Some useful results from probability theory

The **conditional probability** of A given B is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Law of total probability

If B_n form a partition of the sample space, i.e., the B_n are disjoint and the union is the whole sample space, then

$$Pr(A) = \sum_n Pr(A|B_n)Pr(B_n)$$

In particular, if X and Y are discrete valued stochastic variables, then

$$Pr(X = x) = \sum_y Pr(X = x|Y = y)Pr(Y = y)$$

Law of total expectation (when Y takes discrete values y)

$$E(X) = \sum_y E(X|Y = y)P(Y = y)$$