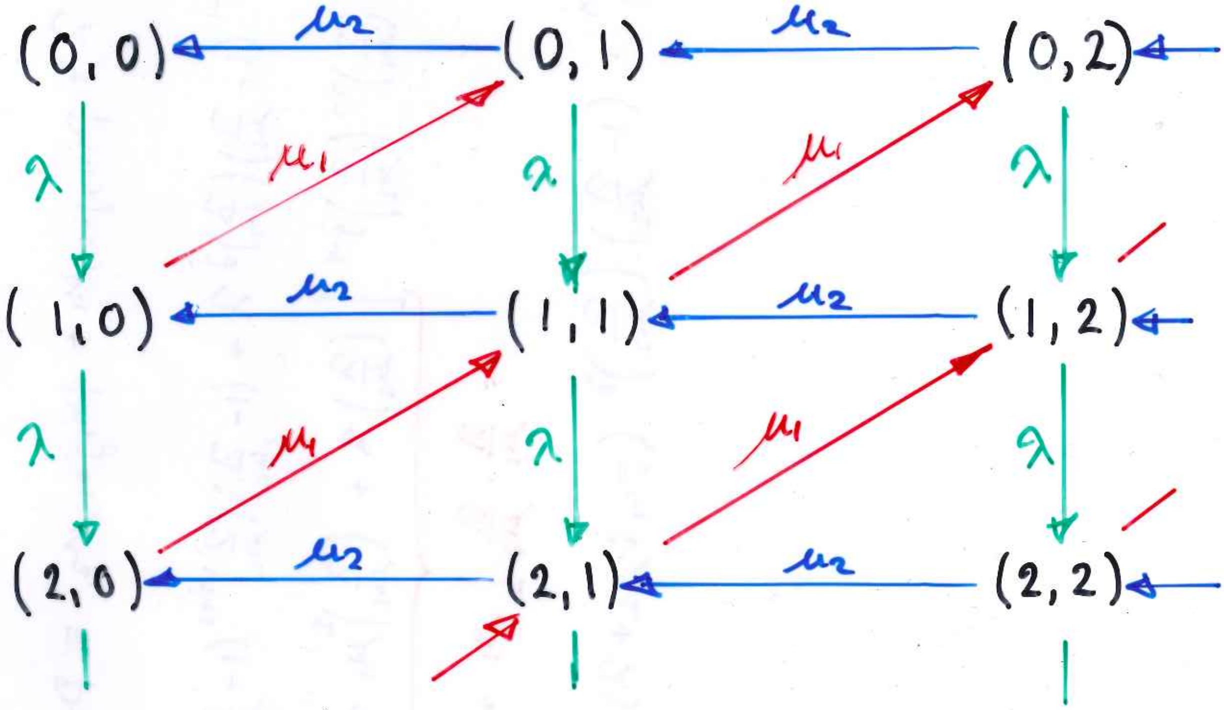
Queuing systems in series

Ex

Let the state $X_t = (N_1, N_2)$ $\qquad$ N_i = # clients in system i (at time t)



Conservation of flow :

$$p_{ij}(\lambda + \mu_1 + \mu_2) = p_{i-1,j}\,\lambda + p_{i+1,j-1}\,\mu_1 + p_{i,j+1}\,\mu_2$$

(for nodes "inside")

It is possible to show that

$$p_{ij} = P(N_1 = i, N_2 = j) = P(N_1 = i)\,P(N_2 = j)$$

$$= \left[\left(1 - \frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_1}\right)^i\right] \cdot \left[\left(1 - \frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\mu_2}\right)^j\right]$$

if we have two M|M|1 - queueing systems.