How to compute $c'(y)$?

$$\boxed{\frac{d}{dy}\int_{u(y)}^{v(y)} g(y,z)\,dz = g(y,v(y))\,\frac{dv(y)}{dy} - g(y,u(y))\,\frac{du(y)}{dy} + \int_{u(y)}^{v(y)} \frac{\partial}{\partial y}\, g(y,z)\,dz} \quad *$$

cancel

$$\frac{d}{dy}\, c(y) = C + h\,\frac{yT}{2} f_z(y)\cdot 1 - 0 + h\int_0^y T f(z)\,dz$$

$$+\, 0 - h\cdot \frac{y^2T}{2y} f_z(y) + h\int_y^\infty \frac{yT}{z} f(z)\,dz$$

$$+\, 0 - \frac{p}{2}\underbrace{\left(yT - 2yT + \frac{y^2T}{y}\right)}_{=0} f_z(y) + \frac{p}{2}\int_y^\infty \left(-2T + \frac{2yT}{z}\right) f(z)\,dz$$

concels

so

$$c'(y) = C + h\int_0^y T f(z)\,dz + h\int_y^\infty \frac{yT}{z} f(z)\,dz + p\int_y^\infty \left(-T + \frac{yT}{z}\right) f(z)\,dz$$

can skip

check convexity, use eq * for all integrals

$$c''(y) = h\,\cancel{T f(y)}\cdot 1 - 0 + 0$$

$$+\, 0 - h\cdot \cancel{T\cdot f(y)}\cdot 1 + h\int_y^\infty \frac{T f(z)}{z}\,dz$$

$$+\, 0 - p\,\cancel{(T - T)}\, f(y)\cdot 1 + p\int_y^\infty \frac{T}{z} f(z)\,dz$$

$$= T(h+p)\underbrace{\int_y^\infty \frac{1}{z}\left(f(z)\right)dz}_{>0}$$

>0, since $\int_y^\infty f(z)\,d(z) = P(z>y) > 0$

C is convex $\Rightarrow$ the minimum y_0 is given by $C'(y_0) = 0$

$$C'(y_0) = c + hT\int_0^{y_0} f(z)\,dz + (h+p)Ty_0\int_{y_0}^{\infty}\frac{1}{z}f(z)\,dz$$

$$- pT\underbrace{\int_{y_0}^{\infty} f(z)\,dz}_{}$$

$$= 1 - \int_0^{y_0} f(z)\,dz \quad \leftarrow \text{since} \quad \int_0^{\infty} f(z)\,dz = 1$$

$$C'(y_0) = c + T(h+p)\int_0^{y_0} f(z)\,dz + Ty_0(h+p)\int_{y_0}^{\infty}\frac{1}{z}f(z)\,dz - pT = 0$$

$$\Rightarrow \int_0^{y_0} f(z)\,dz + y_0\int_{y_0}^{\infty}\frac{1}{z}f(z)\,dz = \frac{p - c/T}{h+p} \qquad (*)$$

↑ this is an eq to find y_0
however need to consider

The optimal y is then given by

$$\hat{y} = \begin{cases} y_0 & , \text{ if } y_0 > x \\ x & , \text{ if } y_0 \le x \end{cases}$$

← we could find a y_0 which is less than the original inventory level, x so