that $x_1 + x_2 + x_3 \leq 8$ (6p)
- (c) What is the optimal solution to

$$\left[\begin{array}{ll} \text{minimize} & f(x) \\ \text{s.t.} & g(x) \leq 24 \\ & x_k \in \{1, 2, 3, \dots\}, \quad k = 1, 2, 3. \end{array} \right]$$

..... (3p)

5. Each day Frasse drives his car the same route, and consumes half a tank of gas. The price of gas fluctuates between $L = 6$ and $H = 8$ according to a Markov process where the transition probabilities are 0.8 for price L followed by price L , and 0.6 for price H followed by price H .

At the end of each day he decides to fill up the tank to half-full or to full, for the current price.

- (a) Determine states and control that defines the Markov decision process described above. (4p)
- (b) Guess the optimal policy for Frasse describing how he should fill up his tank. Determine the optimal expected cost for the process running with the decisions made by that policy. (4p)
- (c) Verify using the policy iteration algorithm that the guessed policy is optimal, or show that it is not optimal. (4p)

Good luck!