KTH Mathematics

Exam in SF2863 Systems Engineering
Monday December 19, 2011, 08.00–13.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Frasse and Heathcliff has started a tech support center for mobile telephone apps. called “Eye-phone experts”. Customers are calling the support center reception with intensity 16 callers per hour. At the reception the bureaucratic twins Jo and John are working and it can be modelled as a $M|M|2$ queue with service intensity 16 callers per hour for each server. When the customer has been served at the reception they are with prob. 1/3 transfered to Frasse, with prob. 1/2 transfered to Heathcliff, and with prob. 1/6 they are disconnected.

The service of Frasse is modelled as a $M|M|1$ queue with service intensity 10 callers per hour and when served by Frasse the customers gets the solution to their problem with prob 3/4 and leaves the system, or, with prob. 1/4 they are transfered back to the reception.

The service of Heathcliff is modelled as a $M|M|1$ queue with service intensity 16 callers per hour and when served by Heathcliff the customers gets the solution to their problem with prob 1/2 and leaves the system, or, with prob. 1/2 they are transfered back to the reception.

(a) Will the queues in the network converge to some stationary state?
What is the probability that a random customer that calls the center actually gets the problem solved? ....(5p)

(b) Determine the average time it takes from making a call to that you are either disconnected or have received the solution to your problem. ....(5p)

2. Frasse has been going to the casino to play blackjack frequently, but now he has decided that he is only going to play two more days. He currently has 1000\$ in his pocket and each day he will play with 500\$. Frasse has decided that each day he plays he will play until he either doubles the 500\$ he started with or until he loses the 500\$. Based on his latest statistics, he estimates that he wins 60 % of the days. He has discovered that if he counts cards he can increase the probability of winning to 70 %, but then there is a 10 % risk that he will loose and a 20 % risk that he will get caught in which case he will be thrown out of the casino, loosing the 500\$ he was playing with and any money in his pocket, and he will not be welcome back.

(a) Frasse wants to find the policy that maximizes his expected winnings after the two days.

Define the optimal value function and derive the DynP equation, including boundary conditions, for the given problem. .... (4p)

(b) Solve the problem, *i.e.*, compute the optimal value function and determine the optimal policy explicitly. (i.e. what will his decisions be at any possible scenario) .... (4p)

3. For this christmas Frasse has rationalized his christmas present shopping and bought 8 of his favourite chocolate boxes. He wants to distribute the boxes to his 3 closest friends; Heathcliff, Ludwig W and Cheapskate-Charlie. Frasse wants to maximize the appreciation that his friends will show to him, and he has estimated that the appreciation of each friend can be quantified by the numbers in the table below where n signifies the number of boxes the person receives and the total appreciation is given by $p(n_H, n_L, n_C) = p_H(n_H) + p_L(n_L) + p_C(n_C)$.

n	$p_H(n)$	$p_L(n)$	$p_C(n)$
0	0	0	0
1	5	3	7
2	8	5	12
3	10	6	15
4	11	7	16

It can be assumed that no person receives more than 4 boxes.

(a) Does p satisfy the conditions necessary to use Marginal allocation? ...... (3p)

(b) Determine all efficient distributions of the boxes such that $5 \leq n_H + n_L + n_C \leq 8$. I.e. for 5,6,7 and 8 boxes you should determine the allocations that maximizes the appreciation. If there is no unique maximizer it is enough to determine one of the maximizers. You should also determine the total appreciation for each of those maximizing allocations. .... (7p)

Solving the problem by total enumeration will not give any points.

4. Frasse's brother-in-law Heathcliff has been working a bit too much lately, and his mood has been going up and down a lot, and since this has started to affect their business Frasse has decided he has to do something. Frasse puts up a Markov chain model for the mood of Heathcliff, where if Heathcliff is up one day he will be up also the next day with probability 0.8, and if he is down he will be down also the next day with probability 0.6. When Heathcliff is up his work generates an income of 60$ per day, and when he is down it is 30$. Frasse has observed that he can change the transition probabilities in two ways.

If he awards Heathcliff with an employee of the day medal, which costs 5$, then the up-to-up probability increases to 0.9 and the down-to-down probability decreases to 0.4.

If he takes Heathcliff out on after-work, spending 30$, then Heathcliff will be up with probability 1 the next day.

(a) Formulate this as a Markov decision problem for maximizing the expected revenue (=income -cost) per day.

Use the policy iteration algorithm to *improve on* the starting policy to do nothing, if possible. How much will the new policy increase the expected revenue? (8p)

(b) Determine the matrices and vectors that define the linear program that solves the MDP described above. "Solve" the LP by randomly choosing the entries in the vector y with values that correspond to a deterministic policy and determine that policy. (the vector y does not have to satisfy the equality constraints, but should have zeros in reasonable positions, so that you can demonstrate that you know how to determine the optimal policy from the LP solution) ... (4p)

5. The new company "Eye-phone experts" is facing an uncertain future. Frasse believes that the value of the stock on the market at the end of the year can be modelled well by a uniformly distributed stochastic variable on the interval $(0, 1000)$. He has now decided to start selling a special kind of options. The price for the option will be $5\$ \cdot S$, where $S \in \mathbb{R}^+$ is a parameter that determines the "target-price" of the option. The holder will, at the end of the year, receive 8$ for each dollar that the price of the stock is below S and 2$ for each dollar that the price of the stock is above S.

Help Frasse to determine the target-price S so that the expected profit for each option is maximized.

What is the expected profit per option for the optimal target-price? ....(8p)

Good luck!